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#### Title:

# OPTIMIZATION OF LOGISTIC ROUTES THROUGH INFORMATION SHARING POLICIES: A GAME THEORY-BASED APPROACH

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## Optimization of logistic routes through information sharing policies: A game theory-based approach

#### Abstract

Regulatory restrictions about the maximum number of FADs by vessel have been established in the tuna fishing industry during the last years, which incorporate new constraints on tuna fishing companies that need, however, to make profitable their high inversions. Based on real-data and argued with the scope of game theory, we address a new way of working for these companies related to the use of FADs between vessels, proving that sharing FADs maximizes both the fuel and time to entire fleets. Our findings show that, with the correct incentives, all stakeholders –company, skipper, and environment– can improve their results jointly when information is shared.

#### **Keywords**

FAD restrictions, Tuna fishing industry, Economic incentives for sharing, Fuel consumption reduction, Game theory, Sustainability

#### 1 Introduction

The performance of the tropical tuna fishing industry is, more than ever, bound to the use of drifting fish aggregating devices (FADs), which have become widespread since 1991 (Ariz, Delgado, Fonteneau, Gonzales Costas, & Pallarés, 1992). With the scope of new regulations affecting the tuna industry, this paper addresses a study from the point of view of efficiency using game theory as theoretical framework.

The global tuna fishery is one of the largest in the world. The most widely used and fastest-growing fishing gear for targeting tuna is the purse seine (PS). Tropical PS started to operate in the Atlantic Ocean in the 1960s and were introduced into the Indian Ocean in the early 1980s. The tuna species are skipkack (*Katsuwonus pelamis*), yellowfin (*Thunnus albacares*), and bigeye (*Thunnus obesus*), and they tend to associate with objects floating at the surface of the ocean (Castro, Santiago, & Santana-Ortega, 2001; P. Fonteneau & Pianet, 2000). The aggregate behavior of tuna with floating objects was first observed with natural floating objects (FOBs) from river mouths. With the aim of imitating FOBs, fishers started deploying large numbers of their own FOBs. These

human-made drifting FADs generally consisted of bamboo with large pieces of net hanging below for stability in the surface currents, and they would stay adrift for up to 2 months (Ménard, Stéquert, Rubin, Herrera, & Marchal, 2000).

The increasing use of FADs concurrently resulted in apparent increases in PS catches per unit effort (CPUE) over time (A. Fonteneau, Chassot, & Bodin, 2013; Maufroy et al., 2016). The extensive use of FADs by the PS fishery industry increases the possibility of a number of negative impacts, including a reduction in yield per recruitment of target tuna species, increased by-catch and perturbation of the pelagic ecosystem balance, and alteration of the normal movements of the species associated with FADs (Bromhead, Foster, Attard, Findlay, & Kalish, 2003; P. Fonteneau & Pianet, 2000), however these effects are difficult to estimate (Lopez, Moreno, Sancristobal, & Murua, 2014).

Due to the increased use of FADs, recent efforts from regional fisheries management organizations (RFMOs) have produced regulations on the number of FADs that a PS can manage. Other restrictions affect the global marine fisheries, like the minimization of bycatch and discards (Gilman, 2011; Zeller, Cashion, Palomares, & Pauly, 2018).

With these newly implemented restrictions, it is mandatory that the tuna fishing industry optimize the use of FADs. Although many studies have been published regarding the use of FADs and their implications, little research exists in how to help the tuna fishing industry optimize their fishing practices (Groba, Sartal, & Vázquez, 2015).

In this context, our work addresses a new way of working for the tuna fishing companies related to the use of FADs between vessels, proving that sharing FADs maximizes fuel efficiency and use of time and decreases CO<sub>2</sub> emissions across entire fleets. First, with a foundation in game theory and the well-being assessment in particular, two different theoretical mechanisms were developed: one without compensation and the other with compensation. The first mechanism (without compensation) explains why many vessels do not like sharing FADs, and the second mechanism (with compensation), shows an equilibrium where all vessels want to share FADs. Second, this theoretical approach is evaluated empirically with real-data through simulations, and the expected result emerges: There is a situation in which all the players –company and skippers– win, proving that best route optimization takes place when FADs information is shared between vessels. Data for this study come from different groups of tuna vessels retrieving their FADs in the Indian Ocean during April 2017.

The paper is organized as follows. The next section provides a review of the literature. Section 3 describes the game theory approach. Section 4 introduces the data, the experimental design and discusses the results. Finally, Section 5 concludes by highlighting the paper's main contributions and implications.

#### 2 Background

The use of FADs by PS has had evolved over the years to improve fishing efficiency. The FAD itself has undergone improvements in shape, materials, and the lengths of nets,

both to drift with the currents of interest and also for minimizing the risk of entangling turtles and other non-targeted species (Girard, Benhamou, & Dagorn, 2004). FADs have also evolved technologically. Since the beginning, the use of artificial FADs has relied on tracking buoys to know where the FADs are. The first buoys were radio-based, and each vessel used secret frequencies to locate its own floating objects (Ménard et al., 2000). In 1996, GPS buoys with a virtually unlimited range appeared on the market and positively affected the expansion of fishing areas (Morón, Areso, & Pallares, 2001).

During the 2000s, satellite technologies, including Inmarsat D+ and Iridium SBD, became affordable alternative for the buoys (Moreno, Dagorn, Sancho, & Itano, 2007). The use of satellite communications was a revolution for the tuna fishing industry because, although each buoy had a monthly airtime fee, there were many advantages compared to the previous buoys based on radio technology. Some of these advantages included receiving FAD positions at any distance and commanding the buoys from the vessel. Further, satellite buoys did not use large carbon antennas for transmission like the radio buoys did. Additionally, satellite buoys were difficult to detect by radar, making them less likely to be stolen, which was a big advantage over radio buoys. Finally, satellite communication technology provided vessels the capacity to share their FADs positions with other vessels, which allowed vessels to work together with the aim of improving their fishing efficiency.

The last buoy improvement was the development of echo-sounders, which were introduced around 2008 to monitor the amount of biomass aggregated beneath the FAD (Lopez et al., 2014). This new technological device reduced the searching time (i.e., enables remote identification of FADs with associated tunas) and provided new information for fishers to learn more about the location and behavior of tuna and other associated species. Several indicators suggest that echo-sounder buoys could be as important or more important than other significant technological developments in the fishery industry, such as the introduction of sonar (Lopez et al., 2014).

The echo-sounder technology embedded in the buoy was a game-changer for the tropical tuna industry in terms of optimization. Before this improvement, PS traveled from FAD to FAD searching for tuna. After the introduction of the echo-sounder tuna buoys, they only traveled to FADs that had fish beneath, which improved their fishing efficiency by saving time and fuel and by discovering new fishing areas. Such efficiency made PS want more echo-sounder buoys in the water (then FADs), with the aim of constant fishing.

The consequences of this were an increase in the use of FADs. For example, in the Atlantic Ocean (A. Fonteneau, Chassot, & Gaertner, 2015), the total number of FADs increased 730%, from 1,175 FADs active in January 2007 to 8,575 in August 2013. In the Indian Ocean this number increased 458%, from 2,250 FADs in October 2007 to 10,300 FADs in September 2013 (Maufroy et al., 2016). This increase has resulted in regulation from RFMOs of the number of FADs that a PS can manage, for example, in the Indian Ocean, where the number of FADs as defined in Resolution 15/08, paragraph 7, will be no more than 350 active instrumented buoys and 700 acquired annually instrumented

buoy per vessel per year (IOTC circular 2017-061). Currently, this limitation is also being followed in the Atlantic Ocean through the International Commission for the Conservation of Atlantic Tunas (ICCAT), and all indicators predict that the Pacific Ocean will follow the same initiative via the Inter-American-Tropical-Tuna-Commission (IATTC) (P. Fonteneau & Pianet, 2000).

Regarding PS activity with FADs, it is noteworthy that skippers have important economic incentives depending on how many tons they fish. Meanwhile, tuna fishing companies or firms pay these incentives with the aim of maximizing the number of fished tons of the whole company. The costs of the entire fleet are assumed by the firm, including salaries, goods, and fuel, among others.

In terms of fishing management and efficiency, Salas and Gaertner (2004) showed how essential it is for effective management to know the dynamics of the fisheries. Bez, Walker, Gaertner, Rivoirard, and Gaspar (2011) used a vessel monitoring system (VMS) to measure tuna fishing efforts to study and quantify the spatial dynamic of the tropical tuna PS fishing activity. In terms of fuel consumption, Parker, Vázquez-Rowe, and Tyedmers (2015) analyzed fuel performance and the carbon footprint of the global PS tuna fleet. Meanwhile Hospido and Tyedmers (2005) employed life cycle assessment (LCA) to quantify the scale and importance of emissions that result from the range of industrial activities associated with contemporary Spanish PS fisheries. Gaertner and Dreyfus-Leon (2004) analyzed the shape of the relationship between CPUE and abundance in a tuna PS fishery, using a simulation with artificial neural networks. In terms of fuel consumption efficiency, Groba et al. (2015) showed how important it can be to optimize the route of a tuna vessel retrieving FADs.

There are different studies in terms of efficiency in different fisheries, for example Belhabib, Greer, and Pauly (2018) compared the artisanal fisheries versus the industrial fisheries, showing that the catches per unit effort (CPUE) of the artisanal fisheries was 11 times lower than industrial CPUE. Guijarro, Ordines, and Massutí (2017) studied the bottom trawl fishery in order to improve the efficiency in the Western Mediterranean. Another example is given by Rust, Yamazaki, Jennings, Emery, and Gardner (2017), discussing the excess capacity and efficiency in the quota managed Tasmanian Rock Lobster Fishery, or the efficiency in the sardinian fisheries cooperatives (Madau, Furesi, & Pulina, 2018).

In the case of tropical tuna fishery efficiency, the literature is scarce. For this reason, and with recent RFMO regulations in mind, the proper use of FADs by tuna vessels is a matter of great importance for tuna fisheries.

In this paper, tuna fishing vessels behaviours using FADs is studied for first time from the point of view of game theory. Indeed, through real-data simulations, this paper shows that there are policies that change ways tuna vessels work with FADs. If tuna fishing companies settle on these new policies they could improve their overall efficiency.

### 3 The tuna fishing vessels problem: A game theory approach

#### 3.1 The tuna fishing vessels problem

Tuna skippers have important economic incentives that directly depend on how many tons they fish. These incentives also depend on the tuna ton price (Jeon, Reid, & Squires, 2008). Because of this, it is important for skippers to maximize the number of ton fished; the more a vessel fishes in less time, the better. A tuna vessel faces, for instance, an optimization problem regarding which route to follow to increase fishing using its FADs, which drift in the ocean (Groba et al., 2015).

Each vessel has its own limited number of FADs to fish. Tuna vessels usually work in small groups, of two or three fishing vessels assisted by a supply vessel to deploy and retrieve FADs from the ocean (Arrizabalaga et al., 2001). In these cases, FADs are shared among the group of fishing vessels, and incentives are shared as well. Groups of vessels that fish together are share confidence, which is one reason they are typically small.

By contrast, firms want to maximize the overall company profits, which means that vessels have to fish as much as they can and that variable costs, such as fuel, crew costs, and equipment must be minimized. This means optimizing fishing of n vessels (vessels that the firm owns) with m FADs (the sum of FADs from all the vessels of the company).

In this scenario, there is a trade-off between firm's and the skippers' interests, highlighted by the limitation of FADs as dictated by RFMOs. Facing this situation, a new scenario for analysis and improvement appears, hinging on how to maximize the profits for all agents. The aim of this paper is to study this equilibrium in detail with real-data and explain how and why tuna fisheries currently operate. Further, this paper presents a new proposal for FAD sharing policies, which shows improvements for both individual and collective performance and reduction of CO<sub>2</sub> emissions.

#### 3.2 A game theory approach

A theoretical model was introduced to study the problem described previously. We considered two different mechanisms that the firm can use to incentivize vessels to share FADs. When vessels share FADs, the total distance traveled by all vessels is reduced, which produces cost savings for the firm. Our analysis was conducted through a non-cooperative game with incomplete information following the model of Aumann (1976), which we believe is the most suitable for this case. We also consider the Bayesian Nash equilibria (BNE) (Nash, 1951), the most standard solution for these kind of games (Harsanyi, 1967).

Let  $N = \{1, ..., n\}$  the set of tuna vessels, briefly, vessels. We assume that all tuna vessels work for the same firm, which we denote by f.

There is a finite number of FADs (or buoys) that have been assigned to the vessels

following some criteria. We assume that each FAD is assigned to a single vessel. Thus, each vessel  $i \in N$  has an initial endowment  $b_i = \{(b_i^k)\}_{k=0}^{n_i} = \{(x_i^k, y_i^k)\}_{k=0}^{n_i}$ . The interpretation is the following. Vessel i has been assigned to handle  $n_i$  FADs  $\{b_i^1, \ldots, b_i^{n_i}\}$ . The position of each FAD k with  $k = \{1, \ldots, n_i\}$  is given by  $(x_i^k, y_i^k)$  where  $x_i^k$  denotes the latitude and  $y_i^k$  the longitude. Besides we denote by  $b_i^0 = (x_i^0, y_i^0)$  the position of vessel i at the beginning of the process. We also assume that FADs are numbered in the order of recovering by vessel i. Namely, vessel i is located in position  $(x_i^0, y_i^0)$ . Thus, it moves to FAD  $b_i^1$  and recover the tuna in such FAD. Next, vessel i moves to position FAD  $b_i^2$  and so on.

Therefore, we make the following assumptions:

- Each vessel knows the position of all FADs to which has been assigned. Each vessel does not know the location of the FADs assigned to other vessels.
- In the theoretical model, we assume that each vessel has a cost c per mile traveled between FADs. This cost is paid by the firm. In our simulations, we compute c by assuming that the average vessel speed is 15 knots. Thus, we estimate a fuel cost of \$29 US per nautical mile traveling between FADs.
- Vessels cannot know in advance the amount of tuna they will find at each FAD. We denote by q the expected amount of tuna by FAD. We denote by  $q_i^k$  the amount of tuna recovered by skipper i in FAD  $b_i^k$ . These amounts will be known only after fishing.

In our simulations we take q = 6.1 tons for every skipper i and every FAD  $b_i^k$ .

• Each vessel i cannot know in advance the amount of time  $t_i^k$  for recovering the tuna of FAD  $b_i^k$ .

In our simulations we assume that  $t_i^k$  is 3 hours for every vessel i and every FAD  $b_i^k$ .

• Each skipper receives a price p by each amount of tuna fished. In our simulations we consider several values for p.

Once vessel (skipper) i has recovered all of its FADs, the utility obtained is computed as the amount fished multiplied by the price paid by the firm. Namely,

$$p\sum_{k=1}^{n_i} q_i^k$$

The utility of the firm is

$$(p_f - p) \sum_{i=1}^{n} \sum_{k=1}^{n_i} q_i^k - c \sum_{i=1}^{n} d(b_i)$$

where  $d(b_i)$  is the distance traveled by vessel *i* for recovering all FADs in  $b_i$ . Namely, the firm pays a unit price of p to every vessel and sells the fish at the price  $p_f$ . Additionally, the firm has to pay costs associated with the travel of the vessels.

As skippers do not pay for fuel, they do not have an incentive to share their FADs to minimize the distance traveled. Nevertheless, the firm has incentives. If the cost is reduced (and all FADs are recovered), then the total utility of the firm will be increased.

We consider two possible mechanisms that firms can use to induce skippers to share their FADs. We model such mechanisms as two games with incomplete information following Aumann's model. Additionally, we study the Bayesian Nash equilibria (BNE) of both games, which provide predictions of the behavior of rational agents when facing such situations.

In the Appendix, we theoretically study both mechanisms, and we formally present the games for modeling both mechanisms. We also compute the BNE associated with both mechanisms (Propositions 1 and 2).

For now, we present the results in a more informal way. The basic idea of both mechanisms is the same. First, the vessels or skippers decide independently if they want to share its FADs or not. If a vessel says no, then this vessel fishes with its FADs. For the vessels that say yes, the firm reassigns their FADs among the cooperating. Next, every vessel fishes in its reassigned FADs.

Mechanism 1: Reassigning FADs without compensation. The firm will pay to the skippers according to the FADs each vessel has been assigned. Suppose that vessel i initially had 20 FADs, decided to share its FADs, and was reassigned with 18 FADs. The firm will pay skipper i according to the amount of fish obtained by the 18 reassigned FADs. If vessel i is reassigned with the same or more FADs than it initially had, then vessel i will be paid also according with the number of assigned FADs.

In Proposition 1 of the Appendix we theoretically study this mechanism. Here we discuss the practical implications of Proposition 1. According to part (a), if each vessel decides not to share its FADs (as in Example 2 of the Appendix), then we have a BNE, and the firm cannot save in fuel. In other cases (as in Example 1 of the Appendix), there could exist a different BNE, wherein some vessels share FADs and the firm saves fuel costs.

By part (b) of Proposition 1, we realize that the utility of each skipper i in any BNE will always be the same and coincide with the utility skipper i obtains when it does not share FADs. This result is independent of the number of FADs, the position of the FADs, and the information the vessels have over the position of the FADs. Thus, skipper i does not have an incentive to share its FADs in any circumstance because skipper i cannot improve its expected utility by sharing instead of not sharing. If skipper i shares its FADs, it could be the case that skipper i receives more FADs than it initially had, but it could also receive less. The average will be the same.

Our theoretical results prove that under this mechanism, skippers do not have

incentives to share their FADs under any circumstance. This helps explain why tuna vessels work alone or in small groups. Nevertheless, this mechanism is not the most beneficial for the firm.

Mechanism 2: Reassigning FADs with compensation. The firm will guarantee to skippers that share their FADs to pay, at minimum, according with the number of FADs the vessel initially had. For example, suppose that vessel i initially had 20 FADs, decided to share its FADs, and is reassigned with 18 FADs. The firm will pay skipper i according to the amount obtained by vessel i would receive if it recovered 2 more FADs. If vessel i is reassigned with the same or more FADs than it initially had, it will be paid according to the number of reassigned FADs.

In Proposition 2 of the Appendix, we theoretically study this mechanism. Here we discuss the practical implications of Proposition 2. According to part (a), we know that there is a BNE when every skipper decides not to share its FADs. The same applies to Mechanism 1. Per part (c), there is also a BNE when every skipper shares its FADs and the firm reorganizes all the FADs optimally. Further, the utility of the firm and each skipper under part (c) is greater than or equal to when no vessels share FADs.

Next we asked, when is the BNE of part (c) different from that of part (a)? We also asked the extent of these differences. In Example 3 of the Appendix, both BNE are essentially the same. Thus, from a theoretical point of view, the answer to our question is that it depends on the characteristics of the problem. We then offered (in the next section) a practical answer to both questions. After developing simulations based on real data, our results showed that, in all cases studied, the BNE of part (c) was different from the one of part (a). Additionally, both were quite different in terms of utility obtained by the skippers and the firm. In this case, the firm clearly benefits more than the skippers.

Part (b) of Proposition 2 says the following: Suppose that skipper i decides between sharing or not sharing its FADs. Independent of the position of its FADs or the decision taken by other skippers, the expected utility obtained sharing its FADs is never smaller than the expected utility obtained not sharing its FADs. This means that the Bayesian Nash equilibria we should observe in practice is the one in which every skipper shares its FADs. Thus, with Mechanism 2, every skipper has incentives to share its FADs, and this mechanism is also suitable for the firm.

#### 4 Data and results

In this section we design an experiment that, based on data from FADs movements, assesses the theoretical propositions made in the previous chapter. It is worth recalling that we used real-data from different tuna fishing companies. To test our model exclusively for scientific purposes, Marine Instruments provided us with anonymous real-data from several tuna vessels fishing in the FAO capture zone no. 57 (Eastern Indian Ocean) from April 9 to April 23, 2017. Specifically this experiment was based on a tuna company composed of 3 vessels (i.e., 3 skippers) with 20 FADs per vessel.

This information was obtained randomly using the MSB software, a platform for receiving and visualizing buoy data, from Marine Instruments. We performed 10 measurements in each experiment, varying the positions of the FADs and the vessels, to obtain representative mean values for each case study. We suppose that tuna vessels navigate at 15 knots and, for simplicity, the expected average of tuna by FAD is 6.1 tons, with \$29 per nautical mile the cost of fuel at this speed. All these working conditions are represented in Table 1 and were obtained from Marine's historical records for vessels working in this area during the last decade:

Table 1: Experiment assumptions

| Description           | Value     |
|-----------------------|-----------|
| Number of vessels     | 3         |
| FADs per vessel       | 20        |
| Vessel speed          | 15  knots |
| Fishing time          | 3 hours   |
| Tons beneath each FAD | 6.1       |
| Cost per ton          | \$1,400   |
| Fuel cost per mile    | \$29      |
| Skipper benefit       | 10%       |

It should also be noted that, for correct interpretation of the results, all skippers have variable benefits that depend on the quantity of fish they catch. While this value may be different from one company to another, we have supposed an average benefit of about 10% of the total amount of tuna fished, which is based on the average tuna stock price. Although the companies did not give us this information, they confirmed that the supposed percentage is a reasonable value, and this percentage does not affect the theory we want to prove. For simplicity, we paid no attention to the firm's fixed expenditures, such as crew costs, supplies, fishing licenses, etc.

Considering these conditions and following the same structure as in the previous theoretical section, a total of three different scenarios were considered. The first scenario (Table 2) describes the current situation where the skippers do not share their FADs. In the second scenario (Table 3) the three skippers share the FADs without compensation. Finally, in the last scenario (Table 4), the same situation is proposed but with compensation for the skippers to share. Next, each of these three situations is analyzed in detail.

In the first scenario we assumed that the skippers did not share their FADs. In these conditions, therefore, each skipper only knows the position of their own FADs. The results obtained are shown in Table 2, where we can observe the money earned by each skipper (vessel) and the money earned by the firm (owner of the three tuna vessels) within the conditions (tons per FAD, cost per ton, fuel cost, etc.) previously illustrated in Table 1. To obtain a representative average value (Avg.), we have repeated each simulation 10 times representing different FADs situations. It is worth recalling that we

assume the same quantity of fish beneath of each FAD. Therefore, the expected earned money for each skipper is the same for each simulation but it is not for the firm, because totals also depend on how many miles the vessels navigate, and firm benefits depend not only on the amount of tuna captured, but also on the fuel spent; the more miles traveled, the fewer benefits for the firm.

Table 2: Current way of working: Vessels do not share their FADs

|      | Skipper 1 | Skipper 2 | Skipper 3 | Ship owner |
|------|-----------|-----------|-----------|------------|
| 1    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 248,267 |
| 2    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 247,448 |
| 3    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 244,741 |
| 4    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 255,185 |
| 5    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 255,032 |
| 6    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 256,515 |
| 7    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 258,642 |
| 8    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 255,185 |
| 9    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 244,166 |
| 10   | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 251,943 |
|      |           |           |           |            |
| Avg. | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 251,712 |

In the second scenario (Table 3) we assume that the three skippers (vessels) agree to share their FADs, so the firm makes an optimal distribution of the FADs and assigns them in a smart way from the office to the vessels. This means that sometimes one vessel can have 20 FADs, sometimes more and sometimes less. Table 3 shows these results, and we can observe that skippers 1 and 2 achieve greater benefits than skipper 3 because, on average, they had more FADs during the simulations.

The firm in this scenario would obtain important benefits because of the fuel saved by this smart distribution of FADs, reaching 8.5% improvement compared to the previous scenario. However, the total benefits of the skippers does not change. Skipper 1 improves 4.3%, skipper 2 improves 0.8% but skipper 3 decreases 5.1% (compared to Table 1, which reflects current fishing methods). This results confirm the theoretical results we have seen in the previous section. Skippers do not have not incentive to share their FADs with other vessels because the expected benefit of a skipper when sharing their FADs, is the same that when no sharing the FADs. Thus, as there is no expectation of improvement, it seems very likely that the skippers would not want to take risks and continue working only with their own FADs. Seen from the global point of view of the company (and shareholders), however, the best scenario would involve sharing. Our empirical results corroborate the theoretical assumptions described above and help explain why many tuna vessels work alone. Nevertheless, this mechanism is not the more suitable for the firm.

In the third scenario the firm changes its strategy of incentives for the skippers, as shown in Proposition 2 (see Mechanism 2: Reassigning FADs with compensation in Section 3). In this scenario, the firm guarantees pay to vessels that share FADs at least

Table 3: Mechanism 1: Reassigning FADs without compensation

|      | Skipper 1 | Skipper 2  | Skipper 3 | Ship owner |
|------|-----------|------------|-----------|------------|
| 1    | \$ 17,818 | \$ 16,121  | \$ 16,970 | \$ 281,716 |
| 2    | \$ 17,818 | 1 \$ 6,970 | \$ 16,121 | \$ 266,843 |
| 3    | \$ 19,515 | \$ 15,273  | \$ 16,121 | \$ 269,936 |
| 4    | \$ 16,970 | \$ 16,970  | \$ 16,970 | \$ 276,557 |
| 5    | \$ 17,818 | \$ 16,121  | \$ 16,970 | \$ 285,337 |
| 6    | \$ 16,970 | \$ 18,667  | \$ 15,273 | \$ 258,980 |
| 7    | \$ 17,309 | \$ 18,723  | \$ 14,877 | \$ 269,586 |
| 8    | \$ 17,164 | \$ 19,063  | \$ 14,683 | \$ 270,121 |
| 9    | \$ 19,515 | \$ 16,121  | \$ 15,273 | \$ 279,064 |
| 10   | \$ 16,121 | \$ 16,970  | \$ 17,818 | \$ 272,243 |
|      |           |            |           |            |
| Avg. | \$ 17,702 | \$ 17,100  | \$ 16,107 | \$ 273,038 |
|      |           |            |           |            |
| Diff | 4.3%      | 0.8%       | -5.1%     | 8.5%       |

according to the number of FADs the vessel initially had. In other words, when a skipper has fewer FADs assigned than the average, he or she is automatically compensated by the firm. For example, when a skipper has 2 FADs fewer than the current situation (i.e., 20), the company will still pay for 20 FADs, so there is not any risk for the skipper. But, when the skipper has more FADs assigned than the average (for instance, 21), he or she will keep them, fishing more, and the quantity of fish expected for each vessel will be more than in the first scenario. With this policy of incentives, it seems logical to expect the bosses to be encouraged to collaborate since everybody wins. In fact, as was theoretically proved in Proposition 2, every skipper has incentives to share its FADs; therefore, the Bayesian Nash equilibria we should observe in practice with real-data is the one in which every vessel shares its FADs.

The results are shown in Table 4, where we can clearly see that each skipper enjoys more benefits than in the first scenario where did not share. Further, the firm continues earning more than the first scenario, although less than the second, as it was expected. In this way, our empirical findings complement the previous theoretical section. While, theoretically, we could only predict that it was favorable to share FADs, the simulations performed not only confirm this but also verify that both the company and the skippers get more benefit with this new procedure. In fact, with our data, we can also estimate how much more they will earn on average over time. We can assure, therefore, that this mechanism is suitable for the firm.

We used the nearest neighbor strategy for recovering FADs during the simulations. This means that FAD distribution was based on assigning FADs closer to each tuna vessel. This is a quick and sound distribution method commonly used by the tuna industry at present, but it is far from being optimal. The results could be further improved if this recovery strategy was changed to adapt to the dynamic nature of drifting FADs,

Table 4: Mechanism 2: Reassigning FADs with compensation

|      | Skipper 1 | Skipper 2 | Skipper 3 | Ship owner |
|------|-----------|-----------|-----------|------------|
| 1    | \$ 17,818 | \$ 16,969 | \$ 16,969 | \$ 280,867 |
| 2    | \$ 17,818 | \$ 16,969 | \$ 16,969 | \$ 265,994 |
| 3    | \$ 19,515 | \$ 16,969 | \$ 16,969 | \$ 267,390 |
| 4    | \$ 16,969 | \$ 16,969 | \$ 16,969 | \$ 276,557 |
| 5    | \$ 17,818 | \$ 16,969 | \$ 16,969 | \$ 284,488 |
| 6    | \$ 16,969 | \$ 18,666 | \$ 16,969 | \$ 257,283 |
| 7    | \$ 17,309 | \$ 18,723 | \$ 16,969 | \$ 267,492 |
| 8    | \$ 17,163 | \$ 19,062 | \$ 16,969 | \$ 267,834 |
| 9    | \$ 19,515 | \$ 16,969 | \$ 16,969 | \$ 276,518 |
| 10   | \$ 16,969 | \$ 16,969 | \$ 17,818 | \$ 271,394 |
| Avg. | \$ 17,786 | \$ 17,524 | \$ 17,054 | \$ 271,582 |
| Diff | 4.8%      | 3.3%      | 0.5%      | 7.9%       |

as is shown in Groba, Sartal, and Vázquez (2018). In this case, it was proved that the quantity of miles traveled could be reduced by 21.4% in the case of 3 veseels and 20 FADs per vessel in comparison with the NN strategy, as we use in our approach. It not only indicates that the firm is going to earn more due to the route optimization, but also that fishing time will be reduced, so skippers can fish the same quantity in less time and still gain all the associated economic and environmental implications.

#### 5 Conclusions

Based on data from different groups of tuna vessels retrieving their FADs in the Indian Ocean during 2017, this paper proposes a new, coordinated way of working for the tuna fishing companies related to FAD collection. Situated within well-known game theory, our findings reflect the value of sharing FADs. We demonstrate that, with the correct incentives, there is a situation in which all stakeholders—company, skipper, and environment—obtain better results. Further, global economic profits are realized for the fleet and company, and CO<sub>2</sub> emissions are reduced.

From a scholarly perspective, our work provides empirical evaluation using real-data and supports and applies the adequacy of the proposed theoretical model –a non cooperative game with incomplete information following the model of Aumann considering Bayesian Nash equilibria— to a complex, real-world situation. We assumed two different situations: 1) reassigning FADs without compensation and 2) reassigning FADs with compensation. While in the first situation our results only corroborate theoretical assumptions (and explain why tuna vessels work alone), the empirical portion of the second situation complements the previous theoretical section. While, theoretically, we

could only predict that it was favorable to share FADs, the simulations performed not only allow us to confirm this and verify that both the company and the skippers get more benefit with this new procedure. Additionally, with our data, we can also estimate how much more they each will earn on average over time. We can assure, therefore, that this mechanism is suitable for the firm.

As the firm will enjoy savings due to the route optimization, tuna vessels will reduce their fishing time and fuel consumed. In addition, fuel reduction presents another important advantage: increased storage capacity. This paper opens up, therefore, a set of possibilities for a wide range of real-world problems.

Similarly, from a policy-maker perspective, our work addresses a new, more-efficient way to work with increasing FAD regulations regarding the number of FADs per PS. While, these regulations were first introduced in the Indian Ocean by the Indian Ocean Tuna Commission (IOTC) with Resolution 17/08, it is very likely that these regulatory restrictions will soon extend to the rest of the oceans by the ICCAT and the IATTC. It is mandatory, therefore, for the tuna fishing industry to optimize the use of FADs. As it seems clear that this number will be drastically reduced in the next few years, there is no other way but to use them as efficiently as possible.

From an environmental perspective, our proposal would directly reduce the total current  $CO_2$  emissions. This is a significant improvement, as climate change is one of the main problems facing humanity today (Howard-Grenville, Buckle, Hoskins, & George, 2014). In addition, the development of more sustainable fishing methods using FADs may be possible because of our research.

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#### References

Ariz, J., Delgado, A., Fonteneau, A., Gonzales Costas, F., & Pallarés, P. (1992). Logs and tunas in the eastern tropical atlantic. a review of present knowledge and uncertainties. In *Proceedings of the international workshop on fishing for tunas associated with floating objects, la jolla, ca* (pp. 21–65).

Arrizabalaga, H., Ariz, J., Mina, X., de Molina, A. D., Artetxe, I., Pallares, P., & Iriondo, A. (2001). Analysis of the activities of supply vessels in the indian ocean from observers data. *Doc. IOTC*.

Aumann, R. (1976). Agreeing to disagree. Annals of Statistics, 4, 1236–1239.

- Belhabib, D., Greer, K., & Pauly, D. (2018). Trends in industrial and artisanal catch per effort in west african fisheries. *Conservation Letters*, 11(1), e12360.
- Bez, N., Walker, E., Gaertner, D., Rivoirard, J., & Gaspar, P. (2011). Fishing activity of tuna purse seiners estimated from vessel monitoring system (vms) data. *Canadian Journal of Fisheries and Aquatic Sciences*, 68(11), 1998–2010.
- Bromhead, D., Foster, J., Attard, R., Findlay, J., & Kalish, J. (2003). A review of the impact of fish aggregating devices (fads) on tuna fisheries: Final report to the fisheries resources research fund. Bureau of Rural Sciences Canberra.
- Castro, J. J., Santiago, J. A., & Santana-Ortega, A. T. (2001). A general theory on fish aggregation to floating objects: an alternative to the meeting point hypothesis. *Reviews in fish biology and fisheries*, 11(3), 255–277.
- Fonteneau, A., Chassot, E., & Bodin, N. (2013). Global spatio-temporal patterns in tropical tuna purse seine fisheries on drifting fish aggregating devices (dfads): Taking a historical perspective to inform current challenges,ãÜ. Aquatic Living Resources, 26(1), 37–48.
- Fonteneau, A., Chassot, E., & Gaertner, D. (2015). Managing tropical tuna purse seine fisheries through limiting the number of drifting fish aggregating devices in the atlantic: food for thought. *Collect. Vol. Sci. Pap. ICCAT*, 71(1), 460–475.
- Fonteneau, P., & Pianet. (2000). A worldwide review of purse seine fisheries on fads.
- Gaertner, D., & Dreyfus-Leon, M. (2004). Analysis of non-linear relationships between catch per unit effort and abundance in a tuna purse-seine fishery simulated with artificial neural networks. *ICES Journal of Marine Science*, 61(5), 812–820.
- Gilman, E. L. (2011). Bycatch governance and best practice mitigation technology in global tuna fisheries. *Marine Policy*, 35(5), 590–609.
- Girard, C., Benhamou, S., & Dagorn, L. (2004). Fad: Fish aggregating device or fish attracting device? a new analysis of yellowfin tuna movements around floating objects. *Animal Behaviour*, 67(2), 319–326.
- Groba, C., Sartal, A., & Vázquez, X. H. (2015). Solving the dynamic traveling salesman problem using a genetic algorithm with trajectory prediction: An application to fish aggregating devices. *Computers & Operations Research*, 56, 22–32.
- Groba, C., Sartal, A., & Vázquez, X. H. (2018). Integrating forecasting in metaheuristic methods to solve dynamic routing problems: Evidence from the logistic processes of tuna vessels. *Engineering Applications of Artificial Intelligence*, 76, 55–66.
- Guijarro, B., Ordines, F., & Massutí, E. (2017). Improving the ecological efficiency of the bottom trawl fishery in the western mediterranean: It's about time! *Marine Policy*, 83, 204–214.
- Harsanyi, J. (1967). Games with incomplete information played by ,Äòbayesian,Äô players. *Management Science*, 14 (3), 159–182.
- Hospido, A., & Tyedmers, P. (2005). Life cycle environmental impacts of spanish tuna fisheries. Fisheries Research, 76(2), 174–186.
- Howard-Grenville, J., Buckle, S. J., Hoskins, B. J., & George, G. (2014). Climate change and management. *Academy of Management Journal*, 57(3), 615–623.
- Jeon, Y., Reid, C., & Squires, D. (2008). Is there a global market for tuna? policy

- implications for tropical tuna fisheries. Ocean Development & International Law, 39(1), 32-50.
- Lopez, J., Moreno, G., Sancristobal, I., & Murua, J. (2014). Evolution and current state of the technology of echo-sounder buoys used by spanish tropical tuna purse seiners in the atlantic, indian and pacific oceans. Fisheries Research, 155, 127–137.
- Madau, F. A., Furesi, R., & Pulina, P. (2018). The technical efficiency in sardinian fisheries cooperatives. *Marine Policy*, 95, 111–116.
- Maufroy, A., Kaplan, D. M., Bez, N., De Molina, A. D., Murua, H., Floch, L., ... editor: Jan Jaap Poos, H. (2016). Massive increase in the use of drifting fish aggregating devices (dfads) by tropical tuna purse seine fisheries in the atlantic and indian oceans. *ICES Journal of Marine Science*, 74(1), 215–225.
- Ménard, F., Stéquert, B., Rubin, A., Herrera, M., & Marchal, É. (2000). Food consumption of tuna in the equatorial atlantic ocean: Fad-associated versus unassociated schools. *Aquatic living resources*, 13(4), 233–240.
- Moreno, G., Dagorn, L., Sancho, G., & Itano, D. (2007). Fish behaviour from fishers, Äô knowledge: the case study of tropical tuna around drifting fish aggregating devices (dfads). Canadian Journal of Fisheries and Aquatic Sciences, 64(11), 1517–1528.
- Morón, J., Areso, J. J., & Pallares, P. (2001). Statistics and technical information about the spanish purse seine fleet in the pacific. In 14th meeting of the standing committee on tuna and billfish, noumea, new caledonia (pp. 9–16).
- Nash, J. (1951). Non-cooperative games. The Annals of Mathematics, 54 (2), 286–295.
- Parker, R. W., Vázquez-Rowe, I., & Tyedmers, P. H. (2015). Fuel performance and carbon footprint of the global purse seine tuna fleet. *Journal of Cleaner Production*, 103, 517–524.
- Rust, S., Yamazaki, S., Jennings, S., Emery, T., & Gardner, C. (2017). Excess capacity and efficiency in the quota managed tasmanian rock lobster fishery. *Marine Policy*, 76, 55–62.
- Salas, S., & Gaertner, D. (2004). The behavioural dynamics of fishers: management implications. Fish and fisheries, 5(2), 153–167.
- Zamir, S. (2013). Bayesian games: Games with incomplete information. in: Meyers r. (eds) encyclopedia of complexity and systems science. Springer.
- Zeller, D., Cashion, T., Palomares, M., & Pauly, D. (2018). Global marine fisheries discards: a synthesis of reconstructed data. Fish and Fisheries, 19(1), 30–39.

#### A Appendix

We now introduce some well known concepts of non cooperative game theory. We refer to Zamir (2013) for a detailed discussion of such concepts.

An Aumann model of incomplete information (Aumann (1976)) is a tuple

$$(I, X, (\pi_i)_{i \in I}, P)$$

where I is the set of agents; X is the set of states of the world; for each  $i \in I$ ,  $\pi_i$  is a partition of X; and P is a probability distribution over X (called common prior).

Given  $x \in X$  and  $i \in I$  we denote by  $\pi_i(x)$  the element of  $\pi_i$  to which x belongs to.

The interpretation is as follows. There is a possible set of states of the world (X) and a probability distribution (P) over X known by all agents. An element  $x \in X$  is randomly selected according with P. Each agent  $i \in I$  has different information about such element, which is given by  $\pi_i$ . We assume that agent i knows that an element of  $\pi_i(x)$  has happened, but he/she can not distinguishes among the elements of  $\pi_i(x)$ .

The Harsanyi's model of incomplete information (Harsanyi (1967) is more popular than the Aumann's model of incomplete information. In this paper we use Aumann's model because it fits better with the problem we are studying.

For each state of the world  $x \in X$  we consider the classical non-cooperative game  $\Gamma^x = \left(I, (A_i^x)_{i \in I}, (u_i^x)_{i \in I}\right)$  played at this state. For each agent  $i \in I$ ,  $A_i^x$  denotes the set of pure actions that agent i can take when the state of the world is x. We assume that  $A_i^x = A_i^{x'}$  when  $\pi_i(x) = \pi_i(x')$ . Besides  $u_i^x : \times_{i \in I} A_i^x \to \mathbb{R}$  represents the utility of agent i

An strategy for agent i is a mapping  $\sigma_i$  assigning to each state of the world  $x \in X$  an action  $\sigma_i(x) \in A_i^x$  such that  $\sigma_i(x) = \sigma_i(x')$  when  $\pi_i(x) = \pi_i(x')$ . We denote by  $\Sigma_i$  the set of all strategies of agent i.

A Bayesian game on X is a triple  $\left(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I}\right)$  where for each  $\sigma = (\sigma_i)_{i \in I}$  and each  $i \in N$ 

$$u_{i}\left(\sigma\right) = \int_{X} u_{i}^{x}\left(\left(\sigma_{i}\left(x\right)\right)_{i \in I}\right) dP$$

A Bayesian Nash equilibria (briefly BNE) is a tuple  $\sigma = (\sigma_i)_{i \in I}$  such that for for each  $i \in I$  and each  $\sigma'_i \in \Sigma_i$  we have that  $u_i(\sigma) \geq u_i(\sigma \setminus \sigma'_i)$  where  $\sigma \setminus \sigma'_i$  is the combination of strategies where agent i plays  $\sigma'_i$  and each agent  $j \in I \setminus \{i\}$  plays  $\sigma_j$ .

Intuitively, in a BNE at each stage of the world x each agent i is playing a best reply against the strategies of the other agents. Thus, a BNE is an extension of the Nash equilibria (Nash (1951)) to this setting.

#### A.1 Mechanism 1. Reassigning FADs without compensation

We consider the Aumann model of incomplete information  $(I, X, (\pi_i)_{i \in I}, P)$  defined as follows.

- $I = \{f, 1, ..., n\}$  where f is the firm and i denotes vessel i for each i = 1, ..., n. In our simulations we will take 3 vessels. Namely, n = 3.
- X is the set of possible locations of the  $\tau = \sum_{i=1}^{n} n_i$  FADs assigned to the vessels. Namely  $X = Z^{\tau}$  where Z denotes the set of places where a FAD can be located. We assume that coordinates 1 to  $n_1$  from  $Z^{\tau}$  refer to the position of the FADs assigned to vessel 1. Coordinates  $n_1 + 1$  to  $n_1 + n_2$  from  $Z^{\tau}$  refer to the position of the FADs assigned to vessel 2 and so on. A generic element of X will be denoted as  $x = (x_j)_{j=1}^{\tau}$ .

In our simulations we take Z as the Indic Ocean. Besides each vessel will have 20 FADs  $(n_i = 20 \text{ for all } i \in N)$  and hence  $\tau = 60$ .

•  $(\pi_i)_{i\in I}$  model the situation where each vessel only knows the position of its FADs and the firm knows the position of all FADs.

Given  $i \in N$  and  $x, x' \in X$  we have that  $\pi_i(x) = \pi_i(x')$  if and only if the position of the FADs assigned to vessel i in x and x' are the same. Namely, for each  $j = \sum_{k=1}^{i-1} n_k + 1, ..., \sum_{k=1}^{i} n_k$  we have that  $x_j = x'_j$ .

For each  $x \in X$ ,  $\pi_f(x) = \{x\}$ .

• P is a probability distribution over X. We do not consider a specific distribution for P because our theoretical results hold for any P.

The non-cooperative game  $\Gamma^x = \left(I, (A_i^x)_{i \in I}, (u_i^x)_{i \in I}\right)$  we consider is defined for modelling the following situation. Each vessel, independently, decides if it share its FADs with other vessels. If a vessel says no, then such vessel remains with the same FADs. Among the vessels that say yes, the firm reassign the FADs of such vessels among themselves. We now formalize this idea.

- $\bullet$  I as above.
- $(A_i^x)_{i\in I}$ . For each  $i\in N,$   $A_i^x=\{YES,NO\}$ . Let  $N^{x,YES}$  the set of vessels that says YES. Let

$$B^{x,YES} = \bigcup_{i \in N^{x,YES}} \bigcup_{k=1}^{n_i} b_i^k$$

be the set of all FADs assigned initially to vessels that said YES.

 $A_f^x$  is the set of all possible real locations of the FADs of  $B^{x,YES}$  among agents in  $N^{x,YES}$ . Namely,

$$A_f^x = \left\{ \begin{array}{l} (B_i)_{i \in N^{x,YES}} : \text{for each } i \in N^{x,YES}, \ \varnothing \subset B_i \subset B^{x,YES}, \\ \bigcup_{i \in N^{x,YES}} B_i = B^{x,YES} \text{ and} \\ B_i \cap B_j = \varnothing \text{ for each } i,j \in N^{x,YES}, \ i \neq j. \end{array} \right\}$$

•  $(u_i^x)_{i\in I}$ . Let  $(a_i^x)_{i\in I} \in \times_{i\in I} A_i^x$ .

Let  $j \in N$  be a vessel that said NO. Then, the vessel continue with the same FADs,  $b_j$ . Hence its utility is  $u_j((a_i^x)_{i \in I}) = pn_jq$ .

Let  $j \in N$  be a vessel that said YES. Then, the vessel has a new set of FADs,  $B_j$ . Hence its utility is  $u_j((a_i^x)_{i\in I}) = p|B_j|q$  where  $|B_j|$  denotes the number of FADs in  $B_j$ .

Finally, the utility of the firm is

$$u_f((a_i^x)_{i \in I}) = (p_f - p) \sum_{i=1}^n n_j q - c \sum_{i \in N \setminus N^{x,YES}} d(b_i) - c \sum_{i \in N^{x,YES}} d(B_i)$$

The utility of the firm has three parts. The first one,  $(p_f - p) \sum_{i=1}^n n_j q$ , corresponds to the benefits of selling the fish. This part is independent of the actions taking by the vessels. The second one,  $-c \sum_{i \in N \setminus N^{x,YES}} d(b_i)$ , corresponds to the cost of the

fuel of the vessels that did not share its FADs. This part depends on the actions of the vessels but not on the action of the firm. The third one,  $-c\sum_{i\in N^{x,YES}}d\left(B_{i}\right)$ ,

corresponds to the cost of the fuel of the vessels that shared its FADs. This part depends on the actions of the vessels and on the action of the firm.

We now make a theoretical analysis of the Bayesian game  $(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I})$  associated to this case.

**Proposition 1.** Let  $(I, X, (\pi_i)_{i \in I}, P)$  be the Aumann model of incomplete information defined as above.

- (a) Let  $\sigma = (\sigma_i)_{i \in I}$  be such that for each  $i \in N$  and for each  $x \in X$ ,  $\sigma_i(x) = NO$ . Then,  $\sigma$  is a BNE of  $(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I})$  and for each  $i \in N$ ,  $u_i(\sigma) = pn_iq$ .
- (b) Let  $\sigma = (\sigma_i)_{i \in I}$  be a BNE of  $(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I})$ . Then, for each  $i \in N$ ,  $u_i(\sigma) = pn_iq$ .

**Proof of Proposition 1**. We first note that for each  $i \in I$  and each  $\sigma = (\sigma_i)_{i \in I}$  we have that

$$u_{i}\left(\sigma\right) = \int_{X} u_{i}^{x}\left(\left(\sigma_{i}\left(x\right)\right)_{i \in I}\right) dP = \sum_{X_{i} \in \pi_{i}} \int_{X_{i}} u_{i}^{x}\left(\left(\sigma_{i}\left(x\right)\right)_{i \in I}\right) dP$$

and  $\sigma_i(x) = \sigma_i(x')$  for all  $x, x' \in X_i$ .

(a) We have to prove that for each  $i \in I$  and each  $\sigma'_i \in \Sigma_i$ , we have that  $u_i(\sigma) \ge u_i(\sigma \setminus \sigma'_i)$ .

Let i = f. Since all vessels are saying NO, firm has nothing to do. Then, for each  $\sigma'_f \in \Sigma_f$  we have that  $u_f(\sigma) = u_f(\sigma \setminus \sigma'_f)$ .

Let  $i \in N$  and  $\sigma'_i \in \Sigma_i$ . Thus,

$$u_{i}\left(\sigma\backslash\sigma'_{i}\right) = \sum_{X_{i}\in\pi_{i}} \int_{X_{i}} u_{i}^{x}\left(\sigma'_{i}\left(x\right),\left(\sigma_{j}\left(x\right)\right)_{j\in I\backslash\{i\}}\right) dP.$$

Let  $X_i \in \pi_i$  be such that  $\sigma'_i(x) = NO$  for each  $x \in X_i$ . Since  $\sigma_i(x) = NO$  for each  $x \in X_i$  we have that

$$\int_{X_{i}} u_{i}^{x} \left( \sigma_{i}'\left(x\right), \left(\sigma_{j}\left(x\right)\right)_{j \in I \setminus \{i\}} \right) dP = \int_{X_{i}} u_{i}^{x} \left( \left(\sigma_{j}\left(x\right)\right)_{j \in I} \right) dP.$$

Let  $X_i \in \pi_i$  be such that  $\sigma_i'(x) = YES$  for each  $x \in X_i$ . Thus,  $N^{x,YES} = \{i\}$  and  $B^{x,YES} = \bigcup_{k=1}^{n_i} b_i^k$ . Hence  $A_f^x$ , the set of all possible reallocations of the FADs of  $B^{x,YES}$  among agents in  $N^{x,YES}$  has a unique element, namely, to assign all the FADs of vessel i to vessel i. Thus,

$$\int_{X_{i}} u_{i}^{x} \left( \sigma_{i}'(x), \left( \sigma_{j}(x) \right)_{j \in I \setminus \{i\}} \right) dP = \int_{X_{i}} u_{i}^{x} \left( \left( \sigma_{j}(x) \right)_{j \in I} \right) dP.$$

Hence,

$$u_{i}\left(\sigma\backslash\sigma'_{i}\right) = \sum_{X_{i}\in\pi_{i}} \int_{X_{i}} u_{i}^{x}\left(\left(\sigma_{j}\left(x\right)\right)_{j\in I}\right) dP = u_{i}\left(\sigma\right).$$

(b) We first prove a couple of statements that will be used in the proof of this part. Statement 1. Let  $\sigma = (\sigma_i)_{i \in I}$  be a BNE of  $(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I})$ . Then, there exists  $X' \subset X$  such that  $\int_{X'} dP = 1$  and for each  $x \in X'$ ,  $\sigma_f(x) = (B_i^*)_{i \in N^{x,YES}}$  where

$$\sum_{i \in N^{x,YES}} d\left(B_i^*\right) = \min \left\{ \sum_{i \in N^{x,YES}} d\left(B_i\right) : \left(B_i\right)_{i \in N^{x,YES}} \in B^{x,YES} \right\}.$$

<u>Proof of Statement 1</u>. For each  $x \in X$  we denote  $\sigma_f(x) = (B_i)_{i \in N^{x,YES}}$ . Besides, we define  $X'' = \{x \in X : \sigma_f(x) \neq (B_i^*)_{i \in N^{x,YES}}\}.$ 

Suppose that the statement does not hold. Then,  $\int_{X''} dP > 0$ .

We now define  $\sigma_{f}'$  such that  $\sigma_{f}'\left(x\right)=\left(B_{i}^{*}\right)_{i\in N^{x,YES}}$  for all  $x\in X.$  Then

$$u_{f}\left(\sigma\backslash\sigma'_{f}\right) = \int_{X} u_{f}^{x}\left(\sigma'_{f}\left(x\right), \left(\sigma_{j}\left(x\right)\right)_{j\in I\backslash\{f\}}\right) dP$$

$$= \int_{X\backslash X''} u_{f}^{x}\left(\sigma'_{f}\left(x\right), \left(\sigma_{j}\left(x\right)\right)_{j\in I\backslash\{f\}}\right) dP$$

$$+ \int_{X''} u_{f}^{x}\left(\sigma'_{f}\left(x\right), \left(\sigma_{j}\left(x\right)\right)_{j\in I\backslash\{f\}}\right) dP.$$

Since  $\sigma_{f}'\left(x\right)=\sigma_{f}\left(x\right)$  for all  $x\in X\backslash X''$  we have that

$$\int_{X\backslash X^{\prime\prime}}u_{f}^{x}\left(\sigma_{f}^{\prime}\left(x\right),\left(\sigma_{j}\left(x\right)\right)_{j\in I\backslash\left\{f\right\}}\right)dP=\int_{X\backslash X^{\prime\prime}}u_{f}^{x}\left(\left(\sigma_{j}\left(x\right)\right)_{j\in I}\right)dP.$$

Besides,

$$\int_{X''} u_f^x \left( \sigma_f'(x), (\sigma_j(x))_{j \in I \setminus \{f\}} \right) dP$$

$$= \int_{X''} \left[ (p_f - p) \sum_{i=1}^n n_j q - c \sum_{i \in N \setminus N^{x,YES}} d(b_i) - c \sum_{i \in N^{x,YES}} d(B_i^*) \right] dP$$

$$> \int_{X''} \left[ (p_f - p) \sum_{i=1}^n n_j q - c \sum_{i \in N \setminus N^{x,YES}} d(b_i) - c \sum_{i \in N^{x,YES}} d(B_i) \right] dP$$

$$= \int_{X''} u_f^x \left( (\sigma_j(x))_{j \in I} \right) dP.$$

Thus,

$$u_{f}\left(\sigma \backslash \sigma_{f}'\right) > \int_{X \backslash X''} u_{f}^{x}\left(\left(\sigma_{j}\left(x\right)\right)_{j \in I}\right) dP + \int_{X''} u_{f}^{x}\left(\left(\sigma_{j}\left(x\right)\right)_{j \in I}\right) dP$$

$$= \int_{X} u_{f}^{x}\left(\left(\sigma_{j}\left(x\right)\right)_{j \in I}\right) dP = u_{f}\left(\sigma\right),$$

which contradicts that  $\sigma$  is a BNE.

<u>Statement 2</u>. Let  $\sigma = (\sigma_i)_{i \in I}$  be a BNE of  $(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I})$ . For each  $x \in X$  and each  $i \in N$  such that  $\int_{\pi_i(x)} dP > 0$  we have that

$$\int_{x'\in\pi_i(x)} u_i^{x'} \left( \left(\sigma_i\left(x'\right)\right)_{i\in I} \right) dP \ge pn_i q \int_{\pi_i(x)} dP.$$

<u>Proof of Statement 2</u>. Let  $x \in X$  and  $i \in N$  such that  $\int_{\pi_i(x)} dP > 0$  and  $\sigma_i(x) = NO$ . Then, vessel i receives its initial FADs and hence

$$\int_{x'\in\pi_i(x)} u_i^{x'} \left( \left(\sigma_i\left(x'\right)\right)_{i\in I} \right) dP = \int_{x'\in\pi_i(x)} pn_i q dP = pn_i q \int_{\pi_i(x)} dP.$$

Let  $x \in X$  and  $i \in N$  such that  $\int_{\pi_i(x)} dP > 0$  and  $\sigma_i(x) = YES$ . Suppose not. Then,

$$\int_{x' \in \pi_i(x)} u_i^{x'} \left( \left( \sigma_i \left( x' \right) \right)_{i \in I} \right) dP$$

Let  $\sigma'_i$  be such that  $\sigma'_i(x') = NO$  when  $x' \in \pi_i(x)$  and  $\sigma'_i(x') = \sigma_i(x')$  otherwise. Now,

$$u_{i}\left(\sigma\backslash\sigma_{i}'\right) = \sum_{X_{i}\in\pi_{i}} \int_{x'\in X_{i}} u_{i}^{x'}\left(\sigma_{i}'\left(x'\right),\left(\sigma_{j}\left(x'\right)\right)_{j\in I\backslash\{i\}}\right) dP$$

$$= \sum_{X_{i}\in\pi_{i}\backslash\pi_{i}(x)} \int_{x'\in X_{i}} u_{i}^{x'}\left(\sigma_{i}'\left(x'\right),\left(\sigma_{j}\left(x'\right)\right)_{j\in I\backslash\{i\}}\right) dP$$

$$+ \int_{x'\in\pi_{i}(x)} u_{i}^{x'}\left(\sigma_{i}'\left(x'\right),\left(\sigma_{j}\left(x'\right)\right)_{j\in I\backslash\{i\}}\right) dP$$

Since  $\sigma'_{i}(x') = \sigma_{i}(x')$  when  $x' \in X_{i} \in \pi_{i} \backslash \pi_{i}(x)$  we have that

$$\sum_{X_{i} \in \pi_{i} \setminus \pi_{i}(x)} \int_{x' \in X_{i}} u_{i}^{x'} \left(\sigma_{i}'\left(x'\right), \left(\sigma_{j}\left(x'\right)\right)_{j \in I \setminus \{i\}}\right) dP = \sum_{X_{i} \in \pi_{i} \setminus \pi_{i}(x)} \int_{x' \in X_{i}} u_{i}^{x'} \left(\left(\sigma_{j}\left(x'\right)\right)_{j \in I}\right) dP.$$

Since  $u_{i}^{x}\left(\sigma_{i}'\left(x'\right),\left(\sigma_{j}\left(x'\right)\right)_{j\in I\setminus\{i\}}\right)=pn_{i}q$  when  $x'\in\pi_{i}\left(x\right)$  we have that

$$\int_{x'\in\pi_{i}(x)} u_{i}^{x'} \left(\sigma_{i}'\left(x'\right), \left(\sigma_{j}\left(x'\right)\right)_{j\in I\setminus\{i\}}\right) dP = pn_{i}q \int_{\pi_{i}(x)} dP$$

$$> \int_{x'\in\pi_{i}(x)} u_{i}^{x'} \left(\left(\sigma_{i}\left(x'\right)\right)_{i\in I}\right) dP.$$

Thus,

$$u_{i}\left(\sigma \backslash \sigma_{i}'\right) > \sum_{X_{i} \in \pi_{i} \backslash \pi_{i}(x)} \int_{x' \in X_{i}} u_{i}^{x'} \left(\left(\sigma_{j}\left(x'\right)\right)_{j \in I}\right) dP + \int_{x' \in \pi_{i}(x)} u_{i}^{x'} \left(\left(\sigma_{i}\left(x'\right)\right)_{i \in I}\right) dP$$

$$= u_{i}\left(\sigma\right),$$

which contradicts that  $\sigma$  is a BNE.

We now prove (b). We know that

$$u_{i}\left(\sigma\right) = \sum_{X_{i} \in \pi_{i}} \int_{X_{i}} u_{i}^{x} \left(\left(\sigma_{i}\left(x\right)\right)_{i \in I}\right) dP = \sum_{X_{i} \in \pi_{i}: \int_{X_{i}} dP > 0} \int_{X_{i}} u_{i}^{x} \left(\left(\sigma_{i}\left(x\right)\right)_{i \in I}\right) dP$$

By statement 2,

$$\sum_{X_{i} \in \pi_{i}: \int_{X_{i}} dP > 0} \int_{X_{i}} u_{i}^{x} \left( (\sigma_{i} (x))_{i \in I} \right) dP \ge pn_{i}q \sum_{X_{i} \in \pi_{i}: \int_{X_{i}} dP > 0} \int_{X_{i}} dP.$$

Since P is a probability,  $\int_X dP = 1$ . Then,

$$pn_i q \sum_{X_i \in \pi_i: \int_{X_i} dP > 0} \int_{X_i} dP = pn_i q \int_X dP = pn_i q.$$

Besides.

$$\sum_{i=1}^{n} u_{i}(\sigma) = \sum_{i=1}^{n} \int_{X} u_{i}^{x} \left( (\sigma_{i}(x))_{i \in I} \right) dP = \int_{X} \sum_{i=1}^{n} u_{i}^{x} \left( (\sigma_{i}(x))_{i \in I} \right) dP$$

$$= \int_{X} \left[ \sum_{i \in N \setminus N^{x,YES}} p n_{i} q + \sum_{i \in N^{x,YES}} p |B_{i}| q \right] dP$$

$$= pq \int_{X} \left[ \sum_{i \in N \setminus N^{x,YES}} n_{i} + \sum_{i \in N^{x,YES}} n_{i} \right] dP$$

$$= pq \int_{X} \left[ \sum_{i \in N} n_{i} \right] dP = pq \sum_{i \in N} n_{i}$$

$$= \sum_{i \in N} p n_{i} q.$$

Since  $u_i(\sigma) \ge pn_iq$  for each  $i \in N$  and  $\sum_{i=1}^n u_i(\sigma) = \sum_{i \in N} pn_iq$  we deduce that  $u_i(\sigma) = pn_iq$  for each  $i \in N$ .

Proposition 1 says nothing about the utility obtained by the firm. Thus, a natural question that arises is the following: is it possible to find BNE where some vessels share its FADs? Notice that if the answer is YES, then the firm can improve its utility by the fuel's savings.

Next examples show that the answer depends on P and the location of the FADs.

**Example 1.** Consider the case where we have two vessels  $(I = \{f, a, b\})$  and each vessels has two FADs. Besides every vessel knows the location of every FAD. Namely, P assign probability 1 to element  $x = (b_a^1, b_a^2, b_b^1, b_b^2)$  and zero to the rest of elements of X. The distances between the FADs and the vessels are the following:

| distances | 1  | 2  | $b_a^1$ | $b_a^2$ | $b_b^1$ |
|-----------|----|----|---------|---------|---------|
| 2         | 50 |    |         |         |         |
| $b_a^1$   | 5  | 45 |         |         |         |
| $b_a^2$   | 35 | 15 | 30      |         |         |
| $b_b^1$   | 15 | 35 | 10      | 20      |         |
| $b_b^2$   | 45 | 5  | 40      | 10      | 30      |

The distances are computed by assuming that vessels are located in a line. From left to right 1 [5]  $b_a^1$  [10]  $b_b^1$  [20]  $b_a^2$  [10]  $b_b^2$  [5] 2. The distance between vessel 1 and FADs  $b_a^1$  is 5; the distance between FADs  $b_a^1$  and  $b_b^1$  is 10 and so on.

Let  $\sigma$  be the BNE where each vessel says NO. Then, each vessels recover its FADs. Vessel a moves to FAD  $b_a^2$  (distance 5), next to FAD  $b_a^2$  (distance 30) and then back (35). The total distance traveled is 70. Similarly, the distance traveled by vessel b is also 70. Then, the utility of each vessel is 2pq and the utility of the firm is  $(p_f - p) 4q - c140$ .

Let  $\sigma$  be such that each vessel says YES and the firm assign FADs  $b_a^1$  and  $b_b^1$  to vessel a and FADs  $b_a^2$  and  $b_b^2$  to vessel b. It is easy to see that  $\sigma$  is a BNE. Besides the utility of the firm is  $(p_f - p) 4q - c60$ . Thus, in this BNE the firm can improve with respect to the initial situation.

**Example 2**. Consider the same case as in Example 1 but now the distances between the FADs and the vessels are the following:

| distances | 1   | 2   | $b_a^1$ | $b_a^2$ | $b_b^1$ |
|-----------|-----|-----|---------|---------|---------|
| 2         | 140 |     |         |         |         |
| $b_a^1$   | 5   | 135 |         |         |         |
| $b_a^2$   | 105 | 15  | 120     |         |         |
| $b_b^1$   | 125 | 35  | 100     | 20      |         |
| $b_b^2$   | 135 | 5   | 110     | 10      | 30      |

The distances are computed by assuming that vessels are located in a line. From left to right 1 [5]  $b_a^1$  [100]  $b_b^1$  [20]  $b_a^2$  [10]  $b_b^2$  [5] 2.

In this example the unique BNE is the one where each vessel says NO. Notice that if both vessels say YES then the firm assign FAD  $b_a^1$  to vessel a and the other FADs to vessel a. Thus, vessel a is better saying NO than saying YES.

#### A.2 Mechanism 2. Reassigning FADs with compensation

We now introduce the theoretical model for analyzing this case. The Aumann model of incomplete information  $(I, X, (\pi_i)_{i \in I}, P)$  associated to this case is the same as above.

The non-cooperative game  $\Gamma^x = \left(I, (A_i^x)_{i \in I}, (u_i^x)_{i \in I}\right)$  we consider is defined bas follows.  $(A_i^x)_{i \in I}$  is the same as in Case 1. Nevertheless  $(u_i^x)_{i \in I}$  will be modified in order to consider the compensation that firm give to vessels that share its FADs. Let  $(a_i^x)_{i \in I} \in \times_{i \in I} A_i^x$ .

Let  $j \in N$  be a vessel that said NO (namely  $a_j^x = NO$ ). Then, the vessel continue with the same FADs,  $b_i$ . Hence its utility is  $u_j\left((a_i^x)_{i\in I}\right) = pn_jq$ . This utility is the same as in Mechanism 1.

Let  $j \in N$  be a vessel that said YES (namely  $a_j^x = YES$ ). Then, the vessel has a new set of assigned FADs,  $B_j$ . Hence its utility is

$$u_j\left(\left(a_i^x\right)_{i\in I}\right) = p\max\left\{\left|B_j\right|, n_j\right\}q$$

where  $|B_j|$  denotes the number of FADs in  $B_j$ . Notice that if vessel j receives at least  $n_j$  FADs, then it will be paid according with the FADs received. If vessel j receives less

than  $n_j$ , then it will be paid as if the vessel receives  $n_j$  FADs. In this part appears clearly the new incentive mechanism.

Finally, the utility of the firm  $u_f\left((a_i^x)_{i\in I}\right)$  is given by

$$(p_f - p) \sum_{i=1}^{n} n_j q - c \sum_{i \in N \setminus N^{x,YES}} d(b_i) - c \sum_{i \in N^{x,YES}} d(B_i) - \sum_{i \in N^{x,YES}, |B_i| < n_i} p(n_i - |B_i|) q$$

The utility of the firm has four parts. The first one,  $(p_f - p) \sum_{i=1}^n n_j q$ , the second one,  $-c \sum_{i \in N \setminus N^{x,YES}} d(b_i)$ , and the third one,  $-c \sum_{i \in N^{x,YES}} d(B_i)$ , are the same as in the previous case. In this case it appears a fourth one,

$$-\sum_{i \in N^{x,YES}, |B_i| < n_i} p(n_i - |B_i|) q,$$

where it appears the compensation that the firm gives to the vessels that say YES and receive less FADs than initially.

We now make a theoretical analysis of the Bayesian game  $(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I})$  associated to this case.

**Proposition 2.** Let  $(I, X, (\pi_i)_{i \in I}, P)$  be the Aumann model of incomplete information defined as above.

- (a) Let  $\sigma^{NO} = (\sigma_i^{NO})_{i \in I}$  be such that for each  $i \in N$  and for each  $x \in X$ ,  $\sigma_i^{NO}(x) = NO$ . Then,  $\sigma^{NO}$  is a BNE of  $(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I})$  and for each  $i \in N$ ,  $u_i(\sigma^{NO}) = pn_iq$ .
- (b) Let  $i \in N$  and  $x \in X$ . We define  $a_i^{x,YES} = YES$  and  $a_i^{x,NO} = NO$ . For each  $\left(a_j^x\right)_{j\in I} \in \times_{i\in I} A_i^x$  we have that

$$\int_{x' \in \pi_i(x)} u_i^{x'} \left( a_i^{x,YES}, \left( a_j^x \right)_{j \in I \setminus \{i\}} \right) dP \ge \int_{x' \in \pi_i(x)} u_i^{x'} \left( a_i^{x,NO}, \left( a_j^x \right)_{j \in I \setminus \{i\}} \right) dP.$$

(c) There exists a BNE  $\sigma^{YES} = \left(\sigma^{YES}_i\right)_{i \in I}$  of  $\left(I, (\Sigma_i)_{i \in I}, (u_i)_{i \in I}\right)$  where for each  $i \in N$  and for each  $x \in X$ ,  $\sigma^{YES}_i(x) = YES$ . Besides, for each  $i \in I$ ,  $u_i\left(\sigma^{YES}\right) \geq u_i\left(\sigma^{NO}\right)$ .

**Proof of Proposition 2**. (a) It is similar to the proof of Proposition 1 (a).

(b) We know that for each  $x' \in \pi_i(x)$  and each  $\left(a_j^x\right)_{i \in I} \in \times_{i \in I} A_i^x$ 

$$u_i^{x'} \left( a_i^{x,NO}, \left( a_j^x \right)_{j \in I \setminus \{i\}} \right) = p n_i q \text{ and}$$

$$u_i^{x'} \left( a_i^{x,YES}, \left( a_j^x \right)_{j \in I \setminus \{i\}} \right) = p \max \left\{ n_i, \left| B_i^{x'} \right| \right\} q$$

where  $B_i^{x'}$  is the set of FADs assigned to vessel *i* after saying YES. Thus, the result holds trivially.

(c) For each  $x \in X$  we take  $\sigma_f^{YES}\left(x\right) = \left(B_i^*\right)_{i \in N^{x,YES}}$  where

$$c \sum_{i \in N^{x,YES}} d(B_i^*) + \sum_{i \in N^{x,YES}, |B_i| < n_i} p(n_i - |B_i^*|) q$$

$$= \min \left\{ c \sum_{i \in N^{x,YES}} d(B_i) + \sum_{i \in N^{x,YES}, |B_i| < n_i} p(n_i - |B_i|) q : (B_i)_{i \in N^{x,YES}} \in B^{x,YES} \right\}$$

We first prove that  $\sigma^{YES}$  is a BNE. We need to prove that for each  $i \in I$  and each  $\sigma_i \in \Sigma_i$  we have that  $u_i(\sigma^{YES}) \ge u_i(\sigma^{YES} \setminus \sigma_i)$ .

Because of the definition of  $\sigma_f^{YES}$  it is clear that for any  $\sigma_f \in \Sigma_f$ ,  $u_f(\sigma^{YES}) \ge u_f(\sigma^{YES} \setminus \sigma_f)$ .

Let  $i \in N$  and  $\sigma_i \in \Sigma_i$ . In the proof of Proposition 1 we have seen that

$$u_i\left(\sigma^{YES}\right) = \sum_{X_i \in \pi_i} \int_{X_i} u_i^x \left(\left(\sigma_i^{YES}\left(x\right)\right)_{i \in I}\right) dP.$$

Let  $X_i \in \pi_i$  be such that  $\sigma_i(x) = YES$  when  $x \in X_i$ . Then,  $\sigma_i^{YES}(x) = \sigma_i(x)$  and hence,

$$\int_{X_{i}} u_{i}^{x} \left( \left( \sigma_{i}^{YES}\left(x\right) \right)_{i \in I} \right) dP = \int_{X_{i}} u_{i}^{x} \left( \sigma_{i}\left(x\right), \left( \sigma_{i}^{YES}\left(x\right) \right)_{i \in I \setminus \{i\}} \right) dP.$$

Let  $X_i \in \pi_i$  be such that  $\sigma_i(x) = NO$  when  $x \in X_i$ . By part (b),

$$\int_{X_{i}} u_{i}^{x} \left( \left( \sigma_{i}^{YES} \left( x \right) \right)_{i \in I} \right) dP \ge \int_{X_{i}} u_{i}^{x} \left( \sigma_{i} \left( x \right), \left( \sigma_{i}^{YES} \left( x \right) \right)_{i \in I \setminus \{i\}} \right) dP.$$

Thus,

$$u_{i}\left(\sigma^{YES}\right) \geq \sum_{X_{i} \in \pi_{i}} \int_{X_{i}} u_{i}^{x}\left(\sigma_{i}\left(x\right), \left(\sigma_{i}^{YES}\left(x\right)\right)_{i \in I \setminus \{i\}}\right) dP = u_{i}\left(\sigma^{YES} \setminus \sigma_{i}\right).$$

We now prove that for each  $i \in I$ ,  $u_i\left(\sigma^{YES}\right) \geq u_i\left(\sigma^{NO}\right)$ . Using part (b) it is straightforward to prove that for each  $i \in N$ ,  $u_i\left(\sigma^{YES}\right) \geq u_i\left(\sigma^{NO}\right)$ .

Since no vessel share its FADs in  $\sigma^{NO}$  we have that for each  $x \in X$ ,

$$u_f^x\left(\sigma^{NO}\right) = (p_f - p) \sum_{i=1}^n n_j q - c \sum_{i \in N} d\left(b_i\right)$$

and hence

$$u_f\left(\sigma^{NO}\right) = \int_X u_f^x\left(\sigma^{NO}\right) dP = (p_f - p) \sum_{i=1}^n n_j q - c \sum_{i \in N} d\left(b_i\right).$$

We know that

$$u_f(\sigma^{YES}) = (p_f - p) \sum_{i=1}^n n_j q - c \sum_{i \in N} d(B_i^*) - \sum_{i \in N, |B_i| < n_i} p(n_i - |B_i^*|) q$$

where  $\{B_i^*\}_{i=1}^n$  is obtained through the minimization problem defined above.

Thus, for proving that  $u_f(\sigma^{YES}) \geq u_f(\sigma^{NO})$  is enough to prove that it exists  $\{B_i\}_{i=1}^n$  such that

$$c\sum_{i\in N}d\left(B_{i}\right)+\sum_{i\in N,|B_{i}|< n_{i}}p\left(n_{i}-|B_{i}|\right)q\leq c\sum_{i\in N}d\left(b_{i}\right).$$

If we take  $B_i = b_i$  for all i = 1, ..., n we realize that the previous inequality holds.

Next example shows that in some cases, the BNE of parts (a) and (c) could be, in a practical way, the same. Nevertheless our simulations based on real-data will show that both BNE could be very different.

**Example 3.** Consider the same case as in Example 1 but now the distances between the FADs and the vessels are the following:

| distances | 1   | 2   | $b_a^1$ | $b_a^2$ | $b_b^1$ |
|-----------|-----|-----|---------|---------|---------|
| 2         | 120 |     |         |         |         |
| $b_a^1$   | 5   | 115 |         |         |         |
| $b_a^2$   | 10  | 110 | 5       |         |         |
| $b_b^1$   | 110 | 10  | 105     | 100     |         |
| $b_b^2$   | 115 | 5   | 105     | 10      | 5       |

The distances are computed by assuming that vessels are located in a line. From left to right 1 [5]  $b_1^1$  [15]  $b_1^2$  [100]  $b_2^1$  [5]  $b_2^2$  [5] 2.

In the BNE described in part (a) each vessels recover its FADs. Then, the utility of each vessel is 2pq and the utility of the firm is  $(p_f - p) 4q - c40$ . In the BNE described in part (c) each vessels share its FADs. Then the firm reassign all FADs. But the optimal solution is to assign to each vessel its initial FADs. Then, every vessel recover its FADs. Hence, the utility of each vessel is 2pq and the utility of the firm is  $(p_f - p) 4q - c40$ . Even from a theoretical point of view both equilibria are different, in a practical way, both are the same.