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Authors:


Jaime Alonso-Carrera

*Universidade de Vigo*

Stéphane Bouché

*Católica Lisbon School of Business and Economics*



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# Dynamic concern for status, altruism and the operativeness of bequest motive*

Jaime Alonso-Carrera [a] and Stéphane Bouché [b]

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[a] *Corresponding author.*
Universidade de Vigo, Departamento de Fundamentos del Análisis Económico.
Address: Facultad de Ciencias Económicas, Campus Lagoas-Marcosende, 36310 Vigo, Spain.
E-mail address: jalonso@uvigo.es.

[b] Católica Lisbon School of Business and Economics.
Address: Palma de Cima, 1649-023 Lisboa, Portugal.
E-mail address: sbouche@ucp.pt.

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## Abstract

We analyze the implications of dynastic altruism in an overlapping generations model with consumption externalities and where parents are able to influence their children's preferences for social status through their saving behavior. In our model, the level of the altruism factor for which individuals leave bequests to their descendants can be both larger or smaller than in an economy without endogenous status effects. The optimal allocation is characterized by a steady-state capital stock larger than the one corresponding to the modified golden rule. Even when the bequest motive is operative, the intertemporal allocation is suboptimal and the optimal policy consists of time-varying estate and capital income taxes. When the bequest motive is inoperative, a time-varying tax on capital income is combined with a pay-as-you-go social security system.



*JEL classification codes:* D11, E21, E62.

*Keywords:* Bequests, endogenous status effects, consumption externalities.

# 1 Introduction

In this paper we aim at analyzing the implications of dynastic altruism in an overlapping generations model (OLG) with consumption externalities and where parents are able to influence their children's preferences for social status through their saving behavior. In our model, agents compare their consumption levels to an external benchmark and the intensity of the comparison is endogenous and depends on the savings of the previous period. In this way, individuals at the time of birth inherit preferences for social position from their parents, and throughout their lifetime these preferences change as a result of their savings decisions and wealth positions. We assume in addition that the external benchmark is composed of a weighted average of the consumption of all the agents living at the same period.

The original result stated by Barro (1974), according to which the Ricardian Equivalence Principle (i.e., the neutrality of public debt) is fulfilled in an OLG model with dynastic altruism, has been widely revisited in the subsequent literature. In particular, Weil (1987) showed that the degree of altruism is not large enough to generate positive bequests in a dynamically-inefficient economy. Some studies have however shown that this conclusion may be affected by economic phenomena like externalities in human capital accumulation (Caballé, 1995) or the incidence of fiscal policy (Caballé, 1998). Moreover, the result might also be affected by the presence of consumption references in utility. This line of research has been followed by de la Croix and Michel (2001) who introduce consumption aspirations into the model, Alonso-Carrera et al. (2007) who focus on both aspirations and habit formation or Alonso-Carrera et al. (2008) who introduce consumption externalities in a way similar to Abel (2005). Our approach is similar to the latter except that we take into account an additional mechanism through which parents can resort in order to affect the future behavior of their children. Beyond leaving bequests to their descendants, parents are able to choose indirectly the intensity with which children compare their consumption to the average one in society.

Our approach is motivated by two main empirical observations. First, the hypothesis that the intensity of the comparison effect is declining in wealth accumulation is supported by a number of empirical studies. Using a European survey, Clark and Senik (2010) show that comparisons are mostly in the upward direction implying that there is much more scope for comparison among the poor. Moreover, the latter tends to care more about status with respect to relative consumption. Heffetz (2011) estimates income elasticities for goods associated to status and finds a negative relationship between the degree of concern for status and income. These empirical observations have been recently formalized by Dioikitopoulos et al. (2019) who introduce endogenous status effects in an infinite horizon model and study the implications of the latter concerning the dynamics of the saving rate and income inequality. In this paper, we wish to focus on the intergenerational transmission of endogenous status effects in an OLG model and its interaction with dynastic altruism.

The second empirical evidence motivating our approach is related to the capacity of parents to affect their children's preferences. In particular, the fact that parents can shape the preferences of their children through their saving decisions is documented empirically. Knowles and Postlewaite (2005) document that parental saving behavior is an important determinant of children's education and saving choices after controlling for a variety of individual characteristics. Charles and Hurst (2003) find a similar result when studying the correlation of wealth between parents and children. Moreover, the correlation is stronger between mothers and children, who are in general more involved in children's education. The influence of parents on children's preferences has been formalized in several contributions. Doepke and Zilibotti (2008) introduce such a feature in order to explain the British Industrial Revolution. In their model, middle-class parents are able to invest in their children's preferences concerning patience and taste for hard work. A similar approach is used by Doepke and Zilibotti (2017) in order to explain differences in parenting styles and the decrease of authoritarian parenting in industrialized and less unequal countries.

In our model, the presence of endogenous status effects, such as defined above, implies that individuals have an additional incentive to save in order to reduce the intensity of future aspirations. This in turn affects the level of the altruism factor for which individuals decide to leave bequests. When savings are large, parents have less incentives to leave bequests since they reduce the impact of conspicuous consumption by reducing the need for status of their children. On the contrary, when savings are low, parents have an additional incentive to leave bequests in order to compensate for not decreasing the need for status through capital accumulation.

Given the additional benefit from saving in our model, the socially optimal allocation is characterized by a steady-state capital stock larger than the one corresponding to the modified golden rule. Even when the bequest motive is operative, the intertemporal allocation is suboptimal since endogenous status effects imply that the intratemporal allocation between generations has intertemporal consequences that are not internalized by the individuals.

The optimal fiscal policy, designed to restore the equilibrium efficiency, also depends on the operativeness of the bequest motive. When the bequest motive is operative, the optimal policy consists of taxing bequest and capital income at time-varying rates. When the bequest motive is inoperative, the optimal policy consists of a capital income tax with time-varying rate and a pay-as-you-go social security system. The tax on capital income is the same as in the case where the bequest motive is operative, whereas the lump-sum transfers between generations living at the same period correct the suboptimality due to the lack of bequests.

The rest of the paper is organized as follows. The model is presented in Section 2, whereas Section 3 characterizes the competitive equilibrium. Section 4 derives the conditions for the bequest motive to be operative. The socially optimal solution is provided in Section 5. Section 6 analyzes the optimality of the competitive equilibrium, and Section 7 derives the optimal taxation that decentralizes the socially optimal allocation. Section 8 concludes the paper. Finally, the appendix includes all the proofs and the more technical analysis.

# 2 The model

We extend the model with consumption externalities proposed by Alonso-Carrera et al. (2008) to consider that the intensity of the externalities are endogenously determined by the stock of capital as was proposed by Dioikitopoulos et al. (2019). In particular, we consider an OLG model where individuals live for two periods. Each generation is composed of a continuum of individuals distributed on the interval $[0, 1]$. Each individual has a child at the end of the first period. As in Diamond (1965), individuals work in the first period of life and are retired in the second. We index each generation by the period in which its members work.

We assume that individuals are altruistic towards their children to whom they may leave bequests. Let us denote by $b_t$ the amount of bequest that an old individual leaves to its child in period $t$. As usual, we impose the constraint that bequests cannot be negative, i.e.,

$$b_t \geq 0. \tag{1}$$

Each young individual supplies inelastically one unit of labor and receives in exchange the wage $w_t$. He then distributes his labor income and his inheritance between consumption and saving. His budget constraint in young age is thus given by:

$$w_t + b_t = c_t + s_t, \tag{2}$$

where $c_t$ is young age consumption and $s_t$ the amount of savings. In old age, the agents receives the interest of their savings and distribute the latter between consumption and bequest for their children. The budget constraint in old age is given by:

$$R_{t+1} s_t = d_{t+1} + b_{t+1}, \tag{3}$$

where $R_{t+1}$ is the gross rate of return on savings and $d_{t+1}$ is old age consumption.

The utility function of an individual born at period $t$ is given by

$$V_t = U(\hat{c}_t, \hat{d}_{t+1}) + \beta V_{t+1}, \tag{4}$$

where $V_{t+1}$ represents the indirect utility of his descendant, $\beta \in [0,1)$ is the altruism factor, and $\hat{c}_t$ and $\hat{d}_{t+1}$ represent the effective consumption levels in the first and second period of life respectively. We assume that individuals derive utility from comparing their consumption level to some given consumption reference. We assume the following functional forms for effective consumptions:

$$\hat{c}_t = c_t - \gamma(s_{t-1})v_t^y, \tag{5}$$

and

$$\hat{d}_{t+1} = d_{t+1} - \gamma(s_t)v_{t+1}^o, \tag{6}$$

where $\gamma(s) \in [0,1)$ provides a measure of the intensity of the consumption reference. Contrary to Alonso-Carrera et al. (2008), the intensity of the reference is endogenous and depends on savings. We are assuming intergenerational transmission of preferences for status.[1] Hence, an individual born at period $t$ inherits the intensity $\gamma$ from his parent, which then depends on savings $s_{t-1}$ choose by the latter. He uses this intensity in assessing the utility from consumption when young. However, the intensity of the concern for status when old will depend on his own savings $s_t$. Following Dioikitopoulos et al. (2019), we assume that (a) $\gamma'(s) < 0$ and $\gamma''(s) > 0$ for all $s$, (b) $\lim_{s\to 0} \gamma'(s) = \gamma_0$, and (c) $\lim_{s\to\infty} \gamma'(s) = \gamma_\infty$, with $\gamma_0 > \gamma_\infty$.

The consumption references are assumed to be weighted arithmetic averages of the consumption of the two living generations. The reference in young age is given by:

$$v_t^y = \frac{c_t + \theta^y d_t}{1 + \theta^y}, \tag{7}$$

where $\theta^y \in [0,1)$ is the weight of the consumption of the representative old agent in the consumption reference of the young agent. The reference in old age is given by:

$$v_{t+1}^o = \frac{\theta^o c_{t+1} + d_{t+1}}{1 + \theta^o}, \tag{8}$$

where $\theta^o \in [0,1)$ is the weight of the consumption of the representative young agent in the consumption reference of the old agent. It should be noted that our assumption on $\theta^y$ and $\theta^o$ implies that we are giving a larger weight to the consumption of agents belonging

[1] Doepke and Zilibotti (2008, 2017) rationalize and provide evidence of intergenerational transmission of preferences.

to the same generation. Expressions (7) and (8) can be rewritten as follows:

$$v_t^y = \epsilon^y c_t + (1-\epsilon^y)d_t, \tag{9}$$

and

$$v_{t+1}^o = \epsilon^o c_{t+1} + (1-\epsilon^o)d_{t+1}, \tag{10}$$

where $\epsilon^y = \frac{1}{1+\theta^y}$ and $\epsilon^o = \frac{\theta^o}{1+\theta^o}$.

Following Abel (1986) and Laitner (1988), we assume that the utility function $U(.,.)$ is additively separable in both arguments. Hence, we consider

$$U(\hat{c}_t, \hat{d}_{t+1}) = u(\hat{c}_t) + \rho u(\hat{d}_{t+1}), \tag{11}$$

where $\rho > 0$ is the temporal discount factor. Furthermore, we also assume that $u(.)$ is twice continuously differentiable with $u'(z) > 0$, $u''(z) < 0$ for $z > 0$, and the Inada conditions $\lim_{z\to 0} u'(z) = \infty$ and $\lim_{z\to\infty} u'(z) = 0$ hold.

In period 0, there is an initial old generation born in period $-1$. The individuals of generation $-1$ have the following utility function in old age:

$$\rho u(\hat{d}_0) + \beta V_0. \tag{12}$$

There is a single commodity $Y_t$ in this economy which can be used either for consumption or investment. This commodity is produced by means of a neoclassical production function $F(K_t, L_t)$ where $K_t$ is the capital stock and $L_t$ the amount of labor used in period $t$. We suppose that capital is fully depreciated after one period.[2] The production function in intensive form is given by $f(k_t)$ and we assume $f'(k) > 0$, $f''(k) < 0$ for all $k$ as well as the Inada conditions $\lim_{k\to 0} f'(k) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$.

# 3 Competitive equilibrium

Given the initial stock of capital $K_0$, a competitive equilibrium in this economy consists of a path of prices $\{R_t, w_t\}$ and a path of quantities $\{K_t, c_t, d_t, s_t, b_t\}$ that are consistent with consumer and firm optimization and with the market clearing conditions: (i) $L_t = 1$, (ii) the capital stock installed in period $t+1$ is equal to the saving of period $t$, i.e.,

$$k_{t+1} = s_t, \tag{13}$$

[2]We can consider that one period of the model consists on about 30 years for the empirical evaluation. Hence, full depreciation of the capital stock seems a realistic assumption.

and (iii) the aggregate resource constraint given by

$$f(k_t) = c_t + d_t + k_{t+1}. \tag{14}$$

The problem faced by each individual belonging to generation $t$ is to maximize (4) with respect to $\{c_t, d_{t+1}, b_{t+1}\}$ subject to (1), (2), (3), (5) and (6). This is equivalent to solving the following dynamic programming problem:

$$V_t(b_t, s_{t-1}) = \max_{s_t, b_{t+1}} \left\{ \begin{array}{l} u(w_t + b_t - s_t - \gamma(s_{t-1})v_t^y) \\ \\ +\rho u(R_{t+1}s_t - b_{t+1} - \gamma(s_t)v_{t+1}^o) + \beta V_{t+1}(b_{t+1}, s_t) \end{array} \right\}. \tag{15}$$

with $b_{t+1} \geq 0$, for $v_t^y$, $v_t^o$, $w_t$ and $R_{t+1}$ given for all $t$. It should be noted that the introduction of endogenous status effects implies that the value function $V_t(b_t, s_{t-1})$ depends not only on $b_t$ but also on $s_{t-1}$, which is not the case when the intensity of aspirations is exogenous. Additionally, to characterize the equilibria, we must also consider the problem faced by the individual belonging to generation $-1$ in period 0, which is equivalent to

$$\max_{b_0} \{\rho u(R_0 k_0 - b_0 - \gamma(k_0)v_0^o) + \beta V_0(b_0, k_0)\}, \tag{16}$$

with $b_0 \geq 0$, for $v_0^o$ and $R_0$ given.

By using the envelope theorem, we obtain that the first-order conditions for problem (15) with respect to $s_t$ is given by

$$\frac{u'(c_t - \gamma(s_{t-1})v_t^y) + \beta\gamma'(s_t)v_{t+1}^y u'(c_{t+1} - \gamma(s_t)v_{t+1}^y)}{(R_{t+1} - \gamma'(s_t)v_{t+1}^o)\rho u'(d_{t+1} - \gamma(s_t)v_{t+1}^o)} = 1, \tag{17}$$

whereas the first-order condition for both problems (15) and (16) with respect to $b_t$ is

$$\rho u'(d_t - \gamma(s_{t-1})v_t^o) \geq \beta u'(c_t - \gamma(s_{t-1})v_t^y), \tag{18}$$

where the latter condition holds with equality if $b_t > 0$. Expression (17) characterizes the optimal allocation of consumption along the lifetime of an individual. It should be noted that the introduction of endogenous status effects increases the marginal cost of consuming in young age since additional savings decrease the intensity of the comparison effect both in old age and for the direct descendant in young age. More precisely, the marginal return to savings in our model exceeds the rate of return $R_{t+1}$. In particular, the effective return on savings is given by

$$R_{t+1} - \rho\gamma'(s_t)v_{t+1}^o u'(d_{t+1}) - \beta\gamma'(s_t)v_{t+1}^y u'(c_{t+1}),$$

which is larger than the market return $R_{t+1}$ because $\gamma'(s_t) < 0$. The last two terms come from the utility gain that savings generates by reducing the status concern. The second term is the gain obtained by the own individuals in old age, whereas the third term is the utility gain derived from the reduction in the status concern of the descendants. Therefore, the fact that the intensity of status concern is endogenous stimulates savings but not necessarily the willingness to leave bequest as we will show below.

The latter implies that the intertemporal allocation depends on the altruism factor $\beta$ even when $b_t = 0$ in contrast to the model of Alonso-Carrera et al. (2008). In other words, in our economy individuals have two instruments to condition the welfare of their descendants: (i) the intergenerational transmission of wealth, i.e., physical bequests $b$; and (ii) the intergenerational transmission of preferences, i.e., their decision on savings will determine their children's concern for status. Hence, even when individuals do not leave bequest, their decision on savings still depends on the altruism factor because, as was mentioned, this determines the preferences of their descendants. If the bequest motive is operative, expression (18) characterizes the optimal allocation of consumption between two consecutive generations. When the bequest motive is operative, the marginal cost of leaving a larger amount of bequest must be equal to the discounted marginal utility of consumption of the direct descendant.

After having presented the equilibrium conditions on the demand side of the economy, we will move to the supply side and we characterize the optimal plan of the firms. Since firms behave competitively, rental prices equal marginal productivities, i.e., we get that

$$R_t = f'(k_t) \equiv R(k_t), \tag{19}$$

and

$$w_t = f(k_t) - f'(k_t)k_t \equiv w(k_t). \tag{20}$$

The competitive equilibrium of this economy is a path $\{k_{t+1}, c_t, d_{t+1}, b_t\}_{t=0}^{\infty}$ that, given an initial value of $k_0$, solves the system of equations composed of (17) and (18), together with (1), (2), (3), (9), (10), (19), (20), (13) and the transversality condition

$$\lim_{t\to\infty} \beta^t u'(\hat{c}_t) b_t = 0. \tag{21}$$

The transversality condition states that the present value of the long-run amount of bequests is equal to zero.

We will focus on steady-state equilibria, that is competitive equilibria where the variables $k_t$, $c_t$, $d_t$ and $b_t$ are constant. At the steady-state, expression (17) becomes

$$h(k, b) \equiv u'(\hat{c})(1 + \beta\gamma'(k)v^y) - \rho u'(\hat{d})(R - \gamma'(k)v^o) = 0, \tag{22}$$

where we know from (2), (3), (19), (20) and (13) that

$$\hat{c} = (1 - \gamma(k)\epsilon^y)(w(k) + b - k) - \gamma(k)(1 - \epsilon^y)(f'(k)k - b),$$

$$\hat{d} = [1 - \gamma(k)(1 - \epsilon^o)](f'(k)k - b) - \gamma(k)\epsilon^o(w(k) + b - k),$$

$$v^y = \epsilon^y(w(k) + b - k) + (1 - \epsilon^y)(f'(k)k - b),$$

and

$$v^o = \epsilon^o(w(k) + b - k) + (1 - \epsilon^o)(f'(k)k - b).$$

The equation $h(k, b) = 0$ is only defined for values of $k > 0$ and $b \geq 0$ for which $\hat{c} > 0$ and $\hat{d} > 0$. The use of Equation (22) depends on whether or not the bequest motive is operative. On the one hand, this equation implicitly defines a relationship between the stationary capital stock and the stationary value of bequests, $k = K(b)$, when the bequest motive is operative. On the other hand, Equation (22) provides the stationary value of the capital stock when the bequest motive is not operative.

To go further with the analysis of the equilibrium properties, we should first characterize the partial derivatives of the function $h(k, b)$. More precisely, the sign of these derivatives will be crucial in proving the existence and stability of the stationary equilibria. We start by focusing on a change in the stationary amount of bequests. Hence, we obtain from (22) that the derivative of $h(k, b)$ with respect to $b$ is given by

$$h_b = \left\{ \begin{array}{l} u''(\hat{c})(1 + \beta\gamma'(k)v^y)[1 - \gamma(k)(2\epsilon^y - 1)] \\ \\ +u'(\hat{c})\beta\gamma'(k)(2\epsilon^y - 1) + \rho u'(\hat{d})\gamma'(k)(2\epsilon^o - 1) \\ \\ +\rho u''(\hat{d})(f'(k) - \gamma'(k)v^o)[1 - \gamma(k)(1 - 2\epsilon^o)] \end{array} \right\}. \tag{23}$$

Observe that $h_b$ is negative for all $b > 0$ if the intensity of consumption externalities is constant, i.e., $\gamma'(k) = 0$. Otherwise, the sign of this derivative is ambiguous. By following a continuity argument, we will assume from now on that $\gamma'(k)$ is sufficiently small in absolute terms such that $h_b < 0$ holds for all $b > 0$. In any case, the next result states sufficient conditions to obtain the later property.

**Proposition 1.** *$h_b < 0$ for all $b > 0$ when at least one of the following conditions holds: (i) $\theta^y = \theta^o = 0$, or (ii) $\theta^y < \theta^o$.*

The first condition in Proposition 1 implies that individuals do not consider the consumption of the other generation living at the same period in forming their consumption reference. Although the second condition admits this intergenerational comparison in consumption, it imposes that old individuals care more for comparing with the consumption of the contemporaneous young individuals than the other way around. However, we will assume in general that $h_b < 0$ without imposing any of these sufficiency conditions. Otherwise, we would be disregarding some interesting scenarios for the Pigouvian taxation as will be shown below.

**Assumption A**. *$h_b < 0$ for all $b > 0$.*

We now focus on a change in the stationary level of the capital stock. Hence, we obtain from (22) that the derivative of $h(k, b)$ with respect to $k$ is given by

$$h_k = \left\{ \begin{array}{l} u''(\hat{c})(1 + \beta\gamma'(k)v^y)\frac{\partial \hat{c}}{\partial k} \\ \\ +u'(\hat{c})\beta\left(\gamma''(k)v^y + \gamma'(k)\frac{\partial v^y}{\partial k}\right) \\ \\ -\rho u''(\hat{d})(f'(k) - \gamma'(k)v^o)\frac{\partial \hat{d}}{\partial k} \\ \\ -\rho u'(\hat{d})\left(f''(k) - \gamma''(k)v^o - \gamma'(k)\frac{\partial v^o}{\partial k}\right) \end{array} \right\}. \tag{24}$$

As was proved by Alonso-Carrera et al. (2008, Lemma 3.2), this derivative $h_k$ is positive in the case of a constant intensity of consumption externalities (i.e., if $\gamma'(k) = 0$) when the following condition holds:

$$|f''(k)| < \min\{1/k, f'(k)/k\}. \tag{25}$$

This condition requires the stationary first period consumption $c$ and the stationary second period consumption $d$ to be decreasing and increasing, respectively, in the stationary capital. However, the sign of $h_k$ is ambiguous when the intensity of the consumption externality is endogenous. Following the same argument of continuity used in characterizing the sign of $h_b$, we will assume from now on that $h_k > 0$ for all $b \geq 0$. In any case, the next result states sufficient conditions to obtain the latter property.

**Proposition 2.** *Consider that Condition (25) holds. If*

$$\frac{f'(k) + f''(k)k}{1 + f''(k)k} < \theta^o, \tag{26}$$

*and*

$$-\gamma'(k) < \frac{(1-\gamma(k)\epsilon^y)(1+kf''(k))+\gamma(k)(1-\epsilon^y)(f'(k)+kf''(k))}{\epsilon^y(w(k)-k)+(1-\epsilon^y)f'(k)k}, \tag{27}$$

*then* $h_k > 0$ *for all* $b \geq 0$.

By following the proof in the appendix we provide the following economic interpretation of the conditions in Proposition 2. Condition (26) requires the weight of the consumption of the representative young in the reference of both generations to be sufficiently large. This condition is equivalent to requiring that the references in both periods of life $v^y$ and $v^o$ decrease in the stationary capital stock. Finally, condition (27) requires the stationary effective consumption level in the first period of life $\hat{c}$ to be decreasing in the stationary capital stock. This condition defines an upper-bound on the impact of the stationary capital stock on the intensity of the comparison effect.

**Assumption B**. *Conditions (25), (26) and (27) hold.*

Assumptions A and B allow to characterize the equilibrium relationship between the stock of capital and the amount of bequests at the steady state. The next result establishes that the stationary stock of capital is an increasing function of the stationary value of bequests in this case.

**Proposition 3.** *Consider that Assumptions A and B hold. Hence,* $K'(b) > 0$.

Let $\overline{k}$ denote the stationary capital stock when there are no bequest. As was mentioned above, this capital stock depends on the altruism factor $\beta$ provided that $\gamma'(k) < 0$, so we can write $\overline{k} = \overline{k}(\beta)$. By applying the implicit function theorem to (22), and using the features of function $h(k, 0)$ that we have stated above, the next result directly characterizes this relationship.

**Proposition 4.** *Consider that Assumption B holds. Hence,* $\overline{k}'(\beta) > 0$ *if* $\gamma'(k) < 0$, *whereas* $\overline{k}'(\beta) = 0$ *if* $\gamma'(k) = 0$.

Following Abel (1987), we are going to assume that the steady-state equilibrium value of the capital stock $\overline{k}$ in an economy without bequests is unique and that $h(k, 0) \lesseqgtr 0$ whenever $k \lesseqgtr \overline{k}$.

**Assumption C**. *There exists a unique strictly positive value of* $\overline{k}$ *satisfying* $h(k, 0) = 0$ *with* $\hat{c} > 0$ *and* $\hat{d} > 0$.

We combine expressions (17), (18) when it is binding and (19), all of them evaluated at the steady-state with no bequests, in order to obtain the threshold value of the altruism factor $\beta$ above which bequests are strictly positive. In particular, we obtain that this threshold, which will be denoted by $\overline{\beta}$ is given by the solution of the following equation:

$$G(\beta) \equiv \beta \left\{ f'[\overline{k}(\beta)] - \gamma'[\overline{k}(\beta)] \left[ v^o(\beta) + v^y(\beta) \right] \right\} - 1 = 0, \tag{28}$$

where we have made use of the fact that the equilibrium without bequest depends on the altruism factor $\beta$ and, therefore, $\overline{k}$, $v^y$ and $v^o$ are functions of this parameter. Equation (28) could have multiple solutions or no solution for $\beta$.[3] Let $\overline{\beta}_i$, with $i = \{1, 2, ..., I\}$, be the roots of Equation (28) with $\overline{\beta}_i < \overline{\beta}_j$ if $i < j$. We define the following set of values for $\beta$:

$$B = \{\beta \mid \beta \in (\overline{\beta}_i, \overline{\beta}_{i+1}), \text{ where } i \text{ is an odd integer}\}.$$

The following result characterizes the operativeness of the bequest motive.

**Proposition 5.** *Consider that Assumptions B and C hold. Hence,* $b > 0$ *if and only if* $\beta \in B$.

Expression (28) for the threshold value $\overline{\beta}$ is clearly different from the one obtained in the case of Weil (1987) or Alonso-Carrera et al. (2008), where the threshold satisfies $\overline{\beta} = 1/f'(\overline{k})$. There is an additional term due to the fact that the intensity of the comparison effect is endogenous. Moreover, the threshold value now directly depends on the consumption references in both periods of life. Therefore, in stark contrast with Weil (1987), the dynamic inefficiency of the economy without altruism may not be sufficient to prevent the bequest motive from being operative when the individuals are altruistic. Since $\overline{k}$ is increasing in $\beta$, then $f'[\overline{k}(0)] > f'[\overline{k}(\beta)]$ for all $\beta > 0$. Hence, if the economy without altruistic motive is dynamically inefficient, i.e., $f'[\overline{k}(0)] < 1$, then it holds that $f'[\overline{k}(\beta)] < 1$ for all $\beta > 0$. However, we observe from (28) that this property is not sufficient to obtain $\overline{\beta} > 1$ in our economy with altruistic motive and where the intensity of status concern depends on saving decisions. This means that positive bequest could appear even if the economy without altruistic motive is dynamically inefficient. We will come back to this discussion in the next section. Before that, we will characterize the

[3] In next section, we provide the solution of (28) for some numerical examples. In all of them, we obtain that this equation only has one root.

stationary equilibrium depending on whether or not the bequest motive is operative (i.e., whether or not $\beta \in B$).

**Proposition 6.** *Consider that Assumptions A, B and C hold. Hence:*

*(a) If $\beta \notin B$, then the unique steady-state equilibrium satisfies the following equations:*

$$\begin{aligned}
&\overline{b} = 0,\\
&h(\overline{k}, 0) = 0,\\
&\overline{c} = f(\overline{k}) - \overline{k} f'(\overline{k}) - \overline{k},\\
&\overline{d} = f'(\overline{k})\overline{k},
\end{aligned}$$

*where $\overline{k}$, $\overline{b}$, $\overline{c}$ and $\overline{d}$ are the steady-state values of capital, bequests and consumption in young and old age, respectively.*

*(b) If $\beta \in B$, then the steady-state is unique, exhibits a strictly positive amount of bequests and satisfies the following equations:*

$$\begin{aligned}
&\tfrac{1}{\beta} = f'(k^*) - \gamma'(k^*)(v^o + v^y),\\
&h(k^*, b^*) = 0,\\
&c^* = f(k^*) - k^* f'(k^*) - k^* + b^*,\\
&d^* = f'(k^*)k^* - b^*,
\end{aligned}$$

*where $k^*$, $b^*$, $c^*$ and $d^*$ are the steady-state values of capital, bequests and consumption in young and old age, respectively.*

*(c) Moreover, $k^* > \overline{k}$.*

It should be noticed that the stationary capital stock is always larger when the bequest motive is operative, i.e. $k^* > \overline{k}$.

**Proposition 7.** *Consider that Assumptions A, B and C hold. Hence:*

*(a) The steady-state with inoperative bequest motive is locally saddle path stable for values of $\gamma(k)$ and $\gamma'(k)$ sufficiently close to zero.*

*(b) The steady state equilibrium with operative bequest motive is locally saddle path stable for values of $\gamma'(k)$ sufficiently close to zero.*

We cannot fully characterize the local stability of the steady state. However, Proposition 7 still provides sufficient conditions for the steady-state equilibrium to be locally saddle-path stable. Furthermore, these conditions seem quite realistic. On the one hand, when the bequest motive is operative, the steady state is locally saddle-path stable when the concern for social status has a small sensitivity with respect to savings. As is shown in the proof of Proposition 7, this condition ensures that savings are an increasing function with respect to first-period endowments, i.e., labor income, which positively depends on the capital stock. On the other hand, the local saddle-path stability of the steady state with zero bequests is obtained under Condition (25), which means that second period consumption is increasing in capital, and that the intensity of consumption externalities is small and has a small sensitivity with respect to savings. Therefore, with respect to the model with constant intensity of the concern for social status (see Alonso-Carrera et al., 2008), we are imposing an additional sufficient condition: the response of the quest for social status to changes in savings should not be too large.

# 4 Operativeness of bequest motive

At this point we will analyze the conditions that lead the bequest motive to be operative, i.e., conditions under which $\beta \in B$. We are specially interested in determining how the fact of having an endogenous intensity of the concern for status affects the operativeness of the bequest motive. In particular, we will check whether or not dynastic altruism is able to generate positive bequests when our benchmark economy is dynamically inefficient. As is well known, this is not the case in an economy where individuals either do not have status concern or if its degree is exogenous (see Weil, 1987; Alonso-Carrera et al., 2008).

Unfortunately we cannot state theoretically the features of the set $B$ of the altruism factor $\beta$ for which bequest motive is operative: (i) how this set of values is affected by the fact that status concern is endogenous; or (ii) whether or not bequest motive is operative when the economy without altruism is dynamically inefficient, as was stated by Weil (1987). However, we will numerically simulate our economy to compute the critical value $\overline{\beta}$ and to derive its properties. In particular, we parametrize the economy as follows. We consider a logarithmic utility function $u(x_t) = \ln x_t$, a Cobb-Douglas production function, $f(k_t) = Ak_t^{\alpha}$, and the following functional form for the intensity of status concern proposed by Dioikitopoulos et al. (2019):

$$\gamma(k_t) = \gamma_\infty + (\gamma_0 - \gamma_\infty)\exp(-\gamma_s k_t), \tag{29}$$

where $\gamma_s \geq 0$ captures the sensitivity of $\gamma(k_t)$ with respect to changes in $k_t$; $\gamma_0 \geq 0$ would be the value of $\gamma(k_t)$ for all $k_t$ when $\gamma_s = 0$; and $\gamma_\infty \in [0, \gamma_0]$ is the asymptotic value of $\gamma(k_t)$ and, moreover, it would be the value of $\gamma(k_t)$ for all $k_t$ when $\gamma_0 = \gamma_\infty$. Finally, these parameters $\gamma_0$ and $\gamma_\infty$ delimit the image set of the function $\gamma(k_t)$ as $[\gamma_\infty, \gamma_0]$

Before proceeding with the simulation, we next discuss how the operativeness of bequest motive reacts to the marginal introduction of the dependence of the status concern on saving decisions, i.e., to a marginal change of $\gamma_s$ from zero. We show in Appendix F that this change in $\gamma_s$ increases the range of values of the altruism factor $\beta$ for which bequest motive is operative if the parameters $\gamma_0$ and $\gamma_\infty$ are both sufficiently small. This condition seems empirically plausible because it requires the status concern to slightly depend on savings and, moreover, this status concern should be sufficiently small for any value of savings or capital.

For the parametrization of our economy, we have shown that the marginal introduction of the dependence of the status concern on saving decisions may increase the range of values of the altruism factor in which the bequest motive is operative. We next proceed with the numerical simulation to see that this result may arise even with no marginal changes of the sensitivity of status concern with respect to the capital stock. After that, we will come back to the economic interpretation of this result.

Coming back to the numerical simulation, we set the benchmark values of the parameters as follows. First, we consider that each period in the model corresponds to 35 years. Second, we fix all the parameters, except for those driving the status concern, by following the standard procedure (see, e.g., de la Croix and Michel, 2002). We normalize the technological scale $A$ to unity. We also assume that the share of capital income out of aggregate income is $\alpha = 1/3$. Finally, we set $\rho = 0.1765$, so that the saving rate amounts to 15%, which is in line with the evidence provided by NIPA for the US economy during the last half century.[4]

In the absence of precise data on the formation of preferences for status concern, we set the values of the parameters driving this process in our economy as follows. First, we assume that the consumption references $v^y$ and $v^o$ are symmetric in the sense that $\theta^y = \theta^o$ and, moreover, we set their values to 0.8. Second, we consider the following values for the parameters in the function of the status concern: $\gamma_s = 1.25$, $\gamma_0 = 0.45$ and $\gamma_\infty = 0.25$.

We simulate the benchmark economy and we compare the results with two counterfactual economies: an economy with exogenous intensity of status concern, i.e., with $\gamma_s = 0$ and, so, $\gamma'(k) = 0$ for all $k$; and an economy with $\alpha = 1/4.5$, which leads the economy to be dynamically inefficient. Table 1 shows the results from these numerical simulations. In particular, the table provides the simulated threshold $\overline{\beta}$ of the altruism factor determining the operativeness of bequest motive, the stationary capital stock and the long-run and the short-run rates of return on capital.[5] For all the simulations,

[4]Note that the propensity to save out of labor income is equal to $\rho/(1+\rho)$ under logarithmic preferences.

[5]We denote by long-run rate of return the value of this rate in each period of the model (35 years), i.e., $R_{LP} = R = f'(k)$. On the contrary, the short-run rate of return would be the corresponding annual rate, i.e., $R_{CP} = (R_{LP})^{1/35} - 1 + \delta$, where $\delta$ is the annual depreciation rate of the capital stock that we set to 0.1.

Equation (28) has a unique root, so that there exists a unique threshold $\overline{\beta}$. Furthermore, we derive the following conclusions from the results in Table 1. First, when the economy is dynamically efficient (i.e., $\alpha = 1/3$ and, so $f'(\overline{k}) > 1$), the value of $\overline{\beta}$ is smaller than unity and very similar in the benchmark economy and in the counterfactual economy with $\gamma_s = 0$ and, so, $\gamma'(k) = 0$. On the contrary, when the economy is dynamically inefficient (i.e., $\alpha = 1/4.5$ and, so $f'(\overline{k}) < 1$), we observe that $\overline{\beta} < 1$ in our benchmark economy, whereas $\overline{\beta} > 1$ in the counterfactual economy with $\gamma_s = 0$ and, so, $\gamma'(k) = 0$.

[Insert Table 1]

Contrary to the model without status effects, i.e., with $\gamma = 0$ (see Weil, 1987), and to the model with exogenous concern for status, i.e., with $\gamma'(k) = 0$ (see Alonso-Carrera et al., 2008), we conclude from our numerical analysis that in our economy with $\gamma'(k) \neq 0$, the bequest motive can be operative (i.e., $\overline{\beta} < 1$) even if the economy without bequest is dynamically inefficient, i.e., when $f'(\overline{k}) < 1$. In general, we can then state that the endogeneity of the concern for social status has an ambiguous effect on the operativeness of the bequest motive. For the same values of the structural parameters, the range of values for the altruism factor $\beta$ under which the bequest motive is operative (i.e., $b > 0$) in the benchmark economy (i.e., when $\gamma'(k) > 0$) can be either larger or smaller that the range for which this happens in the economy with exogenous concern for social status (i.e. with $\gamma'(k) = 0$). In any case, it is also important to focus on how endogenous status effects affect the threshold value. When $\gamma'(k)$ is large in absolute value, the range of the altruism factor $\beta$ for which bequest motive is operative increases. It is then more probable that individuals leave bequests to their descendants.

The economic intuition of the result in this section directly follows from the fact that the intensity $\gamma$ of the quest for social status is a decreasing and convex function of savings (and, so, of the capital stock). This reinforces the negative correlation between savings and bequest left to the descendants. The smaller the amount of income that individuals allocate to savings $s_t$, the larger is the intensity $\gamma(s_t)$ of the search for social position that they transmit to their children. Hence, those individuals are willing to leave a larger amount of bequest to their children to compensate them for the larger inherited concern for social status. When savings increase, the intensity of the search for social status rapidly decreases, and so the willingness to leave bequests to the descendants.

In the next sections, we will analyze the optimality of the equilibrium path and we will also characterize the Pigouvian taxation when this path is socially suboptimal. To this end, we will abandon the functional form (29) considered in this section for the status concern $\gamma(k)$ and we will use back the general specification with the properties enumerated in Section 2.

# 5 Socially optimal solution

We assume that the social planner maximizes the discounted sum of utilities and the intergenerational discount factor coincides with the private altruistic factor $\beta$. The social planner then chooses the allocations $\{c_t, d_t, k_{t+1}\}$ to maximize the following objective function:

$$\hat{U}_0 = \left\{ \begin{array}{l} \beta^{-1}\rho u[(1-\gamma(k_0)(1-\epsilon^o))d_0 - \gamma(k_0)\epsilon^o c_0] \\ \\ +\sum_{t=0}^{\infty}\beta^t[u((1-\gamma(k_t)\epsilon^y)c_t - \gamma(k_t)(1-\epsilon^y)d_t)] \\ \\ +\sum_{t=0}^{\infty}\beta^t[\rho u((1-\gamma(k_{t+1})(1-\epsilon^o))d_{t+1} - \gamma(k_{t+1})\epsilon^o c_{t+1})] \end{array} \right\},$$

subject to the aggregate resource constraint (14), and with $k_0$ given.

By the standard procedure, we find the first-order conditions of the previous problem in Appendix G, and then rearrange the expressions to obtain that the necessary conditions for the socially optimal solution are summarized by the following two conditions:

$$\frac{\rho u'(\hat{d}_t)}{\beta u'(\hat{c}_t)} = \frac{1-\gamma(k_t)(2\epsilon^y - 1)}{1-\gamma(k_t)(1-2\epsilon^o)} \equiv I_t. \tag{30}$$

and

$$\frac{\beta u'(\hat{c}_{t+1})[1-\gamma(k_{t+1})(\epsilon^y + I_{t+1}\epsilon^o)]f'(k_{t+1}) - u'(\hat{c}_t)[1-\gamma(k_t)(\epsilon^y + I_t\epsilon^o)]}{\gamma'(k_{t+1})\beta u'(\hat{c}_{t+1})(v^y_{t+1} + I_{t+1}v^o_{t+1})} = 1. \tag{31}$$

The socially optimal solution is then a path $\{k_{t+1}, c_t, d_t\}_{t=0}^{\infty}$ that, for a given initial value $k_0$, solves the system of differential equations (14), (30), (31) together with the transversality condition

$$\lim_{t\to\infty}[(1-\gamma(k_t)\epsilon^y)\beta^t u'(\hat{c}_t) - \gamma(k_t)\epsilon^o\beta^{t-1}\rho u'(\hat{d}_t)]k_{t+1} = 0. \tag{32}$$

The next result characterizes the existence and unicity of the steady-state solution of the social planner's problem.

**Proposition 8.** *There is a unique steady-state solution to the social planner problem. Moreover, this steady-state is given by*

$$\gamma'(k)(v^y + Iv^o) = [1-\gamma(k)(\epsilon^y + I\epsilon^o)]\left(f'(k) - \frac{1}{\beta}\right),$$

$$\frac{\rho u'(\hat{d})}{\beta u'(\hat{c})} = \frac{1-\gamma(k)(2\epsilon^y - 1)}{1-\gamma(k)(1-2\epsilon^o)} = I,$$

*and*

$$f(k) = c + d + k,$$

*where $k$, $c$ and $d$ are the steady-state values of capital, consumption in young and old age, respectively.*

It can be noticed that due to endogenous status effects, the planner's economy exhibits a steady-state capital stock larger than the one corresponding to the modified golden rule.

# 6 Optimality of the competitive equilibrium

In this economy there are two sources of suboptimality of the competitive equilibrium: the consumption externalities and the possible inoperativeness of the bequest motive. These sources interact in a non trivial way. In this section we disentangle the effects of both of them by comparing separately the socially optimal solution with the competitive equilibrium when the bequest motive is and is not operative.

## 6.1 Suboptimality when the bequest motive is operative

When the bequest motive is operative (i.e., when $\beta \in B$ such that $b_t > 0$), the only sources of deviation between the competitive and the socially optimal allocations are the consumption externalities. As was pointed out by Abel (2005), the existence of consumption externalities when there are agents with different ages living at the same period implies that the intratemporal allocation between the two generations is in general suboptimal. However, the presence of endogenous status effects also implies that the socially optimal marginal rate of substitution between young and old consumption is time-dependent as can be seen from expression (30). Therefore, contemporaneous consumption externalities may not be only a source of intratemporal (but intergenerational) source of inefficiency, but also a source of intertemporal (but intragenerational) inefficiency.

We first conclude from (18) that the intratemporal allocation of aggregate consumption in the competitive economy (i.e., the allocation of consumption between the two living generations at period $t$) is given by

$$\frac{u'(\hat{d}_t)}{u'(\hat{c}_t)} = \frac{\beta}{\rho}. \tag{33}$$

By comparing expressions (30) and (33) we see that the marginal rates of substitution between consumption for the young and the old does not coincide unless $I_t = 1$. By using the definitions of $\epsilon^y$ and $\epsilon^o$, we obtain that $I_t = 1$ if and only if $\theta^y = \theta^o$. Therefore, the intratemporal equilibrium allocation between the two living generations is inefficient unless the two generations give the same weight to the consumption of the other generation in forming their consumption reference. Obviously, this inefficiency results in a suboptimal path of bequest. Observe also that this inefficiency does not depend on the fact that the concern for status is endogenously determined by savings.

We now focus on the intertemporal allocation of consumption. By combining (17) and (18), we obtain that the intertemporal allocation in the competitive equilibrium is given by

$$\beta u'(\hat{c}_{t+1})f'(k_{t+1}) - u'(\hat{c}_t) = \gamma'(k_{t+1})\beta u'(\hat{c}_{t+1})(v^y_{t+1} + v^o_{t+1}). \tag{34}$$

By comparing expressions (31) and (34), we see that the equilibrum and the socially optimal allocations do not coincide contrary to the case without endogenous status effects studied by Alonso-Carrera et al. (2008). It should be noticed that both allocations coincide if $\gamma'(k_t) = 0$ for all $k_t$ and, therefore, we recover the result obtained by these authors.[6] Observe that, contrary to what happens with the intragenerational allocation of consumption, the intertemporal allocation of consumption in the competitive equilibrium is also inefficient even in the case where $\theta^y = \theta^o$ provided that $\gamma'(k_t) \neq 0$.

Therefore, we conclude that in our economy with endogenous status effects the path of capital is also suboptimal even when the bequest motive is operative. The competitive equilibrium is suboptimal since endogenous status effects imply that the marginal rate of substitution between consumption for the young and the old has intertemporal consequences which are not internalized by the individuals. This derives from the fact that the external effects of consumption are time-dependent because the intensity of status concern changes with the capital stock.

## 6.2 Suboptimality when the bequest motive is not operative

When the bequest motive is not operative (i.e., when $\beta \notin B$ such that $b_t = 0$), the intertemporal allocation in the competitive equilibrium is given by (18), which can be rewriten for the purpose of this subsection as

$$\frac{\rho u'(\hat{d}_{t+1})\left[f'(k_{t+1}) - \gamma'(k_{t+1})v^o_{t+1}\right]}{u'(\hat{c}_t) + \beta\gamma'(k_{t+1})u'(\hat{c}_{t+1})v^y_{t+1}} = 1. \tag{35}$$

To facilitate the efficiency analysis, we combine (30) and (31), so that we obtain that the socially optimal intertemporal allocation is given by the following condition:

$$\frac{\rho u'(\hat{d}_{t+1})\left\{f'(k_{t+1})\left[\frac{1-\gamma(k_{t+1})(\epsilon^y + I_{t+1}\epsilon^o)}{I_{t+1}}\right] - \gamma'(k_{t+1})v^o_{t+1}\right\}}{u'(\hat{c}_t)\left[1-\gamma(k_t)(\epsilon^y + I_t\epsilon^o)\right] + \beta\gamma'(k_{t+1})u'(\hat{c}_{t+1})v^y_{t+1}} = 1. \tag{36}$$

[6] Observe that $I_{t+1} = I_t = I$ when $\gamma'(k_t) = 0$ for all $k_t$.

By comparing (35) and (36), we conclude that the intertemporal allocation along the equilibrium path in general differs from the socially optimal allocation. In particular, the two intertemporal allocations only coincide if the following two conditions simultaneously hold: (i) the concern for status is exogenous, i.e., $\gamma'(k_t) = 0$; and (ii) the two generations give the same weight to the consumption of the other living generation in forming their consumption reference, i.e., $\theta^y = \theta^o$, since in this case $I_t = 1$. Therefore, as was already stated by Alonso-Carrera et al. (2008), the path of the capital stock is suboptimal if the bequest motive is inoperative even when the intensity of status concern is exogenous, i.e., $\gamma'(k_t) = 0$, provided that $\theta^y \neq \theta^o$.

However, the endogeneity of the status concern reinforces this inefficiency of the capital stock along the equilibrium path. If $\gamma'(k_t) = 0$ the equilibrium inefficiency exclusively arises because the term $I_{t+1}$ does not appear in the numerator of the equilibrium condition (35) for the intertemporal allocation. On the contrary, when $\gamma'(k_t) \neq 0$ the discrepancy between the equilibrium and the socially optimal allocation also arises because of the absence of the terms $1 - \gamma(k_{t+1})(\epsilon^y + I_{t+1}\epsilon^o)$ and $1 - \gamma(k_t)(\epsilon^y + I_t\epsilon^o)$ in the numerator and denominator of (35), respectively. To be clearer, even when $I_t = 1$ for all period $t$ (because, for instance, $\theta^y = \theta^o$), the equilibrium exhibits a suboptimal path of capital if $\gamma'(k_t) \neq 0$ because the two aforementioned terms do not cancel out in Condition (36).

The economic intuition for the previous conclusion is identical to the one given for explaining the inefficiency of the intertemporal allocation in the equilibrium with strictly positive bequest. In our economy the external effects of consumption are time-dependent because the intensity of status concern changes with the capital stock.

# 7 Optimal taxes

In this section, we characterize the optimal tax rates needed to decentralize the socially optimal allocation when endogenous status effects and consumption externalities interact with altruism. We consider a fiscal policy that consists of an estate tax, a capital income tax and a set of lump-sum taxes. Concerning the latter, we assume that young agents pay a lump-sum tax $\tau_t^y$ and the revenues are used to finance a lump-sum transfer to the old $-\tau_t^o$. These tax rates are then related by the following budget constraint:

$$\tau_t^o = -\tau_t^y \tag{37}$$

We also assume that young agents pay an estate tax on the inheritance that they receive from their parents. The revenues from this tax are returned to the young agents by means of a lump-sum subsidy. This second government budget constraint is given by

$$\phi_t^y = \tau_t^b b_t, \tag{38}$$

where $\phi_t^y$ is the lump-sum subsidy to young individuals and $\tau_t^b$ is the estate tax rate. Finally, old agents pay a capital income tax on the returns from saving and the revenues are returned to these agents under the form of a lump-sum subsidy. This third government budget constraint is given by

$$\phi_t^o = \tau_t^k s_t f'(k_t), \tag{39}$$

where $\phi_t^o$ is the lump-sum subsidy to the old individuals and $\tau_t^r$ is the capital income tax rate.

An individual belonging to generation $t$ faces the following dynamic programming problem:

$$V_t(b_t, s_{t-1}) = \max_{s_t, b_{t+1}} \{u\left[c_t - \gamma(s_{t-1})v_t^y\right] + \rho u\left[d_{t+1} - \gamma(s_t)v_{t+1}^o\right] + \beta V_{t+1}(b_{t+1}, s_t)\},$$

with (1), and where

$$c_t = w_t - \tau_t^y + \phi_t^y + (1 - \tau_t^b)b_t - s_t, \tag{40}$$

and

$$d_{t+1} = (1 - \tau_{t+1}^k)R_{t+1}s_t + \phi_{t+1}^o - b_{t+1} - \tau_{t+1}^o, \tag{41}$$

for $v_t^y$, $v_t^o$, $w_t$ and $R_{t+1}$ given for all $t$.

The problem faced by the individuals belonging to generation $-1$ in period 0 is the following:

$$\max_{b_0}\{\rho u\left[(1 - \tau_0^k)R_0 k_0 + \phi_0^o - b_0 - \tau_0^o - \gamma(k_0)v_0^o\right] + \beta V_0(b_0, k_0)\}, \tag{42}$$

with $b_0 \geq 0$, for $k_0$, $v_0^o$ and $R_0$ given.

By following the standard procedure, we first find the first-order condition of the previous two problems, and then rearrange the expressions to obtain that the necessary conditions for the consumption plan at the equilibrium with fiscal policy are the following:

$$u'(\hat{c}_t + \beta\gamma'(k_{t+1})v_{t+1}^y u'(\hat{c}_{t+1}) = \left[(1 - \tau_{t+1}^k)R_{t+1} - \gamma'(k_{t+1})v_{t+1}^o\right]\rho u'(\hat{d}_{t+1}), \tag{43}$$

and

$$\rho u'(\hat{d}_t) \geq (1 - \tau_t^b)\beta u'(\hat{c}_t), \tag{44}$$

where the last condition holds with equality if $b_t > 0$.

Given the initial capital stock $k_0$, and the path of the tax rates $\{\tau_t^b, \tau_t^k, \tau_t^y, \tau_t^o\}$, the competitive equilibrium of this economy with taxes is a path $\{k_t, c_t, d_t, b_t, \phi_t^y, \phi_t^o\}$ that solves the system of difference equations composed of (43) and (44) together with (19), (20), (13), (37), (38), (39), (40), (41) and the transversality condition (21).

Let us denote the path of optimal tax rates as $\{\tilde{\tau}_t^y, \tilde{\tau}_t^o, \tilde{\tau}_t^b, \tilde{\tau}_t^k\}_{t=0}^{\infty}$. The optimal tax rates are such that they make the equilibrium path $\{k_t, c_t, d_t\}$ coincide with the socially optimal path $\{k_t, c_t, d_t\}$ characterized by (14), (30) and (31). Obviously, the socially optimal path of bequests is given by

$$b_t = f'(k_t)k_t - d_t, \tag{45}$$

where $k_t$ and $d_t$ are the socially optimal capital stock and consumption in old age, respectively. If the optimal amount of bequest is positive, the bequest motive is operative along an equilibrium path that attains the first best solution while if the optimal amount of bequests is negative, the bequest motive is inoperative along this equilibrium path. We will need to distinguish between these two cases.

First, the next result characterizes the optimal taxation when the optimal amount of bequest is positive.

**Proposition 9.** *Consider that the optimal amount of bequest is positive. Hence, the optimal rates of taxation satisfy* $\tilde{\tau}_t^b = 1 - I_t$ *and* $\tilde{\tau}_t^k = 1 - \frac{1+\Omega_t}{I_t}$*, where*

$$\Omega_t = \frac{u'(\hat{c}_{t-1})\gamma(k_{t-1})(\epsilon^y + \epsilon^o I_{t-1})}{\beta u'(\hat{c}_t) f'(k_t)} - \gamma(k_t)(\epsilon^y + \epsilon^o I_t). \tag{46}$$

From the proposition we observe the following features of the optimal taxation. First, since $\gamma'(k_t) \neq 0$, then the optimal tax rates are time-varying. If $\gamma'(k_t) = 0$, then $I_t$ is constant for all $t$ as was shown before and $\Omega_t = 0$.[7] Second, the optimal fiscal policy can consist on either taxing or subsidizing capital income depending on the weight that individuals give to the consumption of the two living generations in forming their consumption reference. The next result provides sufficient conditions for having an optimal subsidy on capital income and, moreover, it provides an upper-bound to the tax rate for the case where the optimal policy consists of taxing capital income.

**Proposition 10.** *Consider that the optimal amount of bequest is positive and* $k_t < k_{t+1}$*. The optimal tax rate on capital income satisfies that* $\tilde{\tau}_t^k < 1 - \frac{1}{I_t}$*. Moreover, if* $\theta^y < \theta^0$*, then* $\tilde{\tau}_t^k < 0$*.*

[7] Observe that in this case we can write $\Omega_t$ as follows:

$$\Omega_t = (\epsilon^y + \epsilon^o I)\left[\frac{u'(\hat{c}_{t-1})}{\beta u'(\hat{c}_t) f'(k_t)} - 1\right].$$

The term in brackets is equal to zero when $\gamma'(k_t) = 0$ as follows from (31).

We then conclude that the optimal rate of capital income taxation has an upper-bound for the empirical more plausible scenario where $k_t < k_{t+1}$ . Furthermore, the optimal rate converges to this upper-bound when $\gamma'(k_t)$ tends to zero. This conclusion is a consequence of the fact that the external effects of consumption are smaller when the concern for status is endogenous. In other words, individuals can partially control the external effects that they and their descendants suffer by choosing savings. Proposition 10 also states that the optimal tax rate on capital income is negative when the consumption of young individuals has a larger external effect on the utility of the old individuals than the external effect of the consumption of the latter on the utility of the former. Therefore, the savings and capital stock are suboptimally low, which requires a subsidy to capital income to restore efficiency.

Finally, we characterize the optimal fiscal policy when the optimal amount of bequest is negative.

**Proposition 11.** *Consider that the optimal amount of bequest is negative. Hence, the optimal rates of taxation satisfy* $\tilde{\tau}_t^k = 1 - \frac{1+\Omega_t}{I_t}$, $\tilde{\tau}_t^y = -b_t$ *and* $\tilde{\tau}_t^o = b_t$, *where* $\Omega_t$ *and* $b_t$ *are given by (46) and (45), respectively.*

The proposition states that the optimal rate of capital income taxation is the same as in the case where the optimal amount of bequest is positive. Furthermore, Proposition 9 still applies in this case. However, the optimal fiscal policy now also includes a lump-sum tax on young individuals. While the tax on capital income solves the suboptimality due to consumption externalities, the one due to the inoperativeness of the bequest motive is solved through lump-sum taxes. We use (19), (13), (39) and (41) to obtain

$$d_t = f'(k_t)k_t - \tau_t^o. \tag{47}$$

From the comparison between (45) and (63), we obtain that $\tilde{\tau}_t^o = b_t$ and from (37), we get $\tilde{\tau}_t^y = -b_t$ where $b_t$ is defined from expression (45). Since $b_t < 0$, we obtain that $\tilde{\tau}_t^o < 0$ and $\tilde{\tau}_t^y > 0$. When the optimal bequest is negative, the fiscal policy should then transfer income from young to old individuals to restore the efficiency of the intratemporal (i.e., intergenerational) allocation of consumption. Therefore, the optimal fiscal policy contains in this case a pay-as-you-go social security system.

# 8 Conclusion

In this paper, we have analyzed the implications of dynastic altruism in an OLG model where in addition to the possibility of leaving bequests, parents can shape the future preferences of their children. By increasing savings, individuals are able to reduce the intensity of the concern for status in old age and of the following generation.

Our results highlight the importance of endogenous status effects both for the competitive equilibrium and the socially optimal allocation. The level of the altruism factor for which individuals leave bequests can be both larger or smaller than in a model without endogenous status effects. In particular, endogenous status effects may allow the bequest motive to be operative even when the economy without bequest is dynamically inefficient. The incentive to leave bequests is decreasing in the level of savings since the latter is an alternative instrument that can be used to improve the welfare of the descendants. Concerning the socially optimal allocation, endogenous status effects imply that the intratemporal allocation between generations has intertemporal consequences that are not internalized by the individuals. Due to the latter, even when the bequest motive is operative, the intertemporal allocation is suboptimal. When the bequest motive is operative, the optimal policy consists of estate and capital income taxes with time-varying rates. When the bequest motive is inoperative, the same time-varying capital income tax is combined with a pay-as-you-go social security system.

Finally, future research could focus on other forms of consumption references such as intergenerational aspirations or habit formation. These extensions tend to complicate the problem since individuals are then able to influence as well the reference with which their children will compare themselves.

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# Appendix

# A Proof of Proposition 1

Observe that Condition (18) holds with equality when $b > 0$. In this case, the second line of $h_b$ in (23), i.e., the expression

$$u'(\hat{c})\beta\gamma'(k)(2\epsilon^y - 1) + \rho u'(\hat{d})\gamma'(k)(2\epsilon^o - 1),$$

then becomes

$$2\beta u'(\hat{c})\gamma'(k)(\epsilon^y + \epsilon^o - 1).$$

By the definitions of $\epsilon^y$ and $\epsilon^o$, we can write the last expression as

$$\frac{2(\theta^o - \theta^y)\beta u'(\hat{c})\gamma'(k)}{(1+\theta^y)(1+\theta^o)}.$$

Given the properties of the functions $u(.)$ and $\gamma(.)$, we conclude that the latter expression is not positive when at least one of the conditions in the proposition holds.

# B Proof of Proposition 2

To find sufficient conditions for $h_k > 0$, we impose all the elements in (24) to be positive. First, observe that the effective consumption $\hat{d}$ is always increasing in the stationary capital stock, i.e., $\frac{\partial \hat{d}}{\partial k} > 0$. Second, since $1 + \beta\gamma'(k)v^y > 0$ for the equilibrium condition (22) to be satisfied at an interior steady state, we first impose the effective consumption $\hat{c}$ to be decreasing in the capital stock. After some algebra, we obtain that $\frac{\partial \hat{c}}{\partial k} < 0$ if and only if Condition (27) holds. Finally, we set that both $v^y$ and $v^o$ are decreasing in the capital stock. We obtain from a simple manipulation that $\frac{\partial v^y}{\partial k} < 0$ and $\frac{\partial v^o}{\partial k} < 0$ if and only if

$$\frac{f'(k) + f''(k)k}{1 + f'(k) + 2f''(k)k} < \min\left\{\epsilon^y, \epsilon^o\right\}. \tag{48}$$

We also observe that $\epsilon^y > \epsilon^o$. Moreover, by using the definition of $\epsilon^o$, we obtain that Condition (48) is equivalent to Condition (26). Thus, the proposition directly follows.

# C Proof of Proposition 5

First, combining (17), (18) and (19), all of them evaluated at the steady state with no bequest, we derive (28) and we also conclude that $b = 0$ if and only if $G(\beta) < 0$. Second, note that the case with $\beta = 0$ corresponds to the economy without altruism. In this case we then obtain that $\overline{k} > 0$. Therefore, we conclude that $G(0) = -1$. Finally, we obtain from (28) that the first derivative of $G(\beta)$ is given by:

$$G'(\beta) = \left\{ \begin{array}{l} f'(\overline{k}) - \gamma'(\overline{k})(v^o + v^y) \\ \\ +\beta \left(\frac{\partial \overline{k}}{\partial \beta}\right) \left[ f''(\overline{k}) - \gamma''(\overline{k})(v^o + v^y) - \gamma'(\overline{k}) \left(\frac{\partial (v^o + v^y)}{\partial k}\right)\right] \end{array} \right\},$$

where we know that $\frac{\partial \overline{k}}{\partial \beta} > 0$ by the properties of the equilibrium and $\frac{\partial (v^o + v^y)}{\partial \beta} < 0$ by Assumption B. Observe that $G'(0) > 0$ because of the properties of the function $\gamma(k)$ determining the intensity of the status concern and of the production function $f(k)$. Therefore, the result directly follows.

# D Proof of Proposition 6

The amount of bequests in a steady-state equilibrium $b^*$ is strictly positive only if expression (18) holds with equality. In this case, the steady-state equilibrium is characterized by this equation and $h(k, b) = 0$. From these two equations, it follows that the steady-state capital stock satisfies

$$f'(k^*) - \gamma'(k^*)(v^o + v^y) = \frac{1}{\beta}.$$

The uniqueness of the steady-state follows immediately from the assumptions on the production function, the aspiration function and the consumption references in both periods of life.

By assumption, $h(k, 0) \lesseqgtr 0$ whenever $k \lesseqgtr \overline{k}$ and from Lemma 1, $h_b < 0$. Therefore, $h(k, b) < h(k, 0)$ for all $k > 0$ and $b^* > 0$. The value $k^*$ satisfying $h(k^*, b^*) = 0$ must be strictly larger than $\overline{k}$ or, equivalently

$$f'(k^*) - \gamma'(k^*)(v^o + v^y) < f'(\overline{k}) - \gamma'(\overline{k})(v^o + v^y) \tag{49}$$

The latter condition being equivalent to $\beta > \overline{\beta}$. We thus have proved that $\beta > \overline{\beta}$ if and only if $b^* > 0$ and $f'(k^*) - \gamma'(k^*)(v^o + v^y) = 1/\beta$. Therefore, $\beta \leq \overline{\beta}$ implies that the stationary amount of bequests is equal to zero and the steady-state capital stock satisfies $h(\overline{k}, 0) = 0$. The value $\overline{k} > 0$ exists and is unique by assumption.

# E Proof of Proposition 7

We will characterize the local stability of the steady state equilibrium with and without operativeness of bequest motive.

(a) *The case when the bequest motive is inoperative.* We first derive conditions guaranteeing the local stability of the steady-state with zero bequests characterized by Proposition 6. In a neighborhood of this steady-state, Condition (18) holds with strict inequality, so that $b_t = 0$ for all $t$. The transitional dynamics around the steady-state are fully characterized by (17). Furthermore, by using (2), (3), (7), (8), (19) and (20), we obtain the following implicit relationship $H(k_t, k_{t+1}, k_{t+2}) = 0$. Therefore, the dynamics are fully described by a second order difference equation in $k$. The stability can be characterized by using the characteristic polynomial:

$$P(\lambda) = \left(\frac{\partial H}{\partial k_{t+2}}\right)\lambda^2 + \left(\frac{\partial H}{\partial k_{t+1}}\right)\lambda + \frac{\partial H}{\partial k_t},$$

where

$$\frac{\partial H}{\partial k_t} = -\left[\frac{\sigma(\hat{c})u'(\hat{c})}{\hat{c}}\right]\left(\frac{\partial \hat{c}_t}{\partial k_t}\right),$$

$$\frac{\partial H}{\partial k_{t+1}} = \left\{\begin{array}{l} u'(\hat{c})\left[\frac{\sigma(\hat{c})}{\hat{c}}\left(\frac{\partial \hat{c}_t}{\partial k_{t+1}} + \beta\gamma'(k)v^y\frac{\partial \hat{c}_{t+1}}{\partial k_{t+1}}\right) + \beta\left(\gamma''(k)v^y + \gamma'(k)\frac{\partial v^y_{t+1}}{\partial k_{t+1}}\right)\right] \\ \\ +u'(\hat{c})\left[1+\beta\gamma'(k)v^y\right]\left[\frac{\pi(\hat{d})}{\hat{d}}\frac{\partial \hat{d}_{t+1}}{\partial k_{t+1}} - \frac{R'(k)-\gamma''(k)v^o-\gamma'(k)\frac{\partial v^o_{t+1}}{\partial k_{t+1}}}{R-\gamma'(k)v^o}\right] \end{array}\right\},$$

and

$$\frac{\partial H}{\partial k_{t+2}} = \left\{\begin{array}{l} u'(\hat{c})\beta\gamma'(k)\left[-\frac{\sigma(\hat{c})}{\hat{c}}\frac{\partial \hat{c}_{t+1}}{\partial k_{t+2}}v^y + \frac{\partial v^y_{t+1}}{\partial k_{t+2}}\right] \\ \\ +u'(\hat{c})(1+\beta\gamma'(k)v^y)\left[\frac{\pi(\hat{d})}{\hat{d}}\frac{\partial \hat{d}_{t+1}}{\partial k_{t+2}} + \frac{\gamma'(k)}{R-\gamma'(k)v^o}\frac{\partial v^o_{t+1}}{\partial k_{t+2}}\right] \end{array}\right\},$$

with $\sigma(\hat{c}) = -\hat{c}u''(\hat{c})/u'(\hat{c}) > 0$ and $\pi(\hat{d}) = -\hat{d}u''(\hat{d})/u'(\hat{d}) > 0$. Local stability implies that only one of the roots of this polynomial must belong to the unit circle. Note also that $P(1) = h_k(\overline{k}, 0) > 0$ under Assumption B. Saddle-path stability then requires $P(-1) < 0$, where

$$P(-1) = \left\{ \begin{array}{c} u'(\hat{c}) \left[ \frac{\sigma(\hat{c})}{\hat{c}} \left( \frac{\partial \hat{c}_t}{\partial k_{t+1}} - \frac{\partial \hat{c}_t}{\partial k_t} \right) - \beta \gamma''(k) v^y \right] \\ \\ +u'(\hat{c}) \beta \gamma'(k) \left[ \frac{\sigma(\hat{c})}{\hat{c}} v^y \left( \frac{\partial \hat{c}_{t+1}}{\partial k_{t+1}} - \frac{\partial \hat{c}_{t+1}}{\partial k_{t+2}} \right) + \frac{\partial v^y_{t+1}}{\partial k_{t+2}} - \frac{\partial v^y_{t+1}}{\partial k_{t+1}} \right] \\ \\ +u'(\hat{c})(1 + \beta \gamma'(k) v^y) \left[ \frac{\pi(\hat{d})}{\hat{d}} \left( \frac{\partial \hat{d}_{t+1}}{\partial k_{t+2}} - \frac{\partial \hat{d}_{t+1}}{\partial k_{t+1}} \right) + \alpha(k) \right] \end{array} \right\}, \tag{50}$$

with

$$\alpha(k) = \frac{R'(k) - \gamma''(k) v^o - \gamma'(k) \left( \frac{\partial v^o_{t+1}}{\partial k_{t+1}} - \frac{\partial v^o_{t+1}}{\partial k_{t+2}} \right)}{R - \gamma'(k) v^o}.$$

Note also that

$$\frac{\partial \hat{c}_t}{\partial k_{t+1}} - \frac{\partial \hat{c}_t}{\partial k_t} = \left\{ \begin{array}{c} [1 - \gamma(k)\epsilon^y] \left[ f''(k)k - 1 \right] \\ \\ +\gamma'(k)[\epsilon^y (w(k) - k) + (1 - \epsilon^y)(f'(k)k)] \\ \\ +\gamma(k)(1 - \epsilon^y) \left[ f''(k)k + f'(k) \right] \end{array} \right\},$$

and

$$\frac{\partial \hat{d}_{t+1}}{\partial k_{t+2}} - \frac{\partial \hat{d}_{t+1}}{\partial k_{t+1}} = \left\{ \begin{array}{c} \gamma(k)\epsilon^o \left[ 1 - k f''(k) \right] \\ \\ +\gamma'(k)[\epsilon^o (w(k) - k) + (1 - \epsilon^o)(f'(k)k)] \\ \\ -[1 - \gamma(k)(1 - \epsilon^o)] \left[ f''(k)k + f'(k) \right] \end{array} \right\},$$

which are both increasing in $\gamma(k)$ and negative at $\gamma(k) = 0$. Moreover,

$$\frac{\partial v^y_{t+1}}{\partial k_{t+2}} - \frac{\partial v^y_{t+1}}{\partial k_{t+1}} < 0,$$

while

$$\frac{\partial v^o_{t+1}}{\partial k_{t+1}} - \frac{\partial v^o_{t+1}}{\partial k_{t+2}} > 0.$$

Finally, we can conclude from the inspection of Expression (50) that $P(-1) < 0$ if the values of $\gamma(k)$ and $\gamma'(k)$ are sufficiently close to zero. Therefore, the first statement of Proposition 7 directly follows.

(b) *The case when the bequest motive is operative.* In this case, Condition (18) holds with equality and, therefore, it implicitly defines $\hat{d}_t = \phi(\hat{c}_t)$, where the derivative of the implicit function is

$$\phi' = \left(\frac{u'(\hat{d}_t)}{u''(\hat{d}_t)}\right)\left(\frac{u''(\hat{c}_t)}{u'(\hat{c}_t)}\right) > 0. \tag{51}$$

Next combine

$$\hat{c}_t = [1 - \gamma(k_t)\epsilon^y]c_t - \gamma(k_t)(1 - \epsilon^y)d_t,$$

and

$$\hat{d}_t = (1 - \gamma(k_t)(1 - \epsilon^o))d_t - \gamma(k_t)\epsilon^o c_t,$$

to obtain

$$c_t = \frac{[1 - \gamma(k_t)(1 - \epsilon^o)]\hat{c}_t + \gamma(k_t)(1 - \epsilon^y)\hat{d}_t}{(1 - \gamma(k_t)\epsilon^y)[1 - \gamma(k_t)(1 - \epsilon^o)] - \gamma(k_t)^2\epsilon^o(1 - \epsilon^y)},$$

and

$$d_t = \frac{(1 - \gamma(k_t)\epsilon^y)\hat{d}_t + \gamma(k_t)\epsilon^o\hat{c}_t}{(1 - \gamma(k_t)\epsilon^y)[1 - \gamma(k_t)(1 - \epsilon^o)] - \gamma(k_t)^2\epsilon^o(1 - \epsilon^y)}.$$

Using the previous equations, the resource constraint (14) can be rewritten as

$$k_{t+1} = f(k_t) - \frac{[1 - \gamma(k_t)(1 - 2\epsilon^o)]\hat{c}_t + [(1 - \gamma(k_t)(2\epsilon^y - 1)]\phi(\hat{c}_t)}{(1 - \gamma(k_t)\epsilon^y)[1 - \gamma(k_t)(1 - \epsilon^o)] - \gamma(k_t)^2\epsilon^o(1 - \epsilon^y)}. \tag{52}$$

Finally, combining (17) and (18), we obtain:

$$\beta u'(\hat{c}_{t+1})[f'(k_{t+1}) - \gamma'(k_{t+1})(v^y_{t+1} + v^o_{t+1})] = u'(\hat{c}_t),$$

which implicitly defines the relationship

$$\hat{c}_{t+1} = \kappa(\hat{c}_t, k_{t+1}). \tag{53}$$

The dynamic system composed by the difference equations (52) and (53) completely drives the equilibrium dynamics. The elements of the Jacobian matrix of this system are

$$\frac{\partial k_{t+1}}{\partial k_t} = f'(k) - T(k, \hat{c}),$$

$$\frac{\partial k_{t+1}}{\partial \hat{c}_t} = -\frac{1-\gamma(k)(1-2\epsilon^o) + [(1-\gamma(k)(2\epsilon^y - 1)]\phi'(\hat{c}_t)}{(1-\gamma(k)\epsilon^y)[1-\gamma(k)(1-\epsilon^o)] - \gamma(k)^2\epsilon^o(1-\epsilon^y)} < 0,$$

$$\frac{\partial \hat{c}_{t+1}}{\partial k_t} = -\left(\frac{\partial k_{t+1}}{\partial k_t}\right)\left\{\frac{u'(\hat{c})\left[f''(k) - \gamma''(k)(v^y + v^o) - \gamma'(k)\left(\frac{\partial v^y_{t+1}}{\partial k_{t+1}} + \frac{\partial v^o_{t+1}}{\partial k_{t+1}}\right)\right]}{u''(\hat{c})[f'(k) - \gamma'(k)(v^y + v^o)]}\right\},$$

and

$$\frac{\partial \hat{c}_{t+1}}{\partial \hat{c}_t} = 1 - \left(\frac{\partial k_{t+1}}{\partial \hat{c}_t}\right)\left\{\frac{u'(\hat{c})\left[f''(k) - \gamma''(k)(v^y + v^o) - \gamma'(k)\left(\frac{\partial v^y_{t+1}}{\partial k_{t+1}} + \frac{\partial v^o_{t+1}}{\partial k_{t+1}}\right)\right]}{u''(\hat{c})[f'(k) - \gamma'(k)(v^y + v^o)]}\right\}.$$

where

$$T(k,\hat{c}) = \left\{\begin{array}{c} \frac{-\gamma'(k)[(1-2\epsilon^o)\hat{c} + (2\epsilon^y - 1)\phi(\hat{c})]}{[1-\gamma(k)\epsilon^y][1-\gamma(k)(1-\epsilon^o)] - \gamma(k)^2\epsilon^o(1-\epsilon^y)} \\ \\ + \frac{\gamma'(k)[\epsilon^o + (1-\epsilon^y)(2\epsilon^o - 1)]\{[1-\gamma(k)(1-2\epsilon^o)]\hat{c} + [(1-\gamma(k)(2\epsilon^y - 1)]\phi(\hat{c})\}}{\{[1-\gamma(k)\epsilon^y][1-\gamma(k)(1-\epsilon^o)] - \gamma(k)^2\epsilon^o(1-\epsilon^y)\}^2} \end{array}\right\}.$$

We observe that $T(k,\hat{c}) = 0$ when $\gamma'(k) = 0$. Hence, we conclude by continuity that $\partial k_{t+1}/\partial k_t > 0$ for values of $\gamma'(k)$ sufficiently close to zero. We now define the characteristic polynomial as

$$Q(\lambda) = \lambda^2 - \left[f'(k) - T(k,\hat{c}) + \frac{\partial \hat{c}_{t+1}}{\partial \hat{c}_t}\right]\lambda + f'(k) - T(k,\hat{c}).$$

Observe also that

$$Q(1) = 1 - \frac{\partial \hat{c}_{t+1}}{\partial \hat{c}_t} < 0,$$

and

$$Q(-1) = 1 + \frac{\partial \hat{c}_{t+1}}{\partial \hat{c}_t} + 2\left[f'(k) - T(k,\hat{c})\right].$$

A sufficient condition for $Q(-1) > 0$ is that $\gamma'(k)$ is sufficiently close to zero, because then $T(k,\hat{c})$ is also sufficiently close to zero. In this case, there is a unique root within the unit circle which proves the desired local saddle-path stability. Therefore, the second statement in Proposition 7 directly follows.

# F Operativeness of bequest motive in the parametric example

We derive the conditions under which the threshold of the altruism factor determining the operativeness of bequest motive increases when the parameter $\gamma_s$, which drives the sensitivity of $\gamma(k)$ with respect to $k$ in our parametric example, marginally increases from zero. To this end, we first compute the following derivatives of (28) with respect to $\gamma_s$:

$$\frac{\partial \gamma(k)}{\partial \gamma_s} = -k(\gamma_0 - \gamma_\infty)\exp(-\gamma_s k) < 0,$$

and

$$\frac{\partial \gamma'(k)}{\partial \gamma_s} = (\gamma_s k - 1)(\gamma_0 - \gamma_\infty)\exp(-\gamma_s k),$$

which is negative at $\gamma_s = 0$.

Second, by applying the implicit function theorem to (22) at $\gamma_s = 0$, we compute

$$\frac{\partial k}{\partial \gamma_s} = -\frac{\left[\rho R v^o u''(\hat{d}) - v^y u''(\hat{c})\right]\left[\frac{\partial \gamma(k)}{\partial \gamma_s}\right] + \left[\rho v^o u'(\hat{d}) + \beta v^y u'(\hat{c})\right]\left[\frac{\partial \gamma'(k)}{\partial \gamma_s}\right]}{h_k}, \tag{54}$$

where we have made use of the fact that $\gamma'(k) = \gamma''(k) = 0$ when $\gamma_s = 0$. The sign of this derivative is ambiguous. While the denominator is positive as $h_k > 0$ by Assumption B, we cannot assess the sign of the numerator without imposing additional conditions. We next show that this numerator is negative if $\epsilon_0$ is sufficiently small. By using (18) at $\beta = \overline{\beta}$ (i.e., when that condition holds with equality), and the fact that $\frac{\partial \gamma(k)}{\partial \gamma_s} = k\left(\frac{\partial \gamma'(k)}{\partial \gamma_s}\right)$ at $\gamma_s = 0$, we rewrite the numerator of (54) as

$$\rho u'(\hat{d})\left(\frac{\partial \gamma'(k)}{\partial \gamma_s}\right)\left\{v^y + v^0 + k\left[\frac{R u''(\hat{d}) v^0}{u'(\hat{d})} - \frac{u''(\hat{c}) v^y}{\beta u'(\hat{c})}\right]\right\}. \tag{55}$$

Let us denote the intertemporal elasticities of substitution in the first and second period by $\sigma(\hat{c}) = -\frac{\hat{c} u''(\hat{c})}{u'(\hat{c})}$ and $\pi(\hat{d}) = -\frac{\hat{d} u''(\hat{d})}{u'(\hat{d})}$, respectively. We then transform (55) as

$$\rho u'(\hat{d})\left(\frac{\partial \gamma'(k)}{\partial \gamma_s}\right)\left\{v^y + v^0 + k\left[-\frac{R\pi(\hat{d}) v^0}{\hat{d}} + \frac{\sigma(\hat{c}) v^y}{\beta \hat{c}}\right]\right\}. \tag{56}$$

Note from Proposition 6 that $b = 0$ and $d = Rk$ at $\beta = \overline{\beta}$. Hence, Expression (56), and so the numerator of (54), can be finally written as

$$\rho u'(\hat{d})\left[\frac{\partial\gamma'(k)}{\partial\gamma_s}\right]\left\{v^y\left[1+\frac{\sigma(\hat{c})dv^y}{R\beta\hat{c}}\right]+v^0\left[1-\frac{d\pi(\hat{d})}{\hat{d}}\right]\right\}. \tag{57}$$

Expression (57) is negative for values of $\gamma_0$ sufficiently small. Note that: (i) under logarithmic preferences $\sigma(\hat{c})=\pi(\hat{d})=1$; and (ii) $d=\hat{d}$ when $\gamma_0=\gamma_s=0$ as this implies that $\gamma(k)=0$. Thus, we conclude by continuity that (57) is negative for sufficiently small values of $\gamma_s$. Therefore, we have proved that $\frac{\partial k}{\partial\gamma_s}$ is positive for sufficiently small values of $\gamma_0$.

Finally, applying the implicit function theorem to (28) at $\gamma_s=0$, we obtain

$$\frac{\partial\overline{\beta}}{\partial\gamma_s}=-\frac{M}{N},$$

with

$$M=\beta f''(k)\left(\frac{\partial k}{\partial\gamma_s}\right)-(v^y+v^o)\left[\frac{\partial\gamma'(k)}{\partial\gamma_s}\right]$$

and

$$N=f'(k)+\beta f''(k)\left(\frac{\partial k}{\partial\beta}\right),$$

where we have again made use of the fact that $\gamma'(k)=\gamma''(k)=0$ when $\gamma_s=0$. The sign of $M$ and $N$ are both ambiguous. However, we first conclude that $M>0$ if $\gamma_\infty$ is sufficiently small because in this case $\frac{\partial\gamma'(k)}{\partial\gamma_s}$ may be sufficiently large in absolute values so that the second term of $M$ may dominate. In addition, we observe from the proof of Proposition 5 that $N$ is equal to $G'(\overline{\beta})$ at $\gamma_s=0$. Hence, we also derive from this proof that $N>0$ around $\overline{\beta}_i$ and $N<0$ around $\overline{\beta}_{i+1}$, with $i$ being an odd integer.

Therefore, we can conclude that for any odd integer $i$, $\frac{\partial\overline{\beta}_i}{\partial\gamma_s}<0$ and $\frac{\partial\overline{\beta}_{i+1}}{\partial\gamma_s}>0$ at $\gamma_s=0$ for values of $\gamma_0$ and $\gamma_\infty$ sufficiently close to zero. From Proposition 5 we can finally state that the marginal introduction of $\gamma_s$ increases the size of the set $B$ determining the values of the altruism factor $\beta$ under which the bequest motive is operative when $\gamma_0$ and $\gamma_\infty$ are both sufficiently small.

# G Conditions for the socially optimal allocation

The Lagrangian function associated to the social planner's problem is given by

$$L(c_t,d_t,k_{t+1})=\hat{U}_0+\sum_{t=0}^{\infty}\lambda_t\beta^t[f(k_t)-c_t-d_t-k_{t+1}],$$

where $\lambda_t$ denotes de Lagrangian multiplier. The respective first-order conditions from maximizing the Lagrangian function with respect to $c_t$, $d_t$ and $k_{t+1}$ are the following:

$$(1-\gamma(k_t)\epsilon^y)\beta^t u'(\hat{c}_t) - \gamma(k_t)\epsilon^o \beta^{t-1}\rho u'(\hat{d}_t) = \beta^t \lambda_t, \tag{58}$$

$$-\gamma(k_t)(1-\epsilon^y)\beta^t u'(\hat{c}_t) + [1-\gamma(k_t)(1-\epsilon^o)]\beta^{t-1}\rho u'(\hat{d}_t) = \beta^t \lambda_t, \tag{59}$$

and

$$\gamma'(k_{t+1})[\beta^{t+1}u'(\hat{c}_{t+1})v^y_{t+1} + \beta^t \rho u'(\hat{d}_{t+1})v^o_{t+1}] = \beta^{t+1}\lambda_{t+1}f'(k_{t+1}) - \beta^t \lambda_t. \tag{60}$$

Combining (58) and (59), we obtain (30). Finally, using (58), (59) and (60), we get (31).

# H Proof of Proposition 9

First, by comparing (30) and (44), we directly obtain the optimal rate of estate taxation. Second, we subtract (31) from (43) to obtain

$$\frac{u'(\hat{c}_t)\gamma(k_t)(\epsilon^y + I_t\epsilon^o)}{\beta u'(\hat{c}_{t+1})f'(k_{t+1})} - \gamma(k_{t+1})(\epsilon^y + I_{t+1}\epsilon^o) = (1-\tilde{\tau}^b_{t+1})(1-\tilde{\tau}^k_{t+1}) - 1.$$

Given the optimal tax rate on bequest, we derive from the previous expression the optimal rate of capital income taxation.

# I Proof of Proposition 10

First, note that $I_t < 1$ if and only if $\theta^y < \theta^o$. Second, from (30) we obtain that

$$\frac{\partial I_t}{\partial k_t} = \frac{2(\theta^y - \theta^o)\gamma'(k_t)}{(1+\theta^y)(1+\theta^o)},$$

which is positive if and only if $\theta^y < \theta^o$. Finally, we will prove that $\Omega_t > 0$ when $k_t < k_{t+1}$. On the one hand, if $\theta^y > \theta^o$ we obtain that the following inequality holds:

$$\gamma(k_t)(\epsilon^y + I_t\epsilon^o) > \gamma(k_{t+1})(\epsilon^y + I_{t+1}\epsilon^o), \tag{61}$$

since $\gamma'(k_t)$, $I_t > 0$ and $\frac{\partial I_t}{\partial k_t} < 0$. Given Inequality (61), we also conclude from (31) that

$$u'(\hat{c}_t) > \beta u'(\hat{c}_{t+1})f'(k_{t+1}). \tag{62}$$

Combining (61) and (62) we directly obtain that $\Omega_t > 0$ if $\theta^y > \theta^o$.

On the other hand, we also prove that (61) and (62) also hold for the case $\theta^y < \theta^o$. We first obtain that

$$\frac{\partial \gamma(k_t)(\epsilon^y + \epsilon^o I_t)}{\partial k_t} < 0,$$

if and only if

$$\frac{2(\theta^y - \theta^o)\gamma'(k_t)}{1+\theta^o} < \frac{(1+\theta^o)^2\left[1-\gamma'(k_t)\right] + \theta^o(1+\theta^y)(1+\theta^o)}{1-\gamma(k_t)+\theta^o(1+\gamma(k_t))}.$$

The last condition always hold when $\theta^y < \theta^o$. In this case (61) and (62) also hold and, therefore, $\Omega_t > 0$.

Since $\Omega_t > 0$ for the whole parameter space when $k_t < k_{t+1}$, the result directly follows.

# J Proof of Proposition 11

When the optimal amount of bequest is negative, the equilibrium path associated to the optimal tax rates is characterized by $b_t = 0$, (14), (41) and (43). From the comparison between (31) and (43), we obtain the following expression:

$$\underbrace{\frac{u'(\hat{c}_t)\gamma(k_t)(\epsilon^y + I_t\epsilon^o)}{\beta u'(\hat{c}_{t+1})f'(k_{t+1})} - \gamma(k_{t+1})(\epsilon^y + I_{t+1}\epsilon^o)}_{\Omega_{t+1}} = (1-\tilde{\tau}^k_{t+1})I_{t+1} - 1.$$

We use (19), (13), (39) and (41) to obtain

$$d_t = f'(k_t)k_t - \tau^o_t. \tag{63}$$

From the comparison between (45) and (63), we obtain that $\tilde{\tau}^o_t = b_t$ and from (37), we get $\tilde{\tau}^y_t = -b_t$ where $b_t$ is defined from expression (45).

**Table 1. Numerical simulations of the steady-state equilibrium**

| | Economies | | |
|---|---|---|---|
| | *Benchmark* | $\beta = 0$ and $\varepsilon_s \neq 0$ | $\beta \neq 0$ and $\varepsilon_s = 0$ |
| $\alpha = 1/3$ : | | | |
| $\overline{k}$ | 0.0505 | 0.0498 | 0.0500 |
| $R_{LP}$ | 2.4398 | 2.4638 | 2.4571 |
| $R_{CP}$ | 0.1258 | 0.1261 | 0.1260 |
| $\overline{\beta}$ | 0.4000 | - | 0.4070 |
| $\alpha = 1/4.5$ : | | | |
| $\overline{k}$ | 0.1538 | 0.1492 | 0.1589 |
| $R_{LP}$ | 0.9530 | 0.9758 | 0.9291 |
| $R_{CP}$ | 0.0986 | 0.0993 | 0.0979 |
| $\overline{\beta}$ | 0.9400 | - | 1.0764 |

Notes: (1) The values for the benchmark economy correspond to the case $\beta = \overline{\beta}$.

(2) $R_{LP}$ is the 35-year rate of return: $R_{LP} = R = f'\left(\overline{k}\right)$.

(3) $R_{CP}$ is the annual rate of return: $R_{CP} = \left[f'\left(\overline{k}\right)\right]^{1/35} - 1 + \delta$, with $\delta = 0.1$.