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Economic Growth and Environmental Degradation when Preferences are Non-Homothetic*

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Abstract

We study the dynamics of pollution in an economic growth model with non-homothetic preferences. We characterize the forces that may drive the evolution of the income-pollution relationship along the development process. In particular, we disentangle the standard accumulation mechanism, that determines the intertemporal allocation of pollution, from a mechanism based on the non-homotheticity of preferences, which leads the intratemporal allocation of expenditure between consumption and pollution abatement to depend on income. As the economy develops and aggregate income grows up, the fraction of income devoted to abatement increases if the income elasticity of abatement is larger than unity. In this case, the pollution may decrease with income even when the elasticity of pollution with respect to abatement is smaller than the elasticity of pollution with respect to emissions. We numerically illustrate how this demand-based mechanism determines the dynamic relationship between pollution and aggregate income.

JEL classification codes: Q2; D62; H23
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1. Introduction

A vast literature has recently studied the relationship between environmental degradation and economic growth.[1] Dealing with this issue is very relevant for predicting the long-run effects of economic growth on social and individual welfare. Following this motivation, a large number of theoretical and empirical papers have investigated whether or not pollution is an inescapable consequence of income growth. These studies assume that pollution is a by-product of either production or consumption and that private agents can still devote resources to abate the levels of pollution. At the empirical ground, there is not a consensus about the nature of the relationship between income and pollution, whereas the theoretical studies have focused on different mechanisms that may be behind this relation. In this paper, we analyze the role of capital accumulation and sustained economic growth in the dynamics of pollution and income when preferences on consumption and environmental quality are non-homothetic. By using a dynamic general equilibrium model that incorporates this kind of preferences, we theoretically show that the relationship between pollution and income may follow a non-monotonic path.

The aforementioned literature has mostly considered the so-called Environmental Kuznets Curve (EKC, henceforth) as an empirical hypothesis: the relationship between pollution and income would exhibit an inverted U-shaped path. More precisely, as Stern (2004) states, "during the early stages of economic growth, degradation and pollution increases, but beyond some level of income per capita the trend reverses". The eventual existence of an EKC would not only lead economic growth to be compatible with the preservation of environmental quality, but would even convert economic growth into a tool to improve that quality. This conclusion stresses the importance of empirically testing the existence of this EKC and, in general, of theoretically exploring the potential determinants of the relationship between pollution and aggregate income.

There is a large empirical literature that tries to estimate regularities in the behavior of pollution along the development process (surveys include Copeland and Taylor, 2004; Dasputa et al. 2002; Dinda, 2004). The results from these empirical studies are inconclusive about the relationship between pollution and income. In particular, the empirical evidence is not robust to changes in the econometric specification and in data. In any case, some studies support the existence of an inverted U-shaped (and even an N-shaped) relationship between income per capita and the emissions of some pollutants (see, e.g., Sengupta, 1997). However, one cannot derive the existence of empirical regularities between pollution and income per capita at the aggregate level.

The empirical debate should lead to looking for the existence of possible economic foundations that may drive the relationship between pollution and income along the development process. Following the terminology popularized by Grossman and Krueger (1991), Brock and Taylor (2005) characterize the links between economic growth and environmental quality through the interaction of three channels: scale, composition and technique effects. Economic growth creates a scale effect, as increasing output tends to raise emissions. When income grows, the composition effect also emerges as the sectoral structure changes firstly from agriculture to industry leading pollution to increase, and

[1] See, for instance, Smulders (1999), Brock and Taylor (2005) or Pasten and Figueroa (2012) for a survey of this literature.

subsequently the sectoral structure moves from industry to services provoking a decrease in pollution. Finally, there is a technique effect as technological progress replaces dirty technologies and improves the productivity of pollution abating environmental effort. Dinda (2004) or Kijima et al. (2010) provide an overview to the theoretical literature that tries to understand and disentangle the underlying mechanisms that may drive the relationship between income and pollution.

We contribute to this theoretical literature by analyzing the dynamic evolution of aggregate income and pollution in a dynamic general equilibrium model with capital accumulation. Many studies analyzing the relationship between income and pollution are based on models without feedback from the environment to economic growth. However, as Arrow et al. (1995) have pointed out, the economic activity is also affected by the evolution of environmental quality. Understanding the links between environment and economic growth then requires the use of a dynamic general equilibrium framework. In this paper, we extend the neoclassical growth model by incorporating pollution as a bad that depends on the emissions flowing from production and on the expenditure that individuals uncoordinatedly devote to activities of pollution abatement.

We basically focus on analyzing the aforementioned scale effect relating the growth of aggregate output with the dynamics of pollution. We prove that this scale effect is driving by three forces in the proposed framework: (i) the standard accumulation mechanism based on the decreasing returns in capital that drives the intertemporal allocation of expenditure on abatement; (ii) a demand-based effect from the non-homothetic feature of preferences that leads dynamic adjustment of income to alter the composition of expenditure between consumption and abatement; and (iii) the effect of the exogenous change in aggregate productivity that imposes a trend to pollution. Our aim is to analyze the relative importance of the demand-based mechanism and its interaction with the other two forces. To this end, we first note that the literature has already shown that the static relationship between income and pollution crucially depends on the properties of preferences and the technology describing the production of pollution. For instance, Lopez (1994), Plassmann and Khanna (2006b), Figueroa and Pasten (2013) or Shibayama and Fraser (2014) state conditions for a non-monotonic dependence of pollution on the level of income. We clarify that the dependence of pollution on income is fully determined by the relationship between two types of elasticities: (i) the income-elasticities of consumers' expenditures on consumption and abatement; and (ii) the elasticities of pollution with respect to emissions and abatement. In particular, the emergence of a non-monotonic relationship between income and pollution requires either the pollution technology or the preferences on consumption and environmental quality to be non-homothetic.[2]

The main objective of the paper is then to illustrate that a dynamically non-monotonic relationship between pollution and income may emerge quite easily with non-homothetic preferences and without imposing restrictive conditions on the process of pollution. In particular, we show that the demand-based mechanism is sufficiently

[2]Many of the theoretical models obtaining a negative relationship between income and pollution are based on one of these non-homotheticities though they are often disguised under seemingly different assumptions. For instance, we will show that non-homogeneity of pollution technology is not a necessary condition for obtaining a non-monotonic path of pollution (see, Plassman and Khanna, 2006b).

flexible to allow any shape for the income-pollution path (monotonic, inverted U-shaped or N-shaped) by imposing plausible constraints on the parameter configuration. The key ingredient of our mechanism is the existence of a minimum consumption requirement that leads income-elasticities of consumption and abatement to be different and to vary with income. With these non-homothetic preferences, changes in income alter the composition of consumers' expenditure in favor of abatement. This change in expenditure composition may generate a negative relationship between pollution and income even when the elasticity of pollution with respect to abatement is smaller than the elasticity of pollution with respect to emissions. This result is in stark contrast with the literature, which states that the existence of this negative relationship requires abatement to have a return to scale in the production of pollution larger than the one of emissions (see, for instance, Andreoni and Levinson, 2001; or Plassmann and Khanna, 2006a).

As was mentioned before, we insert the proposed demand-based mechanism in a general equilibrium model with capital accumulation and exogenous technical progress to characterize the dynamics of the income-pollution relationship. Our aim is to disentangle the relative contribution of the three aforementioned forces driving the dynamics of pollution in the proposed economy. These forces operate in the opposite direction when the elasticity of pollution with respect to abatement is smaller than the elasticity of pollution with respect to emissions. We show that the balance between the forces depends on the level of income, which may generate non-monotonic relation between pollution and income. In particular, we show that the trend effect dominates for sufficiently large levels of income. Hence, if the abatement elasticity of pollution is smaller than the elasticity of pollution with respect to emissions, long-run economic growth will inevitably lead to an environmental degradation. This dominance of the trend mechanism, which is often omitted by the literature, confirm that reducing pollution may require at the end some kind of technological change that could revert the trend of pollution (see, e.g., Brock and Taylor, 2010). However, during the transition the demand-based mechanism may dominate, so that a negative relationship between pollution and income may emerge along the middle stages of development process.

Literature on environmental economics has also largely discussed the importance of nonhomothetic preferences in determining the relationship between income level and environmental degradation (see, e.g., Lopez, 1994; Plassmann and Khanna, 2006b; or Shibayama and Fraser, 2014). Our contribution is close to the study in Shibayama and Fraser (2014), who found that a EKC emerges when preferences are exponential because the associated nonhomotheticity results into an elasticity of substitution between consumption and environmental quality that decreases fast with aggregate income. However, the later studied does not consider that individuals can invest in environment by allocating income to abatement activities. It neither incorporates the accumulation mechanism driving the intertemporal allocation of expenditure on abatement nor the trend mechanism derived from technical progress. Therefore, the demand-based mechanism is the unique engine of the income-pollution relationship in the aforementioned studied. Our contribution to this literature is twofold. On the one hand, we prove that the importance of the demand-based mechanism when abatement are considered depends on the relative values of the following elasticities: the income elasticities of the expenditure in consumption and abatement, and the elasticities of pollution with respect emissions and abatement. In addition, we also analyze how this demand-based mechanism interacts with the accumulation and trend mechanisms, which naturally emerges in a growth economy, in generating the dynamics of the environmental degradation.

The paper is organized as follows. Section 2 presents a dynamic general equilibrium model that incorporates pollution and non-homothetic preferences as key ingredients. We derive the equilibrium path of the economy in Section 3. In Section 4 we characterize the dynamics of the relationship between pollution and income, and we disentangle the three mechanisms driving this relationship. We basically focus on explaining the engineering of the demand-based mechanism. In Section 5 we analyze the welfare properties of our model by characterizing the socially optimal solution. Section 6 presents the conclusions and some final remarks. Finally, some technical procedures are included in Appendix.

2. The model economy

We extend the neoclassical growth model to study the dynamic path of pollution. In particular, the economy consists of competitive firms and n infinitely lived, identical consumers. We assume no population growth. We consider that a single good is produced in each period by means of a constant-returns-to-scale technology, which uses labor and capital as inputs. For simplicity in the exposition, and without loss of generality, we consider a Cobb-Douglas production function. Hence, aggregate output y_t is given by

$$y_t = k_t^{\alpha} \left(A_t l_t\right)^{1-\alpha}, \tag{2.1}$$

where $\alpha \in (0,1)$; A_t measures the efficient units of labor that evolves by means of an exogenous technical progress, such that $A_t = (1+\eta)^t A_0$; and where k_t and l_t are the aggregate stock of capital and the aggregate amount of labor, respectively. Furthermore, the production of this single good generates, as a by-product, emissions of pollutants to the atmosphere. We consider that pollution, which we denote by p_t, is an increasing function of output. However, consumers can voluntarily devote an amount m_t of his income to abate the emissions. We then consider that pollution p_t is given by a function $P(y_t, e_t)$, with partial derivatives $P_y > 0$ and $P_e < 0$, and where e_t is the aggregate expenditure on abatement, i.e., $e_t = \sum_{i=1}^{n} m_{it}$. For the dynamics of the pollution will be crucial the relative scale of emissions and abatement in pollution. Since we are interested in analyzing the demand factors on the evolution of pollution, we consider that the pollution function $P(y_t, e_t)$ is homothetic.[3] For simplicity, we follow Rubio et al. (2009) and Fernandez et al. (2010), among others, and we assume that the pollution is given by

$$p_t = \frac{y_t^{\theta}}{e_t^{\pi}}, \tag{2.2}$$

where $\theta > 0$ and $\pi > 0$. Finally, note that the single good can be consumed, invested or used for abating pollution.

[3]Some authors like, for instance, Andreoni and Levinson (2001) have shown, generally in a static setup, that a non-homothetic pollution function may generate a non-monotonic relationship between pollution and income. Plassmann and Khanna (2006b) show the logic of this result. We will also return to this point in the discussion of our results.

Each consumer is endowed with an initial stock of assets a_0 and with a unit of time in each period that inelastically supplies as labor. They derive utility from consumption and environmental quality. Hence, environmental quality has amenity value as a pure public good, i.e., it is a non-rival and non-excludable consumption good. As was said before, consumers can voluntarily contribute to this public good by spending m_{it} units of their income on abating pollution. More precisely, consumers' preferences are represented by the utility function

$$u(c_t, p_t) = \frac{\left[(c_t - \overline{c}_t)^{\gamma} p_t^{-\lambda}\right]^{1-\sigma}}{1-\sigma}, \tag{2.3}$$

with $\gamma > 0$ and $\lambda > 0$, and where c_t is consumption, $\overline{c}_t$ is a time-varying aspiration or minimum requirement in consumption, and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution of the composite good $(c_t - \overline{c}_t)^{\gamma} p_t^{-\lambda}$. Aspiration means that consumer takes an exogenous reference with respect which his own consumption is compared to. We consider that this consumption aspiration permanently grows at the exogenous rate η of technical progress (which corresponds with the stationary growth rate of aggregate output) to guarantee the existence of balanced growth path, i.e.,

$$\overline{c}_t = (1+\eta)^t \, \overline{c}_0, \tag{2.4}$$

with $\overline{c}_0 \geq 0$. We are then going further from the standard biological notion of minimum of subsistence used by the literature on economic development. Following Christiano (1989), we instead assume that aspirations "reflect an increase over time in the minimum acceptable quality of life". Furthermore, the constraint $c_t > \overline{c}_t$ must hold for all t for having a well defining utility function. This constraint imposes a lower bound to the initial stock of capital k_0, which depends on the initial values of aspirations $\overline{c}_0$ and of the efficient units of labor A_0.

The objective of each consumer is to choose consumption c_t, abatement expenditure m_t and the stock of assets a_{t+1} to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t, p_t), \tag{2.5}$$

subject to (2.2) and the budget constraint

$$(1+r_t)\, a_t + w_t = a_{t+1} + c_t + m_t. \tag{2.6}$$

where r_t and w_t are the rental rates of capital and labor, respectively; and $\beta \in (0,1)$ is the subjective discount rate. In solving this maximization problem, consumer takes the emissions y_t^{θ} as given, so that production is a source of inefficiency in this economy.[4] Another source of inefficiency is the non-cooperative behavior of consumers in deciding their private contributions m_t to pollution abatement. We assume that the n consumers play a *Cournot or non-cooperative Nash game* in selecting their individual expenditure on pollution abatement as any consumer i considers that his own expenditure m_{it}

[4] We will show below that this assumption is not relevant for the qualitative conclusions of our analysis.

has no effect on the other consumers' contributions $\sum_{j\neq i} m_{jt}$. This gives rise to a standard freerider problem, so that the expenditure on pollution abatement is socially suboptimal and, moreover, this inefficiency increases with the number of consumers (see, e.g., Olson, 1965). However, as was shown independently by Chamberlin (1974) and McGuire (1974), the aggregate contribution to a public good is increasing in the number of contributors and converges to a finite and strictly positive amount if these contributors are all identical and the public good (in our case, environmental quality) is a normal good. Even when the increase of the number of contributors reduces the individual expenditures, the added expenditure of a new contributor compensates for the decline in the other's contributions caused by the new entry. Therefore, we can characterize the competitive, general equilibrium in our economy by focusing on the problem of a representative consumer, so that we can abstract from the freerider problem, which is not relevant for our purpose in this paper. We then assume from now on, without loss of generality, $n = 1$, and so $e = m$.

3. The equilibrium path

Given the initial values of capital stock k_0, of assets a_0, of efficient units of labor A_0 and of aspirations $\bar{c}_0$, a *symmetric competitive equilibrium* in this economy consists of a set of paths for prices $\{r_t, w_t\}$, allocations $\{c_t, e_t, a_{t+1}\}$ and capital stock $\{k_t\}$, such that: (i) the path $\{c_t, e_t, a_{t+1}\}$ solves the representative consumer's problem; (ii) the path $\{k_t\}$ maximizes the firms' profits; and (iii) the market clearing conditions for capital, labor and goods hold, i.e., $k_t = na_t$, $l_t = n = 1$ and

$$y_t = k_{t+1} + c_t + e_t - (1-\delta)\, k_t, \tag{3.1}$$

where $\delta \in [0, 1]$ is the depreciation rate of capital stock.

In equilibrium, competition among profit-maximizing firms ensures that both production factors are paid their marginal products. Hence, the profit maximization conditions are

$$r_t = \alpha k_t^{\alpha-1} A_t^{1-\alpha} - \delta, \tag{3.2}$$

and

$$w_t = (1-\alpha)\, k_t^{\alpha} A_t^{1-\alpha}. \tag{3.3}$$

The representative consumer's problem involves two margins. First, total income must be allocated between total expenditure and investment. In addition, total expenditure must be distributed between consumption and pollution abatement. These two trade-offs are characterized by the first order conditions of the aforementioned problem. Our point is that environmental degradation are driven by different mechanisms: some of them operate in the intertemporal margin and others operate in the intratemporal margin. Hence, to better illustrate the mechanics behind the dynamics of environmental degradation in this economy, we separate the intertemporal problem from the intratemporal problem faced by the consumer. To this end, we first define the following composite good:

$$v_t = (c_t - \bar{c}_t)^{\frac{\gamma}{\gamma+\lambda\pi}}\, p_t^{-\frac{\lambda}{\gamma+\lambda\pi}}. \tag{3.4}$$

We then show in Appendix A that the representative consumer's problem can be decomposed into the following two problems, which can be solved sequentially:

1. *Intertemporal problem.* Given the initial stock of assets a_0, the representative consumer solves:

$$\max_{\{v_t, a_{t+1}\}} \beta^t \left[\frac{v_t^{(1-\sigma)(\gamma+\lambda\pi)}}{1-\sigma} \right],$$

subject to

$$(1+r_t)\, a_t + w_t = a_{t+1} + q_t v_t + \bar{c}_t,$$

where

$$q_t = (\gamma + \lambda\pi) \left[\gamma^\gamma (\lambda\pi)^{\lambda\pi} \right]^{\frac{-1}{(\gamma+\lambda\pi)}} y_t^{\frac{\lambda\theta}{(\gamma+\lambda\pi)}}, \tag{3.5}$$

is the price index of the composite good v_t. By following a standard procedure, we find in Appendix B the first order conditions, and then rearrange expressions to summarize the solution of this problem by:

$$\left(\frac{v_t}{v_{t+1}} \right)^{(1-\sigma)(\gamma+\lambda\pi)-1} = \beta \left(\frac{q_t}{q_{t+1}} \right) (1 + r_{t+1}) . \tag{3.6}$$

Equation (3.6) is the usual Euler condition stating that the marginal rate of substitution between present and future expenditure must be equal to the future net rate of return on present investment.

2. *Intratemporal problem.* Given the optimal path of v_t defined by (3.6), the representative consumer also solves the following problem by taking aggregate output y_t as given:

$$\max_{\{c_t, e_t\}} \frac{\left[(c_t - \bar{c}_t)^\gamma e_t^{\lambda\pi} y_t^{-\lambda\theta} \right]^{1-\sigma}}{1-\sigma},$$

subject to

$$c_t + e_t = q_t v_t + \bar{c}_t \equiv g_t,$$

where g_t then stands for the total expenditure by definition. By manipulating the first order conditions of this problem, we derive a condition stating that the intratemporal marginal rate of substitution between consumption and abatement must be equal to unit (i.e., the relative price of abatement in terms of consumption) at the equilibrium. Combining this condition with the constraint of the intratemporal problem, we also obtain the following demands for consumption and abatement effort, respectively:

$$c = \frac{\gamma g_t + \lambda\pi\bar{c}_t}{\gamma + \lambda\pi}, \tag{3.7}$$

and

$$e = \frac{\lambda\pi\,(g_t - \bar{c}_t)}{\gamma + \lambda\pi}. \tag{3.8}$$

Observe that both demands are functions of total expenditure g_t that do not emanate from the origin if the aspirations are presented, i.e., if $\bar{c} \neq 0$. Hence, the consumer's allocation of expenditure between consumption and abatement crucially depend on the non-homothetic feature of preferences. In particular, the composition of expenditure will change along the process of growth and development.

Summarizing, the optimal plan of the representative consumer is therefore fully characterized by the equations (3.6), (3.7), (3.8) and the following transversality condition

$$\lim_{t->\infty} \beta^t u_t^c k_t = 0,$$

where u_t^c represents the marginal utility of consumption, i.e.,

$$u_t^c = \gamma \left(c_t - \overline{c}_t\right)^{\gamma(1-\sigma)-1} p_t^{-\lambda(1-\sigma)}. \tag{3.9}$$

Our economy exhibits a *balanced growth path equilibrium*, along which output, the stock of capital, consumption and abatement grow at the constant rate η. In order to proceed with our analysis, we now normalize the variables to remove the consequences of long-run growth. In particular, we introduce the following normalized variables:

$$\widehat{k}_t = k_t \left(1+\eta\right)^{-t}, \ \widehat{c}_t = c_t \left(1+\eta\right)^{-t}, \ \widehat{e}_t = e_t \left(1+\eta\right)^{-t} \text{ and } \widehat{y}_t = y_t \left(1+\eta\right)^{-t}.$$

Note that the normalized variables correspond with the detrented values of the original variables and, therefore, they remain constant along the BGP. We will denote by $\widehat{x}^*$ the stationary value of the detrented variable $\widehat{x}_t$. Observe also that pollution p_t grows at the constant rate $(1+\eta)^{\theta-\pi}$ along the BGP as follows from log-differentiating pollution function (2.2). Based on this conclusion, we directly obtain the next result that characterizes the asymptotic behavior of pollution.

Proposition 3.1. *If* $\pi > \theta$ *then* $\lim_{t\to\infty} p_t = 0$, *whereas* $\lim_{t\to\infty} p_t = \infty$ *when* $\theta > \pi$.

The long-run dynamics of both income and pollution are only driven by the exogenous trend of the technological change. Hence, the relationship between these two economic variables is always monotone along the BGP equilibrium. Furthermore, this relationship is negative (positive) when the elasticity of pollution with respect to emissions is smaller (larger) in absolute terms than the elasticity of pollution with respect to abatement, i.e., $\theta < (>)\, \pi$. However, along the transition the evolution of the economy is also driven by the dynamic adjustment derived from the imbalances in the stock of capital with respect to its long-run trend. This second force includes the adjustment in the composition of consumer's expenditure, which is the basis of our mechanism explaining a potential negative relationship between pollution and output. The differences in the income elasticities of demand for consumption and abatement may lead pollution to decrease with income along the transition even in the case with $\theta > \pi$. Unfortunately, the income-pollution relation along the process of growth cannot be analytically characterized. We will next calibrate our economy to simulate the dynamics of that relationship.

4. Income-pollution dynamics

The model has to be solved and simulated in order to characterize the dynamic relationship between pollution and GDP. To this end, we choose the parameter values to replicate some facts observed in U.S. data. A period in the model corresponds to a quarter in actual data. The parameterization for the production function, the process governing the accumulation of capital, the discount factor and the intertemporal elasticity of substitution are those commonly used in the RBC literature (see, e.g., Cooley and Prescott, 1995.) In particular, we set the values of those parameters to force the BGP equilibrium of our economy to match the share of capital income on GDP, the consumption to GDP ratio, the investment to GDP ratio, the capital stock to GDP ratio and the average growth rate observed in the data. However, it is more difficult to find macroeconomic empirical evidence to set the values of the other parameters, basically the environmental ones. We will follow the existing literature to set these parameters.

Table 1 describes the benchmark values of the parameters that we use in our numerical simulations. With these values, we first obtain that the steady-state ratios of consumption, investment and abatement to GDP are 0.63, 0.27 and 0.1, respectively. As mentioned before, there is no precise evidence about the share of aggregate abatement expenditure on GDP. The existing evidence reduces to the market expenditure on some particular pollutants.[5] In light of this evidence, we might conclude that we obtain an excessive share of abatement expenditure on GDP. However, note that the abatement in the model should match the actual aggregate expenditure on abatement, which includes all domestic and market activities expensively orientated to reduce the level of emissions from all the pollutants. In any case, the aforementioned figures implies that the shares of consumption and investment on the GDP net of expenditure on abatement are 70% and 30%, respectively, which are very close to those observed in the actual data. Secondly, following Kelly (2003) we set the same shares for consumption and pollution on the composite good, i.e., $\gamma = \lambda$. Finally, following Rubio et al. (2009) and Fernandez et al. (2011), we assume that pollution is a concave function of emissions. Furthermore, we assume that the elasticity of pollution with respect to emissions is larger in absolute terms than the elasticity of pollution with respect to abatement, i.e., $\theta > \pi$. We adopt this assumption because this is the worst scenario to obtain a negative relation between pollution and income. Hence, this assumption seems to be convenient to show the quantitative importance of the non-homothetic preferences for the pollution-income relation.

[Table 1]

We simulate the equilibrium dynamics when the initial stock of capital k_0 is such that their normalized value $\widehat{k}_0$ accounts to the 20% of the stationary value of normalized stock $\widehat{k}^*$. We will focus on the dynamic behavior of pollution along the equilibrium path. Figure 1 graphs the simulated relationship between the logarithmic values of pollution and income. The left-hand side panel shows the behavior of the normalized

[5] For instance, Hackett (2011) asserts that the US expenditure on abatement accounts for 2% of GDP.

variables (i.e., the detrended variables), whereas the right-hand side panel gives the dynamic behavior of the trended variables. While in the trended economy we observe a monotonically positive relationship between pollution and income, the detrended values of pollution and output follows an inverted U-shaped path in our benchmark economy. Therefore, we conclude that a negative relationship between pollution and income does not emerge in our benchmark economy. The upward-sloping trend of pollution more than offset the downward sloping part of the path followed by the detrented pollution. Obviously, we can still conclude from confronting the two panels in Figure 1 that a negative relation would arise, at least during the transitional dynamics, if we will reduce the size of the pollution trend, i.e., the exogenous growth rate η of technological progress. We will illustrate this point below.

[Figure 1]

To understand the dynamic behavior of pollution described above, we compute the growth rate of this variable. As shown in Figure 2, the benchmark economy follows an equilibrium path along which the detrended values of capital, income, consumption and abatement monotonously grow at different time-varying rates when $\widehat{k}_0 < \widehat{k}^*$. Instead detrended pollution follows a non-monotone path. To illustrate the mechanics behind this dynamic behavior we characterize the growth rate of pollution. By using (2.1) and (2.2) we get

$$\frac{p_t}{p_{t-1}} = (1+\eta)^{\theta-\pi} \left(\frac{\widehat{k}_t}{\widehat{k}_{t-1}} \right)^{\alpha\theta} \left(\frac{\widehat{e}_t}{\widehat{e}_{t-1}} \right)^{-\pi}. \tag{4.1}$$

We initially distinguish two forces driving the evolution of pollution along the equilibrium path. On the one hand, there is a *trend mechanism*: the exogenous technical progress generates a trend on output and on pollution. As was shown before, while output follows an upward-sloping trend, pollution exhibits a trend with a positive (negative) slope when $\theta > (<)\, \pi$. Hence, the effect of this mechanism on the income-pollution relationship depends on the technological parameters describing the flow of pollution. In our benchmark economy $\theta > \pi$ so that this first mechanism drives pollution up.

In addition to this permanent force, we observe from (4.1) that pollution is also affected by the dynamic adjustment in the ratio $\frac{\widehat{e}_t}{\widehat{k}_t}$ derived from the imbalances in the capital stock with respect to the BGP. Panel (f) of Figure 2 shows the evolution of this ratio in the benchmark economy. This dynamic adjustment in our economy is driving by two mechanisms. Firstly, the evolution of pollution is determined by a standard *accumulation mechanism*: the accumulation of capital along the transition determines the intertemporal allocation of pollution by altering the interest rate. More precisely, we can observe from (3.6) that the time path of expenditure (and so of consumption and pollution abatement) is fully determined by the path of the interest rate. This response of abatement to the changes in the interest rate determines the transitional dynamics of the ratio $\frac{\widehat{e}_t}{\widehat{k}_t}$. Secondly, we also find a *demand-based mechanism* driving the income-pollution dynamics: the non-homothetic feature of preferences leads the intratemporal marginal rate of substitution between consumption and abatement to depend on the level of income. In particular, as Figure 3 shows, the expenditure share on abatement grows up along the transition provided that $\bar{c}_t > 0$. This mechanism then accelerates the dynamic adjustment of abatement and, therefore, it has important consequences for the transitional dynamics of the ratio $\frac{\widehat{e}_t}{\widehat{k}_t}$ and pollution.

[Figures 2 and 3]

We will next characterize these three mechanisms driving the dynamics of pollution and we will also analyze the relative impact of them on the relationship between pollution and income for the benchmark economy. We are especially interested in disentangling the accumulation mechanism and the demand-based mechanism. To this end, we will first eliminate the trend mechanism by focusing on the relationship between the detrended values of pollution and income, $\widehat{p}_t$ and $\widehat{y}_t$. After that, we will study the effect of the exogenous technological progress by going back to the trended economy.

4.1. The accumulation mechanism driving pollution

The path of pollution depends on the evolution of the interest rate, which determines the allocation of income between expenditure and investment in capital and, thus, the dynamic behavior of the ratio $\frac{\widehat{e}_t}{\widehat{k}_t}$. This accumulation mechanism driving pollution is then the standard neoclassical mechanism for the dynamic adjustment of capital imbalances based on the decreasing returns to capital. In order to isolate that mechanism, we assume in this subsection that preferences are homothetic (i.e., $\bar{c}_0 = 0$). Equilibrium conditions (3.7) and (3.8) shows that in this case $\widehat{c}_t = \left(\frac{\gamma}{\lambda\pi}\right)\widehat{e}_t$, so that the expenditure shares in consumption and abatement are constant along time. Therefore, the ratio $\frac{\widehat{e}_t}{\widehat{k}_t}$ dynamically behaves as the ratio $\frac{\widehat{c}_t}{\widehat{k}_t}$ since consumption and abatement exhibit the same growth rate. By using (3.6) we can then characterize the dynamic behavior of these two ratios, i.e., the slope of the policy functions that give the equilibrium values of consumption and abatement as functions of the capital stock.

As Barro and Sala-i-Martin (2004) prove, the ratio $\frac{\widehat{c}_t}{\widehat{k}_t}$ increases, remains constant or decreases along the transition from $\widehat{k}_0 < \widehat{k}^*$ depending on whether σ is smaller than, equal to or larger than α.[6] Therefore, using the aforementioned equivalence between the dynamics of consumption and abatement, we conclude that the ratio $\frac{\widehat{e}_t}{\widehat{k}_t}$ decreases along the transitional dynamics because $\sigma > \alpha$ in our benchmark economy. More precisely, in our calibrated economy with $\bar{c}_0 = 0$, we obtain that

$$\frac{\widehat{k}_t}{\widehat{k}_{t-1}} > \frac{\widehat{e}_t}{\widehat{e}_{t-1}},$$

for all t, and

$$\lim_{t\to\infty}\left(\frac{\widehat{k}_t}{\widehat{k}_{t-1}} - \frac{\widehat{e}_t}{\widehat{e}_{t-1}}\right) = 0.$$

A large value of σ means that consumers have a strong preference for smoothing expenditure over time (and so their components: consumption and abatement). As

[6] This results is derived in a model without pollution (i.e., with $\lambda = 0$ and $\gamma = 1$). However, it is easy to prove that the results still maintain in our model with $\bar{c}_0 = 0$. This proof is available from authors upon request.

a consequence of that, consumers will initially choose a low capital investment and a high expenditure because $\widehat{k}_0 < \widehat{k}^*$ and the interest rate decreases along the transitional dynamics. This means that $\frac{\widehat{e}_t}{\widehat{k}_t} > \frac{\widehat{e}^*}{\widehat{k}^*}$ for all t in our calibrated economy with $\overline{c}_0 = 0$.

Since during the transition capital grows faster than abatement, we conclude from (4.1) that the accumulation mechanism presented in this subsection generates a negative relationship between pollution and income if $\alpha\theta$ is sufficiently smaller than π. In particular, Figure 4 shows that a positive relationship between the detrended values of these variables emerges for the calibrated values when $\overline{c}_0 = 0$. However, panels (b) and (c) shows that a negative relationship can be obtained by reducing the value of θ. In particular, panel (b) of Figure 4 shows that the detrented values of income and pollution exhibits a hump-shaped relationship when $\theta = 0.16 > \pi$. Therefore, even when the elasticity of pollution with respect to emissions is larger than the elasticity of pollution with respect to abatement, the accumulation mechanism can lead pollution to decrease with income. However, this relationship is forced by adopting an ad-hoc assumption on the technology of pollution and, in particular, in the relative value of emission elasticity of pollution. In order to illustrate the importance of the demand-based mechanism, which is our main aim in the paper, we consider a scenario (the benchmark economy) where this accumulation mechanism always generates an increasing income-pollution relationship.

[Figure 4]

4.2. The demand-based mechanism driving pollution

We will turn to consider our benchmark economy, where the minimum consumption $\overline{c}_t$ is strictly positive. The dynamic behavior of pollution is crucially determined by the non-homothetic feature of preferences as they affect the transitional adjustment of the ratio $\frac{\widehat{e}_t}{\widehat{k}_t}$. In particular, as follows form (3.7) and (3.8), this ratio is increasing in total expenditures g_t provided that $\overline{c}_t > 0$. Hence, during the transitional dynamics from $\widehat{k}_0 < \widehat{k}^*$ there is a meaningful change in the composition of expenditure in favor abatement as total expenditures is an increasing function of capital at the equilibrium (see Figure 3). This finally results into an increase in the growth rate of abatement, which reduces (and even may reverse) the growth rate of pollution as follows from (4.1). In fact, as was shown in Figure 1, in our benchmark economy this demand-based mechanism generates a negative relationship between the detrended values of income and pollution even when the elasticity of pollution with respect of emissions is larger in absolute terms than the elasticity of pollution with respect to abatement, i.e., $\theta > \pi$. The increasing fraction of income devoted to abatement more than compensates this relatively small returns on abatement exhibited by the pollution. In order to realize the power of this mechanism, we must note that the accumulation mechanism leads to a positive relationship between those variables in the benchmark economy. The effect of the demand-based mechanism is much larger than the effect of the accumulation mechanism in a part of the transitional dynamics.

Obviously, the demand-based mechanism determines the equilibrium dynamics through the intratemporal margin faced by consumers. Therefore, to disentangle the engineering of the demand-based mechanism we will consider in this subsection a static, partial equilibrium version of our model, i.e., k and A are constant and the resources constraint is then given by $y = c+e$ as in this case total expenditure g is equal to income y because the absence of any saving motive. We first derive general conditions on preferences and technologies that may generate a non-monotonic relationship between pollution and income in this static economy. To this end, we consider for a moment a general form for the utility function and for pollution, i.e., $U(c,p)$ and $p = P(y,e)$ with $U_c > 0$, $U_p < 0$, $P_y > 0$ and $P_e < 0$, where the subindex denotes the variable with respect to which the partial derivative is being taken. Let us denote the income elasticities of expenditure in consumption and in abatement as ε_c^y and ε_e^y, respectively. We assume that both consumption and environmental quality, given by $1/p_t$, are normal goods, so that $\varepsilon_c^y > 0$ and $\varepsilon_e^y > 0$. We also define the elasticity of pollution with respect to income (output) and abatement effort as ε_P^y and ε_P^e, respectively. Given the properties of the pollution function, we observe that $\varepsilon_P^y > 0$ and $\varepsilon_P^e < 0$. By totally differentiating $P(y,e)$, we obtain

$$\frac{\partial p}{\partial y} = P_y + P_e\left(\frac{\partial e}{\partial y}\right). \tag{4.2}$$

By using the definitions of ε_e^y, ε_P^y and ε_P^e, Equation (4.2) can be written as

$$\frac{\partial p}{\partial y} = \left(\varepsilon_P^y + \varepsilon_P^e \varepsilon_e^y\right)\left(\frac{p}{y}\right). \tag{4.3}$$

From (4.3) the following result characterizes the static relationship between pollution and income in terms of the properties of preferences and pollution technology:

Proposition 4.1. $\frac{\partial p}{\partial y} \lesseqgtr 0$ *if and only if*

$$\frac{1}{\varepsilon_e^y} + \frac{\varepsilon_P^e}{\varepsilon_P^y} \lesseqgtr 0. \tag{4.4}$$

From this general condition (4.4) we can derive the specific mechanisms that can drive the pollution-income path. We identify two complementary mechanisms that may generate a non-monotonic relationship between income and pollution: (i) a supply-side mechanism based on a non-homothetic technology for pollution $P(y,e)$;[7] and (ii) a demand-side mechanism based on a non-homothetic utility function $U(c,p)$. Furthermore, we also conclude from (4.4) that a negative income-pollution relationship may not require the elasticity of pollution with respect to abatement to be larger than the elasticity of pollution with respect to output (i.e., emissions). This condition is

[7] This property leads the elasticities of pollution with respect to its determinants to depend on income. Therefore, contrary to Lemma 1 in Plassman and Khanna (2006b), we show that non homogeneity of $P(y,e)$ is not a necessary condition for a non-monotonic relationship between income an polution provided that the utility $u(c,p)$ is homothetic. What is really needed for a monotonic relationship is that the pollution technology is homothetic. For instance, the function $P(y,e) = \theta \log(y) - \pi \log(e)$ is not homogenous but is homothetic and, therefore, the ratio $\frac{\varepsilon_P^e}{\varepsilon_P^c}$ is constant in income. The technology introduced by Andreoni and Levinson (2001) $P(y,e) = y - y^{\alpha}e^{\beta}$ genarates a non monotonic relationship between pollution and income because it is non homothetic if $\alpha + \beta \neq 1$.

only necessary when the income elasticity of the expenditure on consumption is equal to the income elasticity of the expenditure on abatement (i.e., when the utility function is homothetic).

In this paper, we are focusing on the demand-based mechanism by considering that consumers have aspirations in consumption.[8] This forces the marginal rate of substitution between c and e to depend on total expenditures and, therefore, on income. In terms of Condition (4.4), this means that the income elasticities of expenditures on consumption and abatement are different and depend on income. To illustrate how this mechanism operates, we now go back to the static version of our original model where the pollution and the utility functions are given by (2.2) and (2.3), respectively. In this case, the elasticities of pollution with respect to income (output) and abatement are respectively given by $\varepsilon_P^y = \theta$ and $\varepsilon_P^e = -\pi$. If we assume that $\theta > \pi$, then Condition (4.4) requires the income elasticity of abatement to be larger than unity, i.e., $\varepsilon_e^y > 1$.

From (3.7) and (3.8) we obtain that the income elasticities of c and e are respectively given by:

$$\varepsilon_c^y = \frac{\gamma y}{\gamma y + \lambda \pi \bar{c}}, \tag{4.5}$$

and

$$\varepsilon_e^y = \frac{y}{(y - \bar{c})}. \tag{4.6}$$

Note that $\varepsilon_c^y < 1$ and $\varepsilon_e^y > 1$ at the equilibrium. These values of income elasticities clearly determine the responses of consumption and abatement effort to income changes. Hence, these values of income elasticities are crucial to understand the relation between income and pollution along the equilibrium. Since $\varepsilon_e^y > 1$ we obtain from (4.4) that pollution may decrease with income even when $\theta > \pi$. In particular, by plugging (3.8) in (4.3), we obtain

$$\frac{\partial p}{\partial y} = \left[\frac{p}{y\,(y - \bar{c})}\right] \left[(\theta - \pi)\, y - \theta \bar{c}\right]. \tag{4.7}$$

Observe that the pollution decreases with income if the elasticity of pollution with respect to abatement is larger than the elasticity of pollution with respect to emissions (i.e., $\pi > \theta$). However, even when $\theta > \pi$ there exists a threshold level of income given by

$$\widetilde{y} = \left(\frac{\theta}{\theta - \pi}\right) \bar{c}, \tag{4.8}$$

such that the pollution-income relation is negative (positive) for $y < (>)\, \widetilde{y}$.

[8]There are other alternatives ways of generating non-homothetic preferences. For instance, preferences are also non-homothetic when the utility function is additively separable between c and p, with the two parts with different degrees of homogeneity. An example of these preferences is given by Stokey (1998):

$$U(c, p) = \frac{c^{1-\sigma}}{1-\sigma} - B\frac{p^{1-\gamma}}{1-\gamma},$$

with $\sigma \neq \gamma$. Other possible source of non-homothetic preferences is the case where utility depends on consumption and environmental quality, which is given by the difference of an endowment of quality and pollution. In other words, preferences are given by $U(c, \widetilde{q} - p)$, where $\widetilde{q}$ is the endowment of environmental quality. In this case, the marginal rate of substitution between consumption and abatement may depend on income if pollution is not a homogenous function of degree zero.

The relevant contribution of our analysis is that a negative relation between income and pollution can arise even when the elasticity of pollution with respect to abatement is smaller than the elasticity of pollution with respect to emissions (i.e., when $\theta > \pi$). This is in stark contrast with the previous literature that theoretically derived a non-monotonic relation (see, e.g., Andreoni and Levinson, 2001; Plassmann and Khanna, 2006a; or Egli and Steger, 2007). As Figure 1 illustrates, the presence of a minimum consumption requirement allows for a non-monotonic relationship between the detrended values of pollution and income when $\theta > \pi$. The economic intuition of our results is quite simple. Given the income-elasticities of consumption and abatement, the response of the latter to increases in income is relatively larger (see (4.5) and (4.6)). Hence, for sufficiently small levels of income, the ratio $1/\varepsilon_e^y$ is sufficiently small, such that Condition (4.4) determining a negative income-pollution relationship holds. As the economy develops both income elasticities converge to one and, thus, the ratio $1/\varepsilon_e^y$ increases. However, this is not sufficient to change the sign of the relation between elasticities

$$\frac{1}{\varepsilon_e^y} + \frac{\varepsilon_P^e}{\varepsilon_P^c},$$

when the elasticity of pollution with respect to emissions is smaller than the elasticity of pollution with respect to abatement and, therefore, the pollution monotonically converges to zero. Otherwise, the income-pollution path becomes positive when income reaches a sufficiently large level.

4.3. The effect of trend on pollution dynamics

We have just explained the behavior of the detrended pollution $\widehat{p}_t$. However, we must also be interested on whether or not the negative relationship between pollution and income still maintains when we incorporate the trends of those variables. The right-hand side of Figure 1 illustrates this relation in our benchmark economy. We observe that the effect of the trend dominates in all the periods and, thus, pollution increases with income along the entire equilibrium path. Along the transitional dynamics the income-elasticities of consumption and abatement monotonously approximate to unity, so that the difference between the two income-elasticities monotonously reduces during the transition. The change in the composition of final expenditure then vanishes as the economy approaches the BGP.[9] Obviously, the effect of the accumulation mechanism on the evolution of pollution also tends to vanish when the economy approaches the BGP (i.e., the difference between the growth rates of capital and abatement also tends to zero). Therefore, the dynamics of pollution are only driven by the exogenous technical progress in the long run. In our benchmark economy with $\theta > \pi$, the pollution then monotonously increases in the long run.[10]

[9] For simplicity we have assumed that $\overline{c}_t$ grows at the stationary rate η. This implies that the expenditure adjustment only occurs during the transition. If this fundamental was constant, then preferences would be permanently non-homothetic and, therefore, the demand-based mechanism would also work in the long-run. However, this effect would asymptotically vanish, so that the trend effect would still dominate after some level of income.

[10] Obviously, the two aferomentioned forces go in the same direction when $\theta < \pi$, such that the pollution converges to zero.

In the benchmark economy, the trend effect also dominates during the entire transitional dynamics. This dominance would be larger, the larger the exogenous growth rate of productivity η. Obviously, the trend of pollution is an increasing function of η. In addition, the effect of the dynamic adjustment in the expenditure composition (i.e., the demand-based mechanism) decreases with η. As was explained before, this effect tends to vanish as the economy approaches the BGP. In a neoclassical growth model as ours, the speed of convergence is an increasing function of the exogenous rate of technical progress. The larger the growth rate η, the faster the dynamic adjustment to the BGP will be. Hence, the effect of demand-based mechanism may dominate and, therefore, a negative relationship between income and pollution may emerge during the transitional dynamics for sufficiently small values of η. Figure 5 shows that this is the case when η reduces to 0.001. In this case we obtain a N-shaped relation between trended values of income and pollution as some empirical studies derive from data.[11]

[Figure 5]

4.4. Accounting for the relative contribution of each mechanism

Before closing this section, we quantify the relative importance of the three aforementioned mechanisms driving the dynamics of pollution-income relationship in the benchmark economy. We make use of the expression (4.1) that gives the growth rate of pollution. As we can observe, the accumulation and the demand-based mechanism cannot a priory separate because both are behind the evolution of the ratio between detrented capital and detrented abatement expenditure. Thus, we first have to disentangle the accumulation mechanism from the demand-based mechanism. To this end, we build a counterfactual economy where the expenditure shares in consumption and abatement remains constant at the initial value corresponding to the benchmark economy. More precisely, we assume that both consumption and abatement grow in this counterfactual economy at the same rate as the aggregate expenditure $c_t + e_t$ in the benchmark economy. Using this procedure, we isolate the accumulation mechanism driving the pollution dynamics in the benchmark economy because we eliminate the effect from the non-homothetic feature of preferences. Of course, the demand-based mechanism is recovered by comparing the pollution in the benchmark economy with the one in the aforementioned counterfactual economy. Pollution caused by the demand-based mechanism is given by the residual of this comparison.

Once isolated and quantified each of the three mechanisms, we perform an accounting exercise to determine their relative contribution to the simulated pollution along the transitional dynamics of the benchmark economy. Figure 6 illustrates the results of this exercise. The left-hand panel of this figure shows that the accumulation and the trend mechanisms both contribute to a positive growth of pollution, whereas the demand-based mechanism has a clear negative effect on this growth. In the first part of the dynamics the accumulation mechanism is larger than the trend mechanisms, but in the long run the former vanishes and the latter maintains a constant and

[11] For instance, Sengupta (1997) found that the emissions of CO_2 exhibit an N-shaped relation with income. However, as Kijima et al. (2010) point out, there are not theoretical studies explaining the possible fundamentals of this particular income-pollution path.

positive effect. The demand-based mechanism also vanishes in the long run. However, this mechanism has large effect during the transition. In fact, along some part of the transitional dynamics the negative effect of the former mechanism almost fully compensates the positive effect of the other two mechanisms. The right-hand panel of Figure 6 corroborates these conclusions by plotting the path of pollution under the hypothetical cases where only one of the mechanisms was operative. It also compares this counterfactual pollution paths with the actual path derived in the benchmark economy. As was expected, the pollution would follow a clear downward-sloping trajectory if the demand-based mechanism was the unique force driving it. The pollution path would go along an upward-sloping path if any of the other mechanisms were the unique operative force.

[Figure 6]

Finally, observe that the dynamic adjustment of the expenditure composition, derived from the presence of non-homothetic preferences, in general reduces the flow of pollution. In particular, even in the case where η is sufficiently large such that the trend effect always dominates, the presence of the demand-based mechanism ensures that pollution for each level of income is smaller during the transitional dynamics than in the case of an economy with homothetic preferences (i.e., when $\bar{c}_0 = 0$).

5. Social optimum

Before closing our analysis, we will derive the social planned solution of our model in order to characterize the socially optimal relationship between pollution and GDP. This social planned solution is equivalent to a competitive equilibrium where consumers interiorize that their decisions on capital accumulation affect the flow of emissions. Hence, the difference between the competitive and the planned solution is the consumer's margin on the intertemporal allocation of expenditure. This margin is characterized by the following social Euler condition:

$$u_t^c = \beta u_{t+1}^c \left(1 + r_{t+1}\right) + u_{t+1}^p \left(\frac{\partial p_{t+1}}{\partial y_{t+1}}\right) \left(\frac{\partial y_{t+1}}{\partial k_{t+1}}\right), \tag{5.1}$$

where u_t^c is the marginal utility with respect to consumption at period t, which is given by (3.9); and u_{t+1}^p is the marginal utility with respect to pollution at period $t+1$, which is given by

$$u_{t+1}^p = -\lambda \left(c_{t+1} - \bar{c}_{t+1}\right)^{\gamma(1-\sigma)} p_{t+1}^{-\lambda(1-\sigma)-1}.$$

After substituting for the values of v_t and q_t in (3.6), we also derive the following expression for the Euler condition at the competitive equilibrium:

$$u_t^c = \beta u_{t+1}^c \left(1 + r_{t+1}\right).$$

By comparing the later condition and (5.1), we observe that they differ in the second term of the right-hand side of (5.1). In deciding the intertemporal allocation of consumption, a benevolent social planner takes into account that the present investment will reduce future welfare since the corresponding increase in capital stock will drive emissions up. Therefore, one should expect that the social level of investment would be smaller than the level in the decentralized economy. We will show that point by simulating the social planned solution of our economy.

Figure 7 compares the relation between pollution and income in the competitive economy and in the socially planned solution. The shape of this relation is identical in the two solutions. However, the level of pollution is smaller in the socially planned economy than in the competitive one, for any level of income. Hence, the relation between pollution and income does not qualitatively depend on whether or not consumers internalize the external effects derived from their decisions on consumption and investment. By comparing the rates of investment, consumption and abatement over GDP in the two solutions, we can characterize the inefficiency. Figure 8 illustrates this comparison by computing the path of these rates in the competitive economy and in the socially planned solution. Firstly, we observe that the investment rate is always larger in the competitive economy. The social planner then indirectly reduces the level of emissions by choosing a lower investment for a given level of income. Note that the reduction on investment has a permanent effect on the path of pollution because that reduction translates into a smaller stock of capital. This explains why the difference between the rate of investment in the competitive equilibrium and in the socially planned solution increases along time. We also observe that the path of the investment rate has a hump shape in the two solutions. Obviously, this is a consequence of our demand-based mechanism based on the non-homothetic feature of preferences.[12] Secondly, the social planner devotes less effort to abatement than the consumers in the decentralized economy because the level of emissions is always smaller in the socially planned economy. Finally, we observe that the propensity to consume is always larger in the socially planned economy. Moreover, this propensity follows a U-shaped path as a counterpart of the behavior of the investment rate.

[Figures 7 and 8]

It is obvious that the socially planned solution can be decentralized by either taxing capital income or by subsidizing the expenditure allocated to abatement. The first of these policies reduces the level of investment and, therefore, the level of emissions. On the contrary, the later policy alters the intratemporal margin on final expenditure in favor of abatement.

6. Conclusion

We have analyzed the relationship between pollution and aggregate income along the process of economic growth. To this end, we have characterized the equilibrium dynamics of an economic growth model where consumers are subject to a minimum consumption requirement that makes preferences on consumption and environmental quality non-homothetic. In this framework, we have disentangle the standard accumulation mechanism, that determines the intertemporal allocation of pollution,

[12] By using a sample of 24 OCDE countries, Antras (2001) illustrates that there is clear evidence of a hump shape for the investment rate in the series. He also shows that assuming Stone-Geary preferences, the neoclassical growth model could explain this path of the investment rate.

from the mechanism based on the non-homothetic feature of preferences, which leads the intratemporal allocation of expenditure between consumption and pollution abatement to depend on income. As the economy develops, the fraction of income devoted to abatement increases since the income elasticity of abatement is larger than unity. By incorporating this demand-based mechanism into a dynamic general equilibrium model, we have showed that the relationship between pollution and income may follow a non-monotonic path even when the elasticity of pollution with respect to emissions is larger than the elasticity of pollution with respect to abatement.

The income-pollution path emerging in our economy has some interesting implications for the environmental-oriented fiscal policy and for the comparison across countries. Consider three countries with different levels of income, such that they are placed in different sections of the N-shaped relation between income and pollution. First of all, we observe that these countries may still exhibit the same level of pollution. However, the future evolution of this pollution will dramatically be different in each country. The trade-off between economic growth and environmental quality is transitory for low-income countries. The development in those countries will alter the composition of expenditure in favor of pollution abating effort, so that the positive relationship between pollution and income will be reversed at some point. Therefore, pollution is not a great problem for countries with a sufficiently small income, and the best policy to reduce pollution in those countries would be the development aid.

By the contrary, the trade-off between economic growth and environmental quality is permanent in high-income countries. The minimum consumption requirement is not meaningful in those countries, so that the composition of expenditure is almost constant. Therefore, the evolution of pollution is only driven by the trend effect of economic growth. In this scenario, governments in the richest country should then set active environmental policies that reduce the impact of income on pollution by distorting consumers' decisions on consumption and abatement. For instance, the government may either tax consumption or subsidize the effort devoted to abatement. Any of these policies reduces the consumption-abatement ratio, so that the pollution for any level of income is smaller in presence of these public interventions. Obviously, the government can also directly tax the sources of pollution (i.e. production or capital accumulation).

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Appendix

A. Separating consumer's intertemporal and intratemporal problems

We first derive the first order conditions of the full optimization problem faced by the representative consumer. This problem consists in selecting c_t, e_t and k_{t+1} to maximize (2.5) subject to (2.2) and (2.6) after imposing $m_t = e_t$. By following a standard procedure, we then obtain the first order condition for consumption c_t, abatement e_t and assets a_{t+1}:

$$\frac{\gamma\left(1-\sigma\right)u_t}{c_t-\overline{c}_t}=\mu_t, \tag{A.1}$$

$$\frac{\lambda\pi\left(1-\sigma\right)u_t}{e_t}=\mu_t, \tag{A.2}$$

and

$$\mu_t=\beta\mu_{t+1}\left(1+r_{t+1}\right) \tag{A.3}$$

where u_t is the instantaneous utility in period t given by (2.3), i.e., $u_t = u\left(c_t, p_t\right)$, and μ_t is the Lagrangian multiplier associated with the flow of budget constraints (2.6). By adding (A.1) and (A.2), we obtain

$$\left(\gamma+\lambda\pi\right)\left(1-\sigma\right)u_t=\mu_t\left(c_t-\overline{c}_t+e_t\right). \tag{A.4}$$

Similarly, we raise (A.1) and (A.2) to $\gamma/\left(\gamma+\lambda\pi\right)$ and $\lambda\pi/\left(\gamma+\lambda\pi\right)$, respectively, and then we multiply the resulting expressions to obtain:

$$\frac{\gamma^{\frac{\gamma}{\gamma+\lambda\pi}}\left(\lambda\pi\right)^{\frac{\lambda\pi}{\gamma+\lambda\pi}}\left(1-\sigma\right)u_t}{\left(c_t-\overline{c}_t\right)^{\frac{\gamma}{\gamma+\lambda\pi}}e_t^{\frac{\lambda\pi}{\gamma+\lambda\pi}}}=\mu_t. \tag{A.5}$$

We derive from (2.2) that

$$e_t^{\frac{\lambda\pi}{\gamma+\lambda\pi}}=y_t^{\frac{\lambda\theta}{\gamma+\lambda\pi}}p_t^{\frac{-\lambda}{\gamma+\lambda\pi}}.$$

But plugging the latter expression in (A.5), and using the definition of v_t given by (3.4), we directly obtain

$$\frac{\gamma^{\frac{\gamma}{\gamma+\lambda\pi}}\left(\lambda\pi\right)^{\frac{\lambda\pi}{\gamma+\lambda\pi}}\left(1-\sigma\right)u}{y_t^{\frac{\lambda\theta}{\gamma+\lambda\pi}}v_t}=\mu_t. \tag{A.6}$$

Finally, combining (A.4) and (A.6), we directly get total expenditure as follows:

$$g_t=c_t+e_t=q_tv_t+\overline{c}_t.$$

By plugging the later expression into the budget constraint (2.6), we are equipped to directly derive the intertemporal and the intratemporal problems faced by the representative consumer in choosing his optimal plan.

B. Deriving the Euler Condition

By solving the intertemporal problem faced by the representative consumer, we obtain the first order condition for composite consumption v_t assets a_{t+1}:

$$(\gamma + \lambda\pi)\, v_t^{(1-\sigma)(\gamma+\lambda\pi)-1} = q_t \zeta_t, \tag{B.1}$$

$$\zeta_t = \beta \zeta_{t+1} \left(1 + r_{t+1}\right), \tag{B.2}$$

where ζ_t is the Lagrangian multiplier associated with the flow of budget constraints (2.6). By combining (B.1) and (B.2), we directly obtain de Euler condition (3.6).

Obviously this Euler condition can also be derived from the original, complete optimization problem consisting in selecting c_t, e_t and k_{t+1} to maximize (2.5) subject to (2.2) and (2.6) after imposing $m_t = e_t$. By using the definition of price index in (3.5), the composite good in (3.4) and the utility function in (2.3), we write condition (A.6) as follows:

$$(\gamma + \lambda\pi)\, v_t^{(1-\sigma)(\gamma+\lambda\pi)-1} = q_t \mu_t.$$

Combining the later condition with first order condition for assets (A.3) we also derive the Euler condition (3.6). Note that $\zeta_t = \mu_t$.

Table 1. Benchmark economy

A_0	α	θ	π	δ	η	β	σ	γ	λ	$\bar{c}_0$
1	0.36	0.24	0.15	0.025	0.005	0.99	1.5	1/3	1/3	1.048

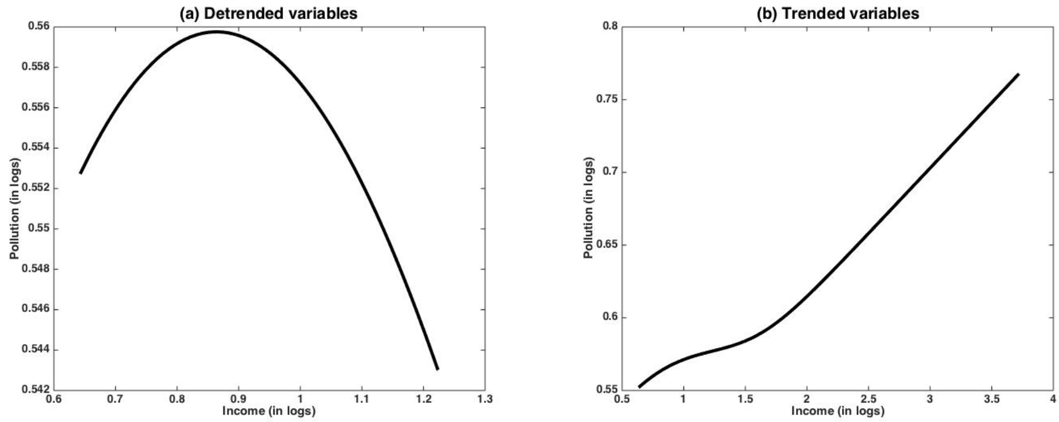


Figure 1. Income-pollution relation in the benchmark economy

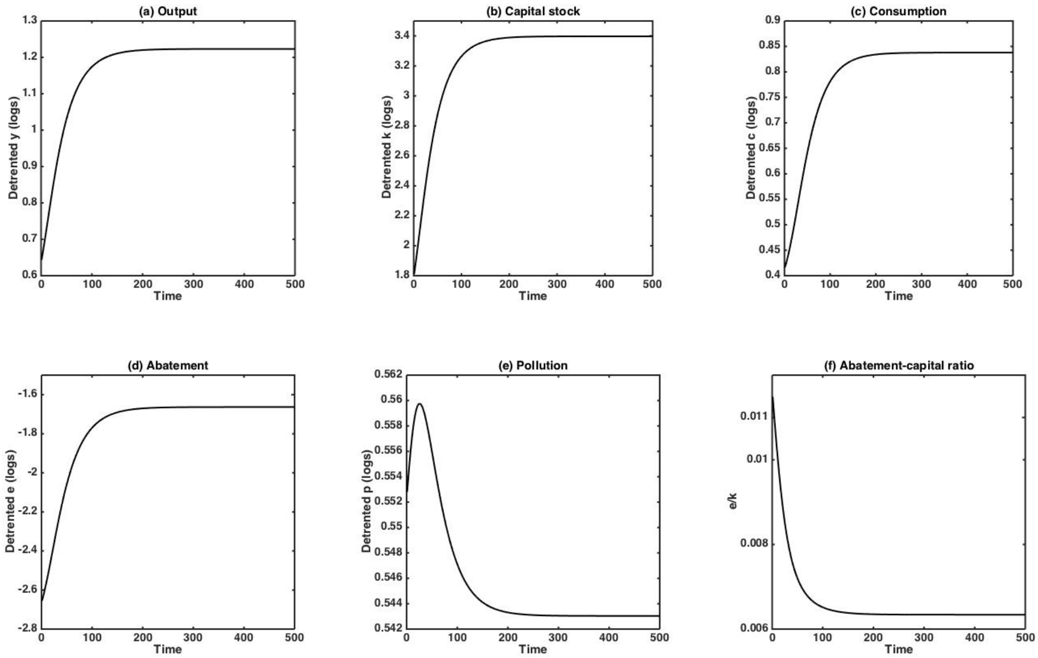
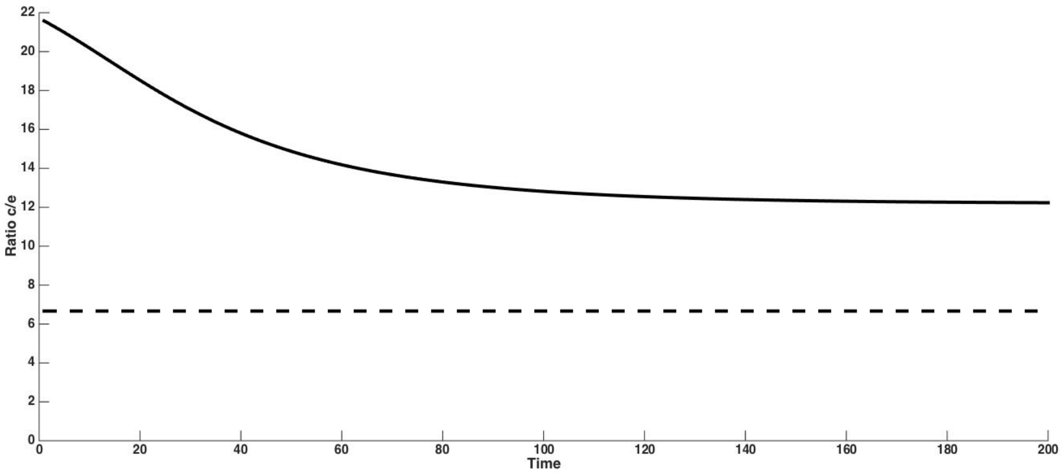
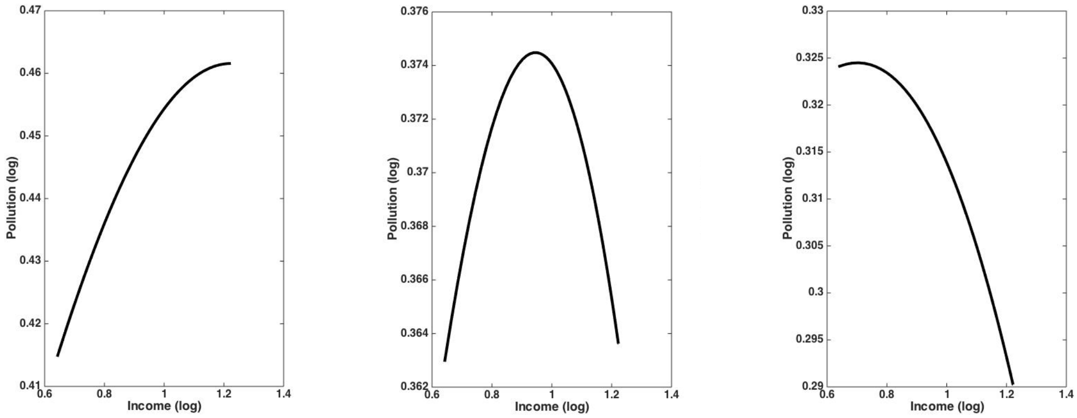


Figure 2. Transitional dynamics of benchmark economy $\left(\widehat{k}_0 = 0.2\widehat{k}^*\right)$



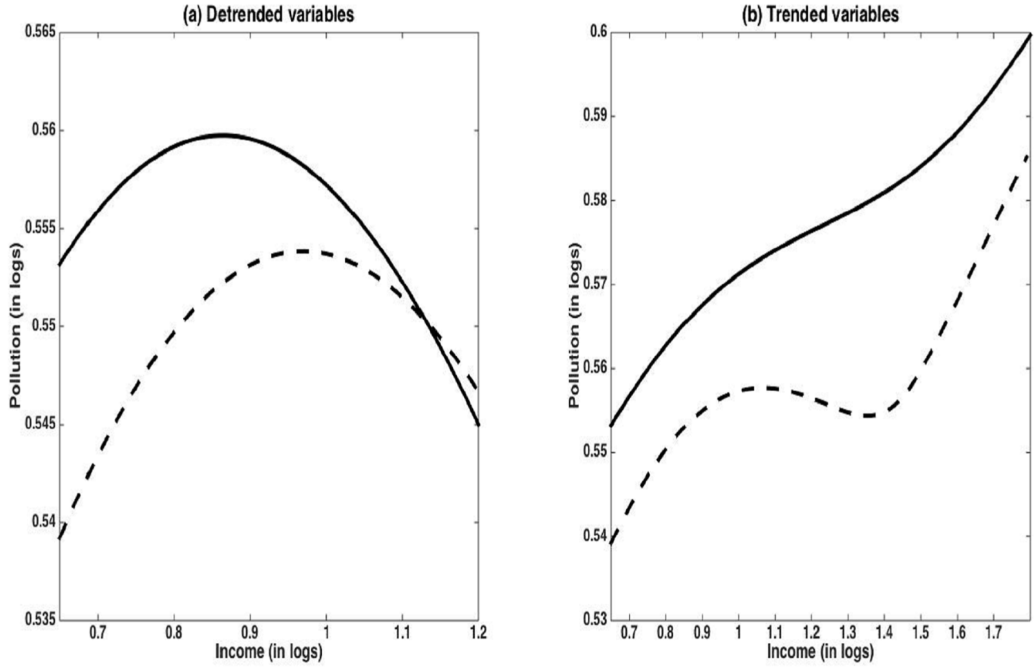
— Benchmark economy - - - Economy with $\bar{c}_0 = 0$

Figure 3. Dynamic adjustment of expenditure composition



(a) $\theta = 0.24$ (b) $\theta = 0.16$ (c) $\theta = 0.10$

Figure 4. Effects of θ on the income-pollution relation ($\bar{c}_0 = 0$)



— Benchmark economy ($\eta = 0.005$) - - - Economy with $\eta = 0.0001$

Figure 5. Effects of stationary growth rate on the income-pollution relation

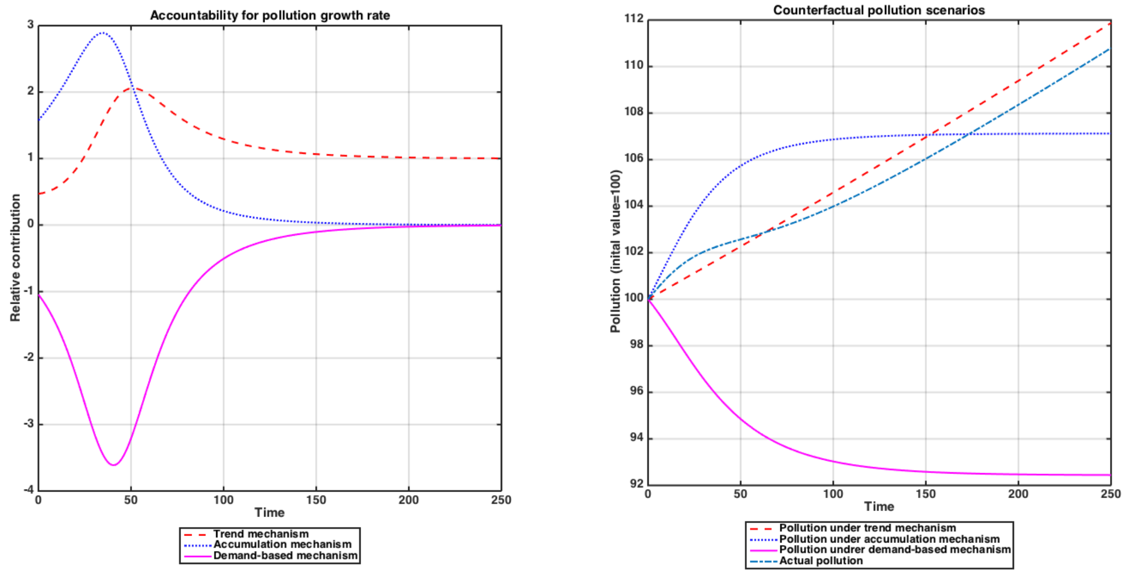
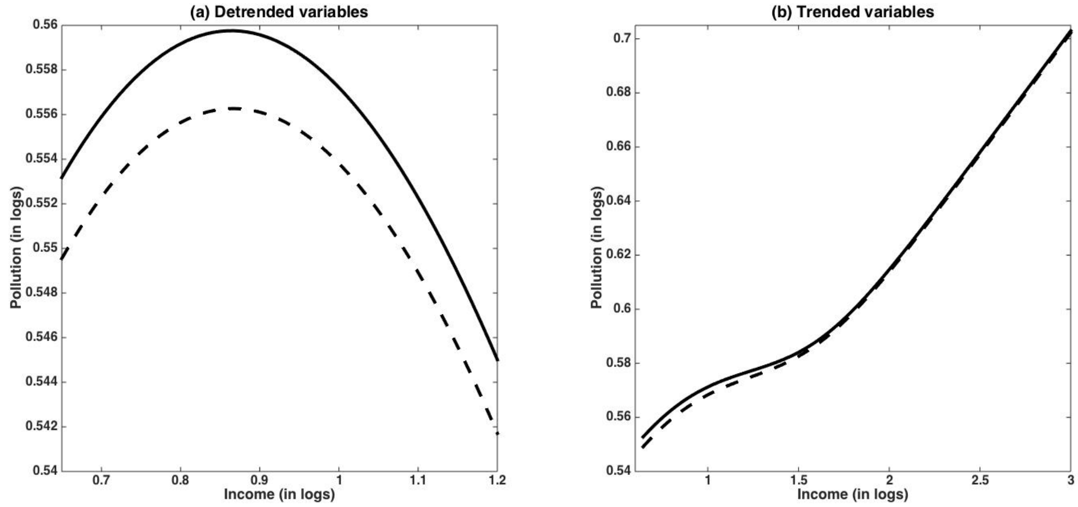
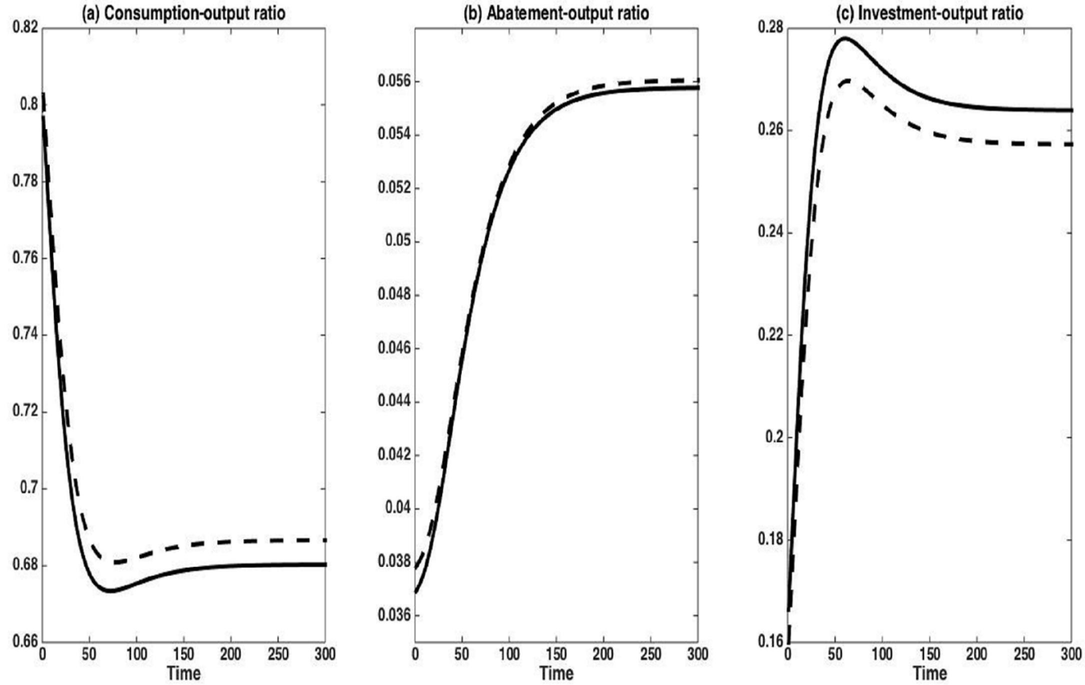


Figure 6. Relative contribution of the mechanisms behind pollution dynamics



— Competitive economy - - - Socially planned solution

Figure 7. Social vs. decentralized income-pollution relation



— Competitive economy - - - Socially planned solution

Figure 8. Dynamic adjustment of the expenditure and investment rates