Title:
THE EUROPEAN POST-WAR RECONSTRUCTION REVISITED

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Abstract

We study the implications of a growth model including social capital and habit formation concerning the post World War II economic experience of European economies. Habits exhibit very low persistence and depend only on last period’s consumption as suggested by empirical evidence. In addition to physical capital, agents invest in social capital which generates both market (production) and non-market (utility) returns. We study an infinite horizon model and compare its implications to a model with habit formation but without social capital. Our framework is more efficient in generating dynamic patterns that replicate the behavior of the main economic variables during the European post-war period. High investment in social capital at the end of the conflict is a key element of our results.

Keywords: economic growth, habit formation, social capital, European economic history

JEL classification codes: O40, O10, E21, D91
1 Introduction

The post-war economic experience of European countries has attracted the attention of the researchers on economic growth due to the particular transition followed by these economies after the destruction of a large part of their physical capital stock. The peculiarity of this recovering transition is illustrated by Figure 1, which presents weighted averages of the output growth rate, the saving rate and the output-capital ratio for five European economies between 1950 and 1985 (France, Germany, Italy, Austria and the Netherlands).\(^1\) The figure presents both the actual data as well as detrended variables using moving averages. The transition is mainly characterized by the following three economic facts. First, the growth rates are initially large and decrease slowly during the adjustment process. Comparing European countries with a control group including US, Canada and Australia, Alvarez-Cuadrado (2008) suggests that the post-war growth rates in Europe have followed a hump-shape with the peak reached several years after the end of the conflict. Papageorgiou and Perez-Sebastian (2006) reports a similar non-monotonic behavior for South Korea and Japan.

![Insert Figure 1](image)

The second main feature of the European recovering transition after World War II is given by the fact that the saving rate also follows a characteristic hump-shaped pattern. It increases monotonically during the first years before reaching its maximum after more than a decade and then slowly decreases. The magnitude of the hump being close to 6%. Maddison (1992) and Antras (2001) find similar evidence for a larger panel of countries while the case of Japan has been documented and discussed by Hayashi (1989) and Christiano (1989).

Finally, the post-war transition in Europe is also remarkable by the non-monotonic behavior of the physical-capital ratio. This aggregate magnitude slowly decreases after the conflict until 1955 before increasing monotonically during the rest of the transition. Christiano (1989) identifies a similar behavior for Japan except that the increase seems to take place during the mid 1960’s. However, to the best of our knowledge, this fact was not widely considered by the literature that documents the post-war experience of European countries.

While the third fact cannot be reconciled with the decreasing returns property of the neoclassical production function, the literature relies on preferences exhibiting non-homotheticity with the introduction of a consumption reference into the utility function to generate the previous first two patterns. This reference can take the form of an exogenous consumption reference (Christiano, 1989; Antras, 2001), consumption externalities (Alvarez-Cuadrado et al., 2004; Alvarez-Cuadrado, 2008) or habit formation (Alvarez-Cuadrado et al., 2004). One

\(^1\)As pointed out by Alvarez-Cuadrado (2008), these five economies are among the ones most affected by the conflict. The data is taken from the Penn World Table.
common feature to all these approaches is the fact that the reference stock is either constant or at least does not adjust immediately. In other words, the reference stock is not only determined by the previous period’s consumption but exhibits some persistence. This time persistence in the consumption reference is a key element since it allows to moderate the growth rate of the reference stock which is necessary in order to replicate accurately the dynamics of the transition. Alvarez-Cuadrado et al. (2004) and Alvarez-Cuadrado (2008) follow the approach of Carroll et al. (1997, 2000) by using a reference stock which is a weighted average of past consumption levels. This generates a reference stock which adjusts slowly in time and combined with a neoclassical production function allows to generate non-monotonic transitional paths. One key element of this approach is the speed at which the reference stock adjusts which determines the relative importance of recent consumption levels.

The previous theory for explaining the observed dynamics of the European economies after World War II has two main drawbacks. On the one hand, the large persistence of the consumption reference required by this theory seems empirically implausible. Concerning more specifically habit formation, while empirical evidence shows that individuals form habits and assess present satisfaction by comparison with standards of living enjoyed in the past (Fuhrer, 2000; Carrasco et al., 2005; Alvarez-Cuadrado et al., 2016), few studies have tried to estimate properly the speed of adjustment of the habit stock. A notable exception is the work of Fuhrer (2000) who finds that the speed of adjustment is very large implying that the reference stock is only determined by last period’s consumption level. Moreover, a large speed of adjustment is also suggested by the equity premium literature (see, e.g., Constantinides, 1990; Boldrin et al., 2001). However, a model without a large habit persistence and a neoclassical production function seems unable to replicate properly the non-monotonic adjustment observed during the post-war era. The model fails by generating growth rates with low persistence and a saving rate that reaches its maximum too early together with a hump that is too large. In addition, with independence of the persistence degree of the consumption reference, the aforementioned theory is unable to replicate the initial decrease in the physical-output ratio and, therefore, its U-shaped dynamic path.

One possible way to improve the model with habits that adjust immediately is to introduce an alternative mechanism that moderates the increase in the habit stock and/or the decrease in the marginal productivity of capital. We borrow the idea of productive consumption by introducing a second consumption good which increases aggregate productivity. However, the intratemporal decision of the individual must have intertemporal consequences in order to affect savings and growth. To this end, we also propose the second consumption good to be durable, i.e., it takes the form of a state variable. We have in mind some intangible goods that have the property of being productive durable consumption goods.

\[\text{See Steger (2002) for a model where the unique consumption good enhances aggregate productivity.}\]
like, for instance, health or social capital. We understand that social capital fits better the historical situation of the considered period because this variable is largely associated to conflicts and aggregate productivity. In particular, empirical evidence suggests that social capital, in any of the definitions considered in the literature, is heavily affected by war and civil conflict since trust, social cohesion and associational membership seem to decrease largely during these events (Becchetti et al., 2013; De Luca and Verpoorten, 2015). This literature also suggests that investment in social capital is large in the years following the conflict implying a fast recovery of the social capital stock. Our model is also in line with this outcome concerning social capital.

Our objective is to show that a model where the stock of habits adjusts immediately combined with social capital accumulation is able to reproduce properly the observed features of the transition during the post-war period. In order to do so, we introduce social capital as a second good into the utility function of the representative agent. We consider that agents accumulate social capital in addition to physical capital. Several definitions of social capital have been proposed in the literature (Putnam et al., 1994; Durlauf and Fafchamps, 2005) but some elements are common to all: the importance of networks, norms and values which characterize social organization and generate externalities at the community level. It is by now acknowledged that social capital shares common features with other types of capital (human and physical) such as its intertemporal dimension and its capacity to generate both benefits and externalities (Agénor and Dinh, 2015). It then seems natural to study the association between social capital and fundamental economic variables. Indeed, social capital seems to be positively correlated with economic growth (Knack and Keefer, 1997; Temple and Johnson, 1998) and can thus influence the behavior of growth and saving rates. Social capital might determine economic outcomes by altering the effective production technology (Fang, 2001), influencing cooperative behavior and facilitating trade (Routledge and Von Amsberg, 2003) or by fostering the accumulation of human capital (Chou, 2006; Bofota et al., 2016).

The presence of a durable good that simultaneously and, more importantly, in a non-rival fashion, enters in the utility and production functions has large consequences on the dynamics of the economy. On one side, the intratemporal allocation between consumption and social capital modifies the intertemporal allocation of consumption. The incentive to accumulate social capital is enhanced by its state variable nature and moderates the increase of the habit stock. On the other side, the rate of return on physical capital is positively affected by social capital which attenuates the neoclassical effect due to decreasing returns. The combination of both mechanisms substitutes the one related to the persistence of the habit stock.

We simulate numerically the model in order to show that such a mechanism is able to reproduce accurately the transition of European countries during the post-war period. More specifically, our numerical results are consistent with
empirical evidence concerning output growth rates, the hump-shaped pattern of the saving rate as well as the initial decrease in the physical capital-output ratio and its subsequent increase. By simulating also the versions of our benchmark model without social capital and with habit persistence, we clearly illustrate the relevance of the proposed mechanisms in replicating the observed dynamics.

The paper is organized as follows. Section 2 presents the model and characterize the competitive equilibrium. Section 3 presents our numerical simulations and compares the results to a model without social capital. We also include in this section a sensitivity analysis concerning initial conditions and habit persistence. Finally, section 5 is devoted to the conclusion.

2 The model

We consider an economy populated by identical and infinitely lived individuals. Population is constant and, therefore, we normalize its size to one. This economy is composed of a unique sector, which produces a homogenous good that consumers use for consumption or to accumulate physical and social capital. The representative agent supplies inelastically one unit of labor at each period and derives utility from consumption $c_t$ as well as from the stock of social capital $e_t$. Consumption is subject to habit formation where the reference stock is only determined by last period’s consumption $c_{t-1}$.

We follow Glaeser et al. (2002) in considering that agents accumulate social capital and that the latter enters directly into the utility function thus generating non-market returns. However, contrary to these authors, we consider that investment in social capital requires units of the final good $m_t$ instead of time allocation. As stated in Bofota et al. (2016), this assumption allows to capture the fact that maintaining social capital might be costly in terms of resources that could be allocated to consumption or physical capital. Another advantage of our formulation is that it does not predict that social capital investment decreases with the wage level contrary to the standard model of Glaeser et al. (2002). In our setup, any private spending that has a social component can be interpreted as social capital investment $m_t$. The accumulation of social capital then proceeds as follows:

$$e_{t+1} = \phi m_t + (1 - \eta)e_t,$$

(1)

where $\eta \in (0, 1)$ is the depreciation rate of social capital and $\phi > 0$ is the scale factor of investing in the accumulation of social capital. Given its intangible characteristics, social capital depreciates quite naturally with time and requires continuous investment efforts. Moreover, social capital is partially community specific implying that when individuals leave their neighborhoods, a part of their accumulated social capital is lost in the process. The mobility of individuals can thus be seen as one of the reasons justifying the introduction of a depreciation term concerning social capital.
The utility that agents derive from social capital also depends on the average social capital in the economy. The idea being that it is the interaction with other individuals that have accumulated social capital that generates well-being to the agent. For example, it is only worth to be member of a specific club if the latter is sufficiently large. Moreover, we consider that social capital can also generate jealousy and envy effects and introduce aspirations in the latter.

In order to take into account the different elements exposed before, we propose the following utility function:

$$U = \sum_{t=0}^{\infty} \beta^t \{ \theta \ln(c_t - \rho_c c_{t-1}) + (1 - \theta) \ln[\varepsilon_t G(\overline{e}_t) - \rho_e \overline{e}_{t-1} G(\overline{e}_{t-1})] \},$$  \hspace{1cm} (2)

where $\theta \in (0, 1)$ is the relative preference for individual consumption and $G(\overline{e}_t)$ is the social capital multiplier which depends on average social capital $\overline{e}_t$. The function $G(\cdot)$ is taken from Glaeser et al. (2002) and captures the importance of interacting with other individuals that have accumulated social capital. We assume that $G(0) = 0$, $G'(\overline{e}_t) > 0$ and the elasticity of $G(\cdot)$ defined as $\varepsilon_G$ is a constant. These assumptions imply that average social capital increases the non-market returns to social capital investment. The parameter governing the intensity of habits in consumption is given by $\rho_c \in (0, 1)$ while the one governing the intensity of aspirations in social capital is given by $\rho_e \in (0, 1)$. It is worth noticing that aspirations concern not only the past average level of social capital but also its interaction with the past social capital multiplier.

The representative consumer faces the following budget constraint:

$$k_{t+1} = w_t + (1 + r_t)k_t - c_t - m_t,$$  \hspace{1cm} (3)

where $r_t$ is the rate of return on physical capital and $w_t$ is the wage rate.

Production takes place through a representative firm which produces the homogenous good with a Cobb-Douglas production function. However, we assume that total factor productivity is an increasing function of the average stock of social capital. We suppose that an economy with more networks and trust among workers is also more productive. The production function takes the following form:

$$f(k_t, \overline{e}_t) = A(\overline{e}_t)k_t^\alpha,$$  \hspace{1cm} (4)

where $\alpha \in (0, 1)$ is the share of physical capital in the production process and $A(\overline{e}_t)$ is total factor productivity which is a function of average social capital. We assume that $A(0) = 0$, $A'(\overline{e}_t) > 0$ and the elasticity of $A(\cdot)$ defined as $\varepsilon_A$ is a constant. We assume perfect competition implying:

$$r_t = \alpha A(\overline{e}_t)k_t^{\alpha - 1} - \delta,$$  \hspace{1cm} (5)

and

$$w_t = (1 - \alpha)A(\overline{e}_t)k_t^\alpha,$$  \hspace{1cm} (6)
where $\delta \in (0, 1)$ is the depreciation rate of physical capital.

The representative consumer faces the problem of choosing consumption $c_t$, investment in social capital $m_t$ and investment in physical capital to maximize (2) subject to (1) and (3), by taking as given $w_t$, $r_t$, $\tau_t$ for all $t$ and initial conditions $k_0, e_0, c_{-1}$. This is a standard dynamic optimization problem with control variables $c_t$ and $m_t$ and state variables $k_t$ and $e_t$. By following the standard procedure, we find in Appendix A the first order conditions and rearrange the expressions to summarize the necessary conditions for optimality by the dynamic system composed of the difference equations (1), (3),

$$\frac{1}{c_t - \rho_c c_{t-1}} - \frac{\beta \rho_c}{c_{t+1} - \rho_c c_t} = \left( \frac{\beta}{c_{t+1} - \rho_c c_t} - \frac{\beta^2 \rho_c}{c_{t+2} - \rho_c c_{t+1}} \right) \left[ 1 + \alpha A(e_{t+1}) k_t^{\alpha-1} - \delta \right], \quad (7)$$

and

$$\left( \frac{\theta}{1 - \rho_e} \right) \left[ \frac{1}{c_t - \rho_c c_{t-1}} - \frac{\beta(1 - \eta + \rho_e)}{c_{t+1} - \rho_c c_t} + \frac{\beta^2 \rho_e (1 - \eta)}{c_{t+2} - \rho_c c_{t+1}} \right] = \frac{\beta \phi G(e_{t+1})}{e_{t+1} G(e_{t+1}) - \rho_e e_{t} G(e_{t})}, \quad (8)$$

together with conditions (5) and (6).

Equation (7) describes the intertemporal allocation of consumption and takes into account habit formation. This intertemporal condition is standard except for the presence of average social capital as a determinant of total factor productivity. The accumulation of social capital will increase the rate of return and moderate the decrease of the latter due to physical capital accumulation. This production externality will be key in generating larger growth rates, a decreasing physical capital-output ratio at the beginning of the transition and a saving rate that reaches its maximum value later than in a model without social capital.

Equation (8) describes the intratemporal allocation between consumption and investment in social capital, taking into account habit formation and the impact of the social capital multiplier. This intratemporal condition will also play a key role in our results and requires some further comments. First, it is interesting to notice that the presence of aspirations in social capital implies that the social capital multiplier influences the dynamic path of the economy. If $\rho_e = 0$, expression (8) does not depend on the social capital multiplier such that the intratemporal allocation between consumption and social capital is independent of the average social capital in the economy.

Second, and most importantly, the intratemporal allocation is influenced by the fact that social capital is a state-variable good that enters the utility function. We can rewrite expression (8) as

$$U_{c_t} - \beta (1 - \eta) U_{c_{t+1}} = \beta U_{e_{t+1}}, \quad (9)$$

where $U_x$ represents the marginal utility of variable $x$. Since social capital is a state variable good, the latter will have an impact on the intertemporal allocation due to the presence of the term $U_{c_{t+1}}$. If $\eta = 1$, the marginal rate of substitution between social capital and consumption is a constant while it becomes variable.
when \( \eta \neq 1 \). In the present case, the marginal rate of substitution is always smaller than in a model where social capital is not a state variable and this translates into a larger social capital consumption ratio in our framework. The state variable nature of social capital induces the agent to invest relatively more in the latter at the expense of consumption and moderates in turn the future stock of habits. This outcome will be important in adjusting the hump of the saving rate by not allowing habits to grow too fast as in the standard model without social capital. The depreciation rate of social capital \( \eta \) plays an important role in our results and an exogenous increase in the latter (other things being equal) induces a substitution from social capital towards consumption. As explained before, at least some part of the depreciation can be interpreted as a consequence of individuals’ mobility. The model predicts that an increase in mobility tends to decrease investment in social capital which is in line with empirical evidence provided by DiPasquale and Glaeser (1999) highlighting the fact that homeowners tend to invest more in social capital.

We can get further insights on the working of the model by combining expressions (7) and (8) in order to obtain the marginal rate of substitution between future consumption and future social capital:

\[
\frac{U_{e_{t+1}}}{U_{c_{t+1}}} = \alpha A (e_{t+1}) k_{t+1}^{\alpha - 1} - \delta + \eta. \tag{10}
\]

When the marginal productivity of physical capital is large, the agent allocates relatively more resources towards savings since future consumption is the most efficient way to generate utility gains. However, as the marginal productivity of physical capital decreases, the agent tends to increase social capital investment at the expense of physical capital accumulation. The change in the relative allocation of both types of capital provides a potential explanation concerning the hump-shaped pattern of the saving rate observed in the data.

We next focus on the existence of a balanced growth path (BGP) equilibrium for this economy. In the present case, a BGP equilibrium is a path along which for any endogenous variable \( x \), we have \( x_{t+1} = g_x x_t \) where \( g_x \) is a constant. When \( g_x = 1 \), we denote the latter as a steady-state. By log-differentiating the production function (4), the budget constraint (3) and the law of social capital accumulation (1), we obtain that along a BGP and a steady-state equilibrium the following conditions hold:

\[
g_m = g_e,
\]

\[
g_k = \left( \frac{\epsilon A}{1 - \alpha} \right) g_e,
\]

and

\[
g_c = \left[ \frac{\epsilon A}{(1 - \alpha) s} - \frac{1 - s}{s} \right] g_e,
\]

where \( s \) is the fraction of total expenditure devoted to consumption along a BGP or a steady-state equilibrium. The next proposition focuses on the existence and
stability of the BGP equilibria of our competitive economy.

**Proposition 1:**

(a) If $\epsilon_A + \alpha < 1$, there is a unique positive steady-state which is locally stable.

(b) If $\epsilon_A + \alpha > 1$, there is a unique positive steady-state which is unstable.

(c) If $\epsilon_A + \alpha = 1$, there is a unique BGP equilibrium where $g_e > 1$ or $g_e < 1$.

**Proof.** See Appendix B

From now on we will focus on the case where $\epsilon_A + \alpha < 1$ guaranteeing the existence of a unique positive and locally stable steady-state. The cases where $\epsilon_A + \alpha \geq 1$ are ruled out for two reasons. First, in the case where $\epsilon_A + \alpha > 1$, the unique positive steady-state is unstable. Second, these cases require an elasticity of social capital in the production function that is too large given the available empirical evidence.

**Assumption 1:** $\epsilon_A + \alpha < 1$.

The steady-state of our competitive economy when Assumption 1 holds is then characterized by the following set of equations:

$$\beta \left[ 1 + \alpha A(e)k^{\alpha-1} - \delta \right] = 1,$$

$$\frac{\theta(1 - \beta \rho_e)(1 - \beta(1 - \eta))}{(1 - \theta)(1 - \rho_c)} = \frac{\beta \phi}{(1 - \rho_e)c},$$

$$A(e)k^\alpha - \delta k = c + m,$$

and

$$e\eta = \phi m.$$
expression (12) defines the optimal consumption-social capital ratio. By combining the steady-state equations we can also explicitly derive the steady-state physical-social capital ratio:

$$\frac{k}{c} = \frac{\alpha \{\theta (1 - \beta \rho_c) [1 - \beta (1 - \eta)] (1 - \rho_e) + (1 - \theta) \beta (1 - \rho_e) \eta\} \phi (1 - \theta) (1 - \rho_c) \{1 - \beta [1 - \delta (1 - \alpha)]\}}{\phi (1 - \theta) (1 - \rho_c) \{1 - \beta [1 - \delta (1 - \alpha)]\}}.$$

An interesting feature of our model is that the steady-state allocation depends on the habit parameter $\rho_e$ which is not the case in a model without social capital. An exogenous increase in the habit formation parameter implies in turn an increase in the physical-social capital ratio. When the habit formation parameter increases, the representative agent decides to invest relatively more in physical capital in order to satisfy future habits. This effect is present even in the long-run contrary to a model without social capital where the steady-state physical capital stock is independent of the habit formation parameter.

3 Numerical analysis

Our objective in this section is to illustrate how our mechanism for explaining the dynamic reconstruction after the conflict operates. To this end, we simulate numerically our theoretical model and we also compare its implications to a model without social capital but with a similar structure concerning habit formation. In the latter case, given $k_0 > 0$ and $c_{-1} > 0$, the economy is fully described by expressions (7) and (3), where $\epsilon_A = m_t = 0$. We will show that contrary to the latter, our framework is able to replicate accurately the dynamic transition of European countries during the post-war period.

To proceed with our numerical analysis, we first need to choose specific functional forms concerning social capital externalities in production and utility.

**Assumption 2:**

(a) The social capital externality in production takes the following form: $A(e_t) = A e_t^{\gamma}$ where $\gamma \in (0, 1)$.

(b) The social capital externality in utility takes the following form: $G(e_t) = e_t^{\sigma}$ where $\sigma \in (0, 1)$.

Under Assumption 2, the elasticity of total factor productivity with respect to social capital is $\epsilon_A = \gamma$, whereas the elasticity of the social capital multiplier in preferences with respect to social capital reduces to $\epsilon_G = \sigma$.

3.1 Calibration

The parameters are calibrated in order to reproduce some of the empirical facts of developed economies. Table 1 describes the benchmark values of the parameters.
that we use in our numerical simulations. First, we fix the parameters that are common in the two economies that we consider: the benchmark economy and the economy without social capital but with habit formation (henceforth, one good economy). The scaling parameter $\Lambda$ governing total factor productivity is arbitrarily set a 3. Following the RBC literature, the discount factor $\beta = 0.96$, the depreciation rate of the physical capital stock $\delta = 0.1$ and the elasticity of physical capital in the production function $\alpha = 0.33$ are chosen in order to obtain a steady-state physical capital-output ratio of 2.3, a steady-state saving rate equal to 23%, and a labor income share in GDP of 67%. The stationary values of these aggregate figures neither depend on the parameters governing the process of habit formation nor on those parameters determining the influence of social capital. Therefore, the values of the previous parameters are the same in all the economies that we will consider in order to obtain comparable results.

Second, we choose the parameter $\rho_c$ governing the process of habit formation. In particular, we set a value of 0.8 which is the one used by Constantinides (1990) and Boldrin et al. (2001) in their respective works on asset pricing. Moreover, this value is in line with empirical evidence provided by Fuhrer (2000) and Fuhrer and Klein (2006). We use this particular value in the two economies we consider: the benchmark economy and the one good economy without social capital.

Finally, we calibrate the parameters that determine the accumulation and the economic influence of social capital: $\theta$, $\gamma$, $\rho_e$, $\sigma$, $\phi$ and $\eta$. We start by discussing the values of the parameters driving the role of social capital in the utility function. The respective weights of consumption and social capital in the utility function are not straightforward to choose since few papers have focused on a model with both goods. In our benchmark case, we set $\theta = 0.6$ which implies that consumption is slightly more important than social capital for the representative agent. The parameter governing aspirations in social capital $\rho_e$ is set at 0.3 which is the value estimated by Alvarez-Cuadrado et al. (2016) concerning consumption envy across individuals. This value is also in line with the estimates provided by Maurer and Meier (2008) as well as with the ones from the experimental literature (see Alpizar et al., 2005). To our knowledge, the parameter governing the magnitude of the social capital externality in utility $\sigma$ is one for which there are no available estimates. Moreover, since the latter does not affect the steady-state outcome, we cannot derive a possible value by using the steady-state equations. We then choose to set a moderate value equal to 0.2.

<table>
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<td>$A$</td>
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<td>$\beta$</td>
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<td>$\eta$</td>
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<tr>
<td>$\sigma$</td>
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</tbody>
</table>

Table 1: Values for the parameters
One of the key parameters governing the dynamics of the economy is $\gamma$ which controls the magnitude of the production externality. Due to the difficulty to define properly social capital, there has been few attempts to estimate the magnitude of social capital externalities in production. Using social development indexes constructed in the early 1960’s, Temple and Johnson (1998) estimate that differences in such indexes can explain between 10 and 40% of the variation in growth rates for developing countries. In addition, the authors find that a part of the impact acts via total factor productivity. Fafchamps and Minten (2002) use data on agricultural traders in Madagascar to estimate the impact of social network capital on productivity and find that the effect is large. For example, a doubling of the number of know traders and potential lenders raises gross margins by 18-22%. Using an augmented-Solow model, Ishise and Sawada (2009) try to estimate the aggregate returns to social capital and obtain values between 10% and 20%. In order to estimate the latter, the authors focus mainly on social connectivity and information sharing. While these are important steps forward, we think that our production externality includes other positive effects of social capital such as the accumulation of human capital or the signaling device used to recruit specific workers. If these elements are important, it is probable that the aggregate returns to social capital are relatively large and we set $\gamma = 0.35$.

We finally need to set the parameters governing the law of motion of social capital. We know that given its intangible nature and the mobility of individuals, the depreciation rate of social capital is probably larger than the one of physical capital. In addition, the European post-war period was characterized by large internal migrations inducing probably important depreciation rates of social capital. Here, we refer to internal migrations due to institutional and technological changes that occurred during the post-war period and not to migrations directly due to the conflict itself. The migration cases of France and Germany have been documented by Saint-Paul (1993) and Giersch et al. (1993) respectively. We then choose to set $\eta = 0.4$. In order to get an idea of what this choice represents, we compute the half-life with which the stock of social capital would adjust to a permanent change in social capital investment. Suppose that the representative agent reduces permanently social capital investment to zero, half of the social capital stock would be depleted in roughly one year and four months.\(^3\) Finally, the parameter governing the impact of investment in social capital $\phi$ is set at 0.9.

### 3.2 Initial conditions

We need to fix the initial values for our three state variables: the physical capital stock, the social capital stock as well as the consumption reference. For most European countries, World War II was responsible for a large destruction of the physical capital stock. The available evidence for war economies suggests a loss

\(^3\)Note that the half-life convergence is given by $t_{1/2} = -\ln2/\ln(1-\eta) = 1.35$ if we set $\eta = 0.4$. 

13
between 30% and 90% of the pre-war physical capital stock depending on the calculation method (Alvarez-Cuadrado, 2008). We then choose an initial value corresponding to 50% of the steady-state physical capital stock as in Alvarez-Cuadrado (2008).

Concerning social capital, as pointed in the introduction, empirical evidence suggests that the latter is also heavily affected by war and civil conflict (Becchetti et al., 2013; De Luca and Verpoorten, 2015). We suppose that the loss is similar to the one experienced by physical capital and fix an initial value corresponding to 50% of the steady-state social capital stock.

Finally, the initial reference for consumption habits is not easy to determine due to the lack of available data concerning consumption levels during the war. We know that the reference must be relatively large in order to generate an increasing saving rate at the beginning of the transition. In order to compare the model with and without social capital we choose to set an initial habit stock corresponding to 80% of its steady-state value. This is slightly below the one used by Alvarez-Cuadrado (2008) who estimates an initial habit stock corresponding to 90% of its steady-state value.

### 3.3 Comparison of both economies

In this section, we simulate the dynamic paths of two models. The first one labelled the one good economy is an economy without social capital but with habit formation. The second one labelled the benchmark economy is our economy with both social capital and habit formation. As was mentioned before the two economies were calibrated in the same way in order to be comparable implying that they exhibit the same steady-state saving rate and physical capital-output ratio. Since we do not focus on long term growth, the steady-state growth rate is zero in both economies.

The results of our simulations are presented in Figure 2. The growth rate of output in our economy with social capital follows a hump-shaped pattern while the one of the one good economy is at first larger and monotonically decreasing. Moreover, growth is more persistent in the model with social capital since the latter is still positive after 25 periods contrary to the model without social capital. Our result is in line with empirical evidence showing that the growth rates peaked several years after the end of the conflict. The difference between both economies is the presence of social capital externalities in production which increase total factor productivity in the benchmark economy. The maximum growth rate generated by the model takes values slightly above 2%. While it is clear that at the beginning of the post-war transition most countries experienced per capita growth rates that are larger than 2%, our model only takes into account the impact of physical capital deepening and social capital externalities. It is evident that other elements affecting long run growth are not taken into account in the current framework. For example, if we follow the approach of Alvarez-Cuadrado
(2008) and introduce additionally an exogenous TFP growth of 2%, our output growth rate is more in line with empirical estimates.

Concerning the physical capital-output ratio, the one good economy exhibits a monotonically increasing ratio as all models based uniquely on consumption references. Our benchmark economy is able to generate a slightly decreasing physical capital-output ratio at the beginning of the transition as suggested by empirical evidence. After this initial decrease, the ratio increases monotonically towards the steady-state. This initial behavior is due to a large accumulation of social capital at the beginning of the transition implying that output grows faster than physical capital. However, this behavior is only temporary and after a few periods, physical capital starts to grow faster than output as in an economy without social capital.

Finally, the behavior of the saving rate is clearly different in both models. The transition in our benchmark economy is characterized by a smaller hump and a maximum value that is reached later in time. The magnitude of the hump in our economy with social capital takes a value around 6% which seems in accordance with empirical evidence (see Figure 1). As explained before, the incentive to accumulate social capital modifies the intertemporal allocation of consumption by moderating the increase in the habit stock. The magnitude of the hump is a direct consequence of the latter since when the habit stock grows faster, the agent needs a larger increase in savings in order to be able to satisfy future consumption references. The saving rate of our model reaches its maximum value later due to the impact of social capital on the rate of return. In both models, at some point during the transition, the neoclassical effect due to a decreasing marginal productivity of physical capital starts to dominate the habit formation effect inducing a decrease in the saving rate. The presence of social capital in the production function delays this outcome and the neoclassical effect only starts to dominate after more than a decade as suggested by empirical evidence.

It is also interesting to focus on the growth rate of social capital. As explained in the introduction, while war and civic conflict affect heavily social capital, empirical evidence suggests that the latter recovers relatively fast after the end of the conflict. Our simulation results deliver a similar outcome with large growth rates concerning social capital at the beginning of the transition. These large growth rates play several roles in our framework: they imply larger output growth rates, delay the decrease in the marginal productivity of physical capital and are the reason why the physical capital-output ratio decreases at the beginning of the transition. This indicates that a different input that is not affected so much by the conflict might not be able to play such a role. For example, human capital which was not subject to destruction levels equivalent to those of physical capital (see for example, Crafts and Toniolo, 1996; Harrison, 2000) might not be able to fulfill this task.
A part of our results are driven by the impact of social capital on aggregate productivity which derives directly from the elasticity of social capital in the production function $\gamma$. The value taken by the gross rate of return in our simulation exercise is a good way to evaluate if this choice is reasonable. In the benchmark economy, the gross rate of return reaches a maximum value around 18% after a few periods before declining monotonically towards its steady-state value of 14%. King and Rebelo (1993) argue that using a standard neoclassical growth model in order to explain the post-war reconstruction would imply extremely high and unrealistic real interest rates. The introduction of habit formation and social capital in our framework allows to reduce the value of real interest rates at the beginning of the transition. However, a sufficiently large value of $\gamma$ is also required in order to generate initial real interest rates that are not too small.

A last method to evaluate the capacity of our framework in replicating accurately the dynamic transition following the conflict is to focus on the speed of convergence of the model. On the empirical side, Islam (1995) and Caselli et al. (1996) estimate that the speed of convergence is close to 9%. We have shown that the benchmark model is characterized by local-stability and the same applies to the one good economy. In our benchmark economy, there are three state variables implying that the speed of convergence of any endogenous variable is a weighted function of the three eigenvalues inside the unit circle. The numerical values of these three stable eigenvalues are given in Table 2. Over time, the weight of the smaller eigenvalues declines so that the largest one inside the unit circle describes the asymptotic value of the speed of convergence. In our benchmark case, the largest eigenvalue inside the unit circle is equal to 0.9252 implying an asymptotic speed of convergence equal to 8.3% (since $\ln(0.9252) = -0.083$), which is in line with empirical evidence. In the one good economy, there are two state variables and the eigenvalues inside the unit circle come in conjugate pairs. In such a case, an appropriate measure of the speed of convergence can be obtained through the modulus of the root which is equal to 0.8443. This in turn implies a speed of convergence equal to 17% which is way above any empirical estimate.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>One good economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2966</td>
<td>$0.8418 + 0.065$</td>
</tr>
<tr>
<td>0.8238</td>
<td>$0.8418 - 0.065$</td>
</tr>
<tr>
<td>0.9252</td>
<td></td>
</tr>
</tbody>
</table>

The introduction of social capital in our model tends to moderate the increase in the habit stock and the decrease in the marginal productivity of physical capital. This additional sluggishness explains why the economy tends to converge at a smaller speed which is much more in line with empirical evidence. Our results
also suggest that our parametrization is fairly consistent with empirical evidence despite the lack of data concerning the parameters related to social capital.

The importance of social capital in our framework leads to the question of the correlation between the latter, growth and the saving rate. Our model is in line with the one of Carroll et al. (2000) which suggests that large growth rates imply in turn large saving rates. The fact that the output growth rate reaches its maximum value before the saving rate and that both variables behave qualitatively in a similar way suggests that this is indeed the case. A related and important question is whether or not fast social capital accumulation implies large growth rates and by extension large saving rates. While we did not proceed to a formal statistical test, our simulations seem to suggest that is this indeed the case with a decrease in social capital growth that precedes the one of output.

3.4 Sensitivity analysis

In this section, we will proceed with two types of sensibility analysis. The first one will focus on the importance of initial conditions for the recovery dynamics while the second will introduce habit persistence into our benchmark model.

3.4.1 Importance of initial conditions

We focus first on the importance of the initial physical capital to social capital ratio concerning the dynamic behavior of our economy. Up to now, we have assumed that both state variables suffered an equivalent destruction during the conflict. In this section, we explore the possibility that social capital might be more affected by war than physical capital. Contrary to human capital which is intrinsic to individuals, social capital depends on the local environment and decreases with the mobility of individuals. In case of an armed conflict, the local environment might be largely modified while individuals might lose part of their social by becoming war refugees. These observations justify the need to explore the consequences of an initial physical capital-social capital ratio larger than in the benchmark case. In order to illustrate this point we set the initial condition for physical capital at a value corresponding to 52% of the steady-state value while the one for social capital is set at 48% of its steady-state value. The results of the numerical simulation are presented in Figure 3 together with the dynamic paths of the benchmark case.

Intuitively, the initial physical capital-output ratio is larger in this economy and the following decrease in the latter is sharper than in the benchmark case. This is related to the faster accumulation of social capital in the present version of the model compared to the benchmark case. The result is reminiscent of the two-sector model literature with both physical and human capital showing that the
relatively scarce input will be accumulated faster during the transition toward the steady-state (Lucas, 1988; Caballé and Santos, 1993). However, our framework differs in the sense that the accumulation of social capital is not directly related to the marginal productivity of the latter (which is an externality in the present model) but depends on the marginal rate of substitution between consumption and social capital investment.

Another important change with respect to the benchmark version is that the initial saving rate is smaller and thus generates a hump that is too large. There seems to be a trade-off between obtaining a sharper decrease in the physical capital-output ratio and a saving rate that is able to replicate accurately the magnitude of the hump. This is not too surprising since the saving rate is by definition decreasing in the physical capital-output ratio.

3.4.2 The model with habit persistence

In this subsection, we introduce habit persistence into our framework in order to study the influence of a low degree of habit persistence on our results. The utility function now becomes

$$U = \sum_{t=0}^{\infty} \beta^t \{ \theta \ln(c_t - \rho_c z_t) + (1 - \theta) \ln[e_t G(\tau_t) - \rho_c \tau_{t-1} G(\tau_{t-1})]\}, \quad (15)$$

where $z_t$ represents the habit reference stock and the dynamics of the latter are given by

$$z_t = \rho z_{t-1} + (1 - \rho)c_{t-1}, \quad (16)$$

with $\rho \in [0, \infty)$. The reference stock in $t$ is a combination between the reference stock and consumption in $t-1$. When $\rho$ is small, persistence is low and the reference is mainly built from last period’s consumption experience. To solve the consumer’s optimization problem of the representative consumer in this case, we can iterate backward the habit stock $z_t$ by using (16), so that we can rewrite the utility function (15) as:

$$U = \sum_{t=0}^{\infty} \beta^t \left\{ \theta \ln(c_t - \rho_c (1 - \rho) \sum_{i=1}^{\infty} \rho^{i-1} c_{t-i}) \right. + (1 - \theta) \ln[e_t G(\tau_t) - \rho_c \tau_{t-1} G(\tau_{t-1})]\right\},$$

and maximize the latter subject to the budget constraint (3) and the law of social capital accumulation (1).

We first compute the marginal utilities of consumption in $t$ and $t + 1$:

$$U_{c_t} = \frac{\theta}{c_t - \rho_c z_t} - \sum_{i=1}^{\infty} \beta^i \theta \rho_c (1 - \rho) \rho^{i-1} c_{t+i} - \rho_c z_{t+i},$$
and

\[ U_{ct+1} = \frac{\theta}{e_{t+1} - \rho c z_{t+1}} - \sum_{i=1}^{\infty} \frac{\beta^i \theta \rho c (1 - \rho) \rho^{i-1}}{e_{t+1+i} - \rho c z_{t+1+i}}. \]

Given initial conditions, \( k_0, e_0 \) and \( z_0 > 0 \), the competitive equilibrium of the economy with habit persistence (i.e., \( \rho \neq 0 \)) is fully described by the dynamic system composed of difference equations (1), (3), (16)

\[ U_{ct} = \beta \left( 1 + \alpha A(e_{t+1}) k_{t+1}^{\alpha-1} - \delta \right) U_{ct+1}, \tag{17} \]

and

\[ U_{ct} = \frac{\beta (1 - \theta) \phi G(e_{t+1})}{e_{t+1} G(e_{t+1}) - \rho c e_t G(e_t)} + \beta (1 - \eta) U_{ct+1}. \tag{18} \]

The steady state is characterized by the following set of equations: \( z = c, e\eta = \phi m, \)

\[ \beta \left[ 1 + \alpha A(e) k^{\alpha-1} - \delta \right] = 1, \]

\[ \frac{\theta [1 - \beta (1 - b)] [1 - \rho c (1 - \rho) \sum_{i=1}^{\infty} \beta^i \rho^{i-1}]}{(1 - \theta)(1 - \rho c)} = \frac{\beta \phi}{(1 - \rho c) e}, \]

and

\[ A(e) k^{\alpha} - \delta k = c + m. \]

At the steady-state, the main difference between the two models concerns the consumption-social capital ratio since the latter depends explicitly on the habit persistence parameter. Comparing the intratemporal allocation in both models we can conclude that the consumption-social capital ratio is larger in the model with habit persistence if and only if

\[ (1 - \rho) \sum_{i=1}^{\infty} (\beta \rho)^{i-1} < 1, \]

or equivalently

\[ \frac{1 - \rho}{1 - \beta \rho} < 1, \]

19
which is always the case since $\beta < 1$. This implies as well a larger physical-social capital ratio in the economy with habit persistence. Given the minor changes implied by habit persistence, it is straightforward to show that there is once again a unique positive steady-state in this case.

In order to simulate this new model, we keep the same values for the parameters but in addition we need to fix the value of the habit persistence parameter $\rho$. Using quarterly data, Fuhrer (2000) estimates that $\rho$ takes values between 0.0015 and 0.052 depending on the estimation method. We will consider two possible values for $\rho$ that maintain a low level of persistence: 0.05 and 0.1. In order to get an idea of what this value represents, we follow the approach of Carroll et al. (2000) and compute the half-life with which habits would adjust toward a permanent change in consumption. When $\rho = 0.05$, this is approximately 3 months (i.e., $t_{1/2} = -\ln 2/\ln \rho = 0.23$), while when $\rho = 0.1$ this is approximately 4 months.

The results of our simulations are presented in Figure 4 which includes the dynamic paths for the three economies that only differ in terms of the habit persistence parameter. The major changes concern the physical capital-output ratio and the saving rate. On one hand, the initial decrease of the physical capital-output ratio observed in the benchmark case tends to disappear when we increase the habit persistence parameter. On the other hand, the initial saving rate tends to increase when we introduce habit persistence in the model. With habit persistence the negative impact of an increase in current consumption on intertemporal utility is larger since the effect is delayed in time and the agent prefers to increase savings at the beginning of the transition. We can finally notice that the maximum growth rates of output and social capital are smaller than in the benchmark case. Given these numerical results, we tend to conclude that including habit persistence in the present model does not seem to increase the accuracy of the recovery dynamics.

Insert Figure 4

We finally compare our benchmark case to a model without social capital but with a degree of persistence in habits sufficiently large to reproduce the stylized facts of the post-war transition. In this case, given $k_0, z_0 > 0$, the economy is fully described by expressions (3), (16) and (17) with $\epsilon_A = m_t = 0$. We follow Alvarez-Cuadrado (2008) and set $\rho = 0.7$ implying a half-life close to two years.\footnote{Alvarez-Cuadrado (2008) focuses on a continuous time framework and sets $\rho = 0.35$. The latter being equivalent to $\rho = 0.7$ in discrete time.} Figure 5 presents the dynamic paths of our key variables for both economies.

Insert Figure 5

Once again, the absence of social capital in the one good economy implies that the growth rate of output is less persistent in this case. Concerning the
saving rate, the one good economy exhibits an initial saving rate that is larger since the representative agent does not need to invest resources in social capital. However, the magnitude of the hump is relatively small compared to the data with a value around 2%. Moreover, the absence of social capital externalities in production implies that the impact of a decreasing marginal productivity of physical capital starts to dominate sooner and the decrease in the saving rate is sharper than in the benchmark case. Finally, due to the absence of social capital, the physical capital-output ratio is monotonically increasing in the one good economy. Moreover, the pattern taken by the latter seems to be in contradiction with the data exposed in Figure 1.

Comparing both frameworks, we would tend to conclude that the model with social capital is more efficient in replicating accurately the post-war reconstruction dynamics of European countries.

4 Conclusion

In this paper, we have studied the dynamic transition of economies which suffer from an important negative capital shock. In order to do so we have focused on the post World War II transition of European countries which has been largely documented empirically. Our theoretical model includes habit formation without persistence, the accumulation of social capital in addition to physical capital as well as social capital externalities in production. While a model relying solely on habits that depend on last period’s consumption is unable to replicate accurately the non-monotonic transitional paths that characterize the post-war era, our model with social capital provides an alternative to models with habit persistence. The latter models require a high degree of persistence which seems implausible given the available empirical evidence.

Our framework is able to replicate accurately the transition concerning the output growth rate, the saving rate as well as the physical capital-output ratio. Our results are driven by the fact that social capital is a state variable consumption good whose intratemporal allocation (relative to the standard consumption good) has intertemporal consequences and by the fact that the rate of return is a increasing function of the social capital stock. The destruction of social capital at levels equivalent to those of physical capital is key for our results and suggests that massive investment in social capital following the conflict might be one of the determinants of high economic growth during the post-war era.

While in the present model this large investment in social capital is entirely determined by private incentives, it seems clear that governments also play a key role in fostering social capital accumulation by implementing an institutional framework which guarantees trust and civic norms. For example, the enforcement of property and contract rights seems in this respect fundamental. In addition, we only have presented social capital as a productive consumption good but the
latter could also play a role as an input in the accumulation of other types of capital such as human or financial capital. In such a case, social capital might also be a determinant of both growth and income inequality.

References


23


**Appendix**

**A The problem of the representative agent**

The Lagrangian function is the following:

\[
L = \sum_{t=0}^{\infty} \beta^t \{ \theta \ln(c_t - \rho_c c_{t-1}) + (1 - \theta) \ln[e_t G(\bar{e}_t) - \rho_c \bar{e}_{t-1} G(\bar{e}_{t-1})] \} \\
+ \sum_{t=0}^{\infty} \beta^t \lambda_t [w_t + (1 + r_t)k_t - c_t - m_t - k_{t+1}] \\
+ \sum_{t=0}^{\infty} \beta^t \mu_t [(1 - \eta)e_t + \phi m_t - e_{t+1}].
\]

The first order conditions of the maximization problem are given by

\[
\frac{\partial L}{\partial c_t} = \frac{\beta^t \theta}{c_t - \rho_c c_{t-1}} - \frac{\beta^{t+1} \theta \rho_c}{c_{t+1} - \rho_c c_t} - \beta^t \lambda_t = 0,
\]

\[
\frac{\partial L}{\partial m_t} = -\beta^t \lambda_t + \phi \beta^t \mu_t = 0,
\]

\[
\frac{\partial L}{\partial k_{t+1}} = \beta^{t+1} \lambda_{t+1} (1 + r_{t+1}) - \beta^t \lambda_t = 0,
\]

and

\[
\frac{\partial L}{\partial e_{t+1}} = \frac{\beta^{t+1} (1 - \theta) G(e_{t+1})}{e_{t+1} G(\bar{e}_{t+1}) - \rho_c e_t G(\bar{e}_t)} + \beta^{t+1} \mu_{t+1} (1 - \eta) - \beta^t \mu_t = 0,
\]

while the transversality conditions for physical and social capital are given by

\[
\lim_{t \to \infty} \beta^t \lambda_t k_t = 0
\]

and

\[
\lim_{t \to \infty} \beta^t \mu_t e_t = 0.
\]
B Proof of Proposition 1

Along a BGP equilibrium, all endogenous variables satisfy \( x_{t+1} = gx_t \) where \( g_x \) is a constant. From expression (7), we obtain

\[
g_c = \beta \left[ 1 + \alpha A(e_{t+1}) k_{t+1}^{\alpha - 1} - \delta \right],
\]

from which we derive

\[
k_t = \left[ \frac{\beta \alpha A(e_t)}{g_c - \beta (1 - \delta)} \right]^{\frac{1}{1-\alpha}}.
\]  \( \text{(19)} \)

From expression (1) we obtain

\[
m_t = \left[ \frac{g_c - (1 - \eta)}{\phi} \right] e_t = \Delta e_t.
\]  \( \text{(20)} \)

We rewrite expression (8) as

\[
\left( \frac{\theta}{c_t} \right) \left[ \frac{1}{1 - \rho_c/g_c} - \beta(1 - \eta + \rho_c) - \frac{\beta^2 \rho_c(1 - \eta)}{g_c(g_c - \rho_c)} \right] = \left( 1 - \frac{\theta}{e_t} \right) \left( \frac{\beta \phi \epsilon_G}{g_{e^{1+\epsilon_G}} - \rho_c} \right),
\]

from which we deduce that

\[
c_t = \Pi e_t,
\]  \( \text{(21)} \)

where \( \Pi > 0 \) is a constant.

Using now expression (3) together with (19), (20) and (21), we obtain

\[
\left[ \frac{\beta \alpha}{g_c - \beta (1 - \delta)} \right]^{\frac{1}{1-\alpha}} \left[ A(e_{t+1})^{\frac{1}{1-\alpha}} - (1 - \delta)A(e_t)^{\frac{1}{1-\alpha}} \right] = \left[ \frac{\beta \alpha}{g_c - \beta (1 - \delta)} \right]^{\frac{1}{1-\alpha}} A(e_t)^{\frac{1}{1-\alpha}} - (\Pi + \Delta) e_t,
\]

which becomes

\[
A(e_{t+1})^{\frac{1}{1-\alpha}} - A(e_t)^{\frac{1}{1-\alpha}} = \left\{ \frac{g_c - \beta [1 - \delta (1 - \alpha) - \theta]}{\beta \alpha} \right\} A(e_t)^{\frac{1}{1-\alpha}} - \Theta e_t,
\]  \( \text{(22)} \)

where \( \Theta > 0 \) is a constant. Let us define the first element on the right hand side of expression (22) as \( \Psi(e_t) \) and compute its derivative to obtain

\[
\Psi'(e_t) = \left\{ \frac{g_c - \beta [1 - \delta (1 - \alpha) + \epsilon]}{\beta \alpha (1 - \alpha)} \right\} \left[ \frac{e_{A A} A(e_t)^{\frac{1}{1-\alpha}}}{e_t} \right].
\]

The limits of \( \Psi'(e_t) \) are given by

\[
\lim_{e_t \to 0} \Psi'(e_t) = \begin{cases} 
+\infty & \text{if } \epsilon + \alpha < 1, \\
0 & \text{if } \epsilon + \alpha > 1, \\
M_1 & \text{if } \epsilon + \alpha = 1,
\end{cases}
\]

26
and

$$\lim_{\epsilon_t \to +\infty} \Psi'(e_t) = \begin{cases} 
0 & \text{if } \epsilon + \alpha < 1, \\
+\infty & \text{if } \epsilon + \alpha > 1, \\
M_2 & \text{if } \epsilon + \alpha = 1,
\end{cases}$$

where $M_1$ and $M_2$ are two constants.

We plot the curves corresponding to the functions $\Psi(e_t)$ and $\Theta e_t$ in Figure 6. A steady-state of the competitive economy is a situation where the functions $\Psi(e_t)$ and $\Theta e_t$ cross implying that $A(e_{t+1})^{1/\alpha} - A(e_t)^{1/\alpha} = 0$ or, equivalently, $e_{t+1} = e_t = e$. This can only occur if $\epsilon_A + \alpha < 1$ or $\epsilon_A + \alpha > 1$. From Figure 6 it is straightforward to conclude that in the case where $\epsilon_A + \alpha < 1$ the steady-state is locally-stable, whereas in the case where $\epsilon_A + \alpha > 1$ the steady-state is unstable. Formally, by applying the implicit function theorem to (22), we obtain that at the steady state:

$$\frac{d\epsilon_{t+1}}{d\epsilon_t} = 1 + (1 - \alpha) e \left[ \frac{\Psi'(e) - \Theta}{\epsilon_A A(e)^{1/\alpha}} \right],$$

which is smaller than one if and only if $\epsilon_A + \alpha < 1$ because $\Psi'(e) < \Theta$ in this case.

Insert Figure 6

When $\epsilon_A + \alpha = 1$, there is no positive steady-state but a BGP equilibrium where endogenous variables grow at constant rates. However, as can be observed from Figure 6, two cases should be distinguished. We first rewrite expression (22) as

$$A(e_{t+1})^{1/\alpha} - A(e_t)^{1/\alpha} = \left\{ \frac{g_c - \beta[1 - \delta(1 - \alpha)]}{\beta \alpha} \right\} - \frac{\Theta e_t}{A(e_t)^{1/\alpha}}.$$ 

Since in this case $\epsilon_A + \alpha = 1$, the last term on the right hand side is a constant.

We can then conclude that $g_c > (\leq) 1$ if and only if

$$\left\{ \frac{g_c - \beta[1 - \delta(1 - \alpha)]}{\beta \alpha} \right\} A(e_t)^{1/\alpha} > (\leq) \Theta e_t.$$ 

When $g_c > 1$, the curve $\Psi(e_t)$ is always above $\Theta e_t$ while the converse is true when $g_c < 1$. 

27
Figure 1: Evolution of key economic variables for five European economies (1950-1985)
Figure 2: Dynamic paths of key economic variables for both economies
Figure 3: Dynamic paths of key economic variables (different initial conditions)
Figure 4: Dynamic paths of key economic variables (different habit persistence)
Figure 5: Dynamic paths of key economic variables for both economies
Figure 6: Existence and stability of steady states

\[ \Psi(\epsilon_t) \]

- If \( \epsilon_A + \alpha = 1 \)
- If \( \epsilon_A + \alpha > 1 \)
- If \( \epsilon_A + \alpha < 1 \)