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An overlapping generations model with endogenous aspirations

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Abstract

In this paper we introduce endogenous aspirations into a standard overlapping generations model. Aspirations decrease with capital accumulation as suggested by empirical evidence. We derive restrictions guaranteeing the strict concavity of the utility function in this case. Under these restrictions, we study the behaviour of the saving rate along the transition which depends on the elasticity of the aspiration function. We also identify conditions guaranteeing the existence of a unique positive steady-state and under which restrictions the latter is asymptotically stable. The competitive economy can exhibit endogenous fluctuations given that aspirations induce the agent to increase young age consumption during expansions and old age consumption during recessions. The study focuses next on the optimal allocation and its decentralization by means of an appropriate tax policy. The optimal capital stock is larger than in the standard Diamond model and the appropriate policy depends on the impact of future aspirations.

Keywords: Overlapping generations, endogenous aspirations, optimal taxation.

JEL classification: D91, E13, E21

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1 Introduction

It is well documented that individuals are concerned with interpersonal comparisons and social status which can be reflected through their consumption choices. In the present paper, we wish to focus on a particular type of interpersonal comparison under the form of inherited aspirations. We consider a general framework where young agents compare their consumption level to the one of their parents in young age and similarly, old agents compare their consumption level to the one of their parents in old age. The novelty of our approach is that we assume that the degree of the aspiration effect is not constant but evolves with time as the economy develops. Specifically, we suppose that our endogenous aspirations decrease with wealth accumulation such that at later stages of development, individuals are less inclined to compare themselves with their parents. We argue that economic development leads to the formation of institutions and social norms that discourage conspicuous consumption activities. There is a large body of literature dealing with aspirations in the standard overlapping generations framework (see, for example, de la Croix, 1996; de la Croix and Michel, 1999; Artige et al., 2004; Alonso-Carrera et al., 2007; Caballé and Moro-Egido, 2014), however, all of these works assume that the degree of aspiration is constant along the growth process.

There is as well large empirical evidence concerning the importance of relative well-being highlighted in works like the ones of Clark and Oswald (1996) or Ferrer-i Carbonell (2005) among others. These authors show that utility depends on present consumption but also on some particular reference point. Additionally, as noted by Becker (1992), individual behavior is affected by inherited tastes that are transmitted from parents to children. For example, Cavalli-Sforza and Feldman (1981) as well as Boyd and Richerson (1988) provide survey evidence concerning the intergenerational transmission of tastes. More recently, Cox et al. (2004) estimate that parental preferences explain between 5% and 10% of their children's preferences after controlling for income while Senik (2009) presents evidence showing that an individual's well-being increases if he has done better in life than his parents. In the same spirit, Charles et al. (2014) find that the intergenerational correlation of consumption ranges between 7% to 9%.

The hypothesis that one's willingness to compare to others decreases with economic development is also supported by a number of empirical studies. Clark and Senik (2010), using a European survey, show that the poor tend to care more about relative consumption and that in rich European countries, citizens find less important to compare their income with the one of others. Banerjee and Duflo (2007) show as well that in poor countries, people care more about status than in richer countries. Heffetz (2011) estimates income elasticities for the consumption of goods exhibiting status, and identifies a negative relationship between the degree of status and income. Boppart (2014) identifies as an empirical regularity the fact that poor households spend a larger fraction of income on goods rather than services compared to rich households. Since services are considered less positional than goods, the willingness to compare to others seems to be decreasing with wealth accumulation. Finally, Charles et al. (2009) consider that since the marginal return from signaling through consumption is decreasing in the income of the reference group, we should observe less conspicuous consumption among individuals who compare themselves with richer reference groups. This latter idea has been empirically formalized by Heffetz and Frank (2011).

To our knowledge, this is the first paper that analyzes the implications of endogenous aspirations decreasing with capital accumulation in an overlapping generations framework. In a recent working paper, Dioikitopoulos et al. (2017) were the first to introduce the idea of endogenous status preferences in an infinite horizon model. In their model, the infinitely-lived individual compares at each point in time its consumption level with the average one, the intensity of the comparison being endogenous and decreasing in capital. While these authors focus on the dynamics of the saving rate and income inequality, we focus on the characterization of both the competitive and the optimal allocations in an overlapping generations model.

An account of the results is as follows. The introduction of an aspiration function decreasing in capital necessitates some additional restrictions in order to ensure the strict concavity of the objective function in the planner's case. These are a sufficiently convex aspiration function as well as a relatively large parameter governing the impact of aspirations on utility. Concerning the competitive equilibrium, we derive a necessary and sufficient condition guaranteeing that the saving rate increases along the transition. A decreasing saving rate can only be obtained if the elasticity of the aspiration function in old age is sufficiently large. We then derive conditions under which the economy is characterized by a unique positive steady-state which is asymptotically stable under some additional restrictions. When the steady-state is dynamically efficient, the dynamical system might exhibit endogenous fluctuations under the form of damped oscillations or through a limit cycle. This is due to the fact that endogenous aspirations induce the agent to increase young age consumption during expansion periods and old age one during recessions giving rise quite naturally to endogenous fluctuations. The result is in line with empirical evidence documenting an increase in savings during recession periods (see, for example, Adema and Pozzi, 2015; Chakrabarti et al., 2013; Crossley et al., 2013). We then proceed with the study of the

optimal allocation by solving the social planner's problem and show that the steady-state capital stock must be larger than the one implied by the standard modified golden rule. We finally derive an optimal tax policy which consists in subsidizing or taxing capital depending on the magnitude of future aspirations and using appropriate lump-sum transfers.
The remainder of this paper is organized as follows. Section 2 introduces the model, derives the restrictions ensuring that our objective function is strictly concave and present the first-order conditions at equilibrium. The intertemporal competitive equilibrium is studied in section 3 where the behavior of the saving rate is analyzed and the issues related to existence, uniqueness and stability of the steady-state are presented. Section 4 focuses on the optimal allocation and its decentralization through an adequate tax policy while section 5 presents a numerical example for both the competitive and the optimal allocations. Finally, section 6 is devoted to the conclusion.

2 The model

We consider an overlapping generations model where a given generation lives for three periods and has perfect foresight. Population is constant and normalized to one. During childhood, the agent does not take any decision and inherits aspirations a_t from the previous young generation (his parents). In young age, the agent supplies inelastically one unit of labor and earns in exchange the real wage w_t. This wage is allocated between consumption c_t and savings s_t:

$$w_t = c_t + s_t.$$

In old age, the agent retires and earns the gross return R_{t+1} on his savings from which he consumes d_{t+1}:

$$d_{t+1} = R_{t+1} s_t.$$

The life-cycle utility function of the representative generation depends on young and old age consumption, as well as on aspirations in both periods and takes the following form:

$$U = \theta \ln(c_t - \gamma(\overline{k}_t) a_t^\sigma) + (1-\theta) \ln(d_{t+1} - \gamma(\overline{k}_{t+1}) d_t^\sigma).$$

where $\theta \in (0,1)$ is the relative preference for young age consumption. Aspirations can be seen as a frame of reference against which consumption in both periods is evaluated. In young age, the agent compares its consumption level to its inherited aspirations while in old age, the agent uses as a reference the consumption level of his parents in old age. We depart from the standard aspiration framework in two respects: first, the intensity of the aspiration effect $\gamma(\overline{k}_t)$ is endogenous and depends on the average (or aggregate) stock of capital in the economy $\overline{k}_t$. In accordance with empirical evidence we assume that $\gamma(\overline{k}_t)$ is decreasing in average capital and as the economy develops, the comparison with the previous generation tends to vanish. The only difference between the aspiration effect in both periods is the time index of average capital. When the economy is growing, the aspiration effect is smaller in old age which is accordance with empirical evidence showing that aspirations are less important for older persons. For example, Clark and Oswald (1996) show that reported satisfaction levels increase with age, older persons putting less weight on comparisons in their welfare evaluations. We do not choose a specific functional form for the aspiration function $\gamma(\overline{k}_t)$ but we impose some restrictions.

Assumption 1: The aspiration function $\gamma(\overline{k}_t)$ satisfies the following properties: $\gamma'(\overline{k}_t) < 0$, $\lim_{\overline{k}_t \to 0} \gamma(\overline{k}_t) = \overline{\gamma} \leq 1$ and $\lim_{\overline{k}_t \to +\infty} \gamma(\overline{k}_t) = 0$.

The aspiration effect is bounded, always lower than one and vanishes in the limit when $\overline{k}_t \to +\infty$. Our second change compared to the standard framework is that effective consumption in both periods depends non-linearly on the level of aspirations if $\sigma \neq 1$ and we should assume for the moment that $\sigma > 0$. The latter assumption will be crucial in order to ensure that the objective function is strictly concave in the planner's case. As in de la Croix (1996) and de la Croix and Michel (1999), we consider that aspirations in young age are equivalent to the consumption level of the previous young generation such that children get used to particular consumption standards when living with their parents:

$$a_t = c_{t-1}.$$

In the present framework, the economy faces an intergenerational externality in both periods under the form of aspirations which are a frame of reference originating in the consumption of the previous generation.
Before proceeding, we will derive conditions guaranteeing that our utility function is strictly concave in all its arguments. In order to do so, we set $\overline{k}_t = k_t$, $\overline{k}_{t+1} = k_{t+1}$ and proceed with the first proposition.

Proposition 1: A necessary and sufficient condition ensuring the strict concavity of the utility function is:

$$\frac{\sigma}{\sigma-1} < \frac{\gamma''(k_t)\gamma(k_t)}{\gamma'(k_t)^2} \quad \forall t,$$

where $\sigma > 1$ and $\gamma''(k_t) > 0$.

Proof. We need to compute the Hessian matrix of our utility function. Since our function is separable across young and old age, we compute two matrices H^c and H^d associated respectively to young and old age. We start by computing the first one which is given by:

$$H^c = \begin{bmatrix} -\frac{\theta}{(c_t-\gamma(k_t)a_t^\sigma)^2} & \frac{\theta\gamma(k_t)\sigma a_t^{\sigma-1}}{(c_t-\gamma(k_t)a_t^\sigma)^2} & \frac{\theta\gamma'(k_t)a_t^\sigma}{(c_t-\gamma(k_t)a_t^\sigma)^2} \\ \frac{\theta\gamma(k_t)\sigma a_t^{\sigma-1}}{(c_t-\gamma(k_t)a_t^\sigma)^2} & H_{22}^c & -\frac{\theta\gamma'(k_t)\sigma a_t^{\sigma-1}c_t}{(c_t-\gamma(k_t)a_t^\sigma)^2} \\ \frac{\theta\gamma'(k_t)a_t^\sigma}{(c_t-\gamma(k_t)a_t^\sigma)^2} & -\frac{\theta\gamma'(k_t)\sigma a_t^{\sigma-1}c_t}{(c_t-\gamma(k_t)a_t^\sigma)^2} & H_{33}^c \end{bmatrix}.$$

where

$$\begin{aligned} H_{22}^c &= -\frac{\theta\gamma(k_t)\sigma(\sigma-1)a_t^{\sigma-2}}{c_t-\gamma(k_t)a_t^\sigma} - \frac{\theta(\gamma(k_t)\sigma a_t^{\sigma-1})^2}{(c_t-\gamma(k_t)a_t^\sigma)^2}, \\ H_{33}^c &= -\frac{\theta\gamma''(k_t)a_t^\sigma}{c_t-\gamma(k_t)a_t^\sigma} - \frac{\theta(\gamma'(k_t)a_t^\sigma)^2}{(c_t-\gamma(k_t)a_t^\sigma)^2}. \end{aligned}$$

We are now able to compute the leading principal minors (Δ) of our matrix H^c:

$$\begin{aligned} \Delta_1 &= -\frac{\theta}{(c_t-\gamma(k_t)a_t^\sigma)^2} < 0, \\ \Delta_2 &= \frac{\theta\gamma(k_t)\sigma(\sigma-1)a_t^{\sigma-2}}{(c_t-\gamma(k_t)a_t^\sigma)^3} > 0, \\ \Delta_3 &= \frac{\theta^3\sigma a_t^{2(\sigma-1)}[-\gamma''(k_t)\gamma(k_t)(\sigma-1)+\sigma\gamma'(k_t)^2]}{(c_t-\gamma(k_t)a_t^\sigma)^4}. \end{aligned}$$

We know that our matrix is negative definite if and only if the leading principal minors alternate in sign starting with $\Delta_1 < 0$. It is straightforward to notice that $\Delta_2 > 0$ if and only if $\sigma > 1$ while $\Delta_3 < 0$ if and only if $\sigma\gamma'(k_t)^2 < \gamma''(k_t)\gamma(k_t)(\sigma-1)$. Given that $\sigma > 1$, the latter restriction can only be satisfied if $\gamma''(k_t) > 0$. This set of conditions then ensures that utility in young age is strictly concave. Given the similarity between utilities in young and old age, we obtain parallel results for H^d where strict concavity is ensured provided that $\sigma\gamma'(k_{t+1})^2 < \gamma''(k_{t+1})\gamma(k_{t+1})(\sigma-1)$ with $\sigma > 1$ and $\gamma''(k_{t+1}) > 0$.
Since the sum of two strictly concave functions is itself strictly concave, we obtain the restriction stated in the proposition. □

The restriction $\sigma > 1$ ensures that the utility function is jointly concave in c_t and a_t as well as in d_{t+1} and d_t. Intuitively, this assumption implies that the negative impact of aspirations on utility is larger for generations whose parents enjoyed larger consumption levels. Moreover, the aspiration function must be sufficiently convex in order to ensure the joint concavity in c_t, a_t and k_t as well as in d_{t+1}, d_t and k_{t+1}. These restrictions should apply to any framework which introduces endogenous aspirations into a utility function similar to ours.

Concerning production, there is a representative firm which produces an homogeneous good with a Cobb-Douglas production function, $y_t = Ak_t^\alpha$, where $\alpha \in (0,1)$ is the share of capital in the production process, $A > 0$ is a time-invariant productivity parameter and we assume complete depreciation after one period. The representative firm maximizes profits in a competitive market that clears:

$$\begin{aligned} R_t &= \alpha A k_t^{\alpha-1}, \\ w_t &= (1-\alpha)Ak_t^\alpha, \\ s_t &= k_{t+1}. \end{aligned}$$

A representative generation faces the following problem:

$$\max_{c_t, d_{t+1}, s_t} \theta \ln(c_t - \gamma(\overline{k}_t)a_t^\sigma) + (1-\theta)\ln(d_{t+1} - \gamma(\overline{k}_{t+1})d_t^\sigma) \tag{1}$$

subject to

$$\begin{cases} w_t = c_t + s_t, \\ d_{t+1} = R_{t+1}s_t, \\ c_t, d_{t+1}, s_t \geq 0, \end{cases}$$

given a_t, d_t, $\overline{k}_t$, $\overline{k}_{t+1}$, w_t and R_{t+1}.

Substituting for c_t and d_{t+1} in expression (1) and taking the derivative with respect to s_t, we obtain the following first-order condition:

$$\frac{\partial U}{\partial s_t} = -\frac{\theta}{c_t - \gamma(\overline{k}_t)a_t^\sigma} + \frac{(1-\theta)R_{t+1}}{d_{t+1} - \gamma(\overline{k}_{t+1})d_t^\sigma} = 0.$$

Concerning second-order conditions, we only need to prove that our utility function is concave in s_t. We directly compute

$$\frac{\partial^2 U}{\partial^2 s_t} = -\frac{\theta}{(c_t - \gamma(\overline{k}_t)a_t^\sigma)^2} - \frac{(1-\theta)R_{t+1}^2}{(d_{t+1} - \gamma(\overline{k}_{t+1})d_t^\sigma)^2} < 0,$$

implying that our utility function is strictly concave and we can proceed with the first-order conditions which at the symmetric equilibrium ($\overline{k}_t = k_t, \overline{k}_{t+1} = k_{t+1}$) can be written as

$$\theta(R_{t+1}k_{t+1} - \gamma(k_{t+1})d_t^\sigma) = (1-\theta)R_{t+1}(c_t - \gamma(k_t)c_{t-1}^\sigma). \tag{2}$$

Expression (2) represents the equality between the marginal benefit of increasing savings and the marginal cost of decreasing present consumption. We can now proceed with the study of the intertemporal equilibrium which is the topic of the next section.

3 Intertemporal competitive equilibrium

By using the first-order conditions as well as the budget constraints, we are able to define the intertemporal equilibrium of this economy.

Definition 1: An intertemporal equilibrium of this economy is a sequence $\{k_t, a_t\}_0^\infty$ with initial conditions $\{k_0, a_0\}$ that satisfies the following difference equations:

$$k_{t+1} = (1-\theta)[(1-\alpha)Ak_t^\alpha - \gamma(k_t)a_t^\sigma] + \theta(\alpha A)^{\sigma-1}\frac{\gamma(k_{t+1})k_t^{\sigma\alpha}}{k_{t+1}^{\alpha-1}}, \quad (3)$$

$$a_{t+1} = (1-\alpha)Ak_t^\alpha - k_{t+1}. \quad (4)$$

From expression (3), we see that the evolution of the capital stock is determined by three elements: the first one is the wage from which savings are generated. The second is the impact of young age aspirations which tends to favor young age consumption and to reduce savings. The third one is the impact of old age aspirations which favors old age consumption and thus increases savings. Expression (4) shows that young age aspirations evolve positively with the wage and the intensity of young age aspirations and negatively with the term related to old age aspirations.
It is interesting to focus on the dynamics of the saving rate $x_t = k_{t+1}/Ak_t^\alpha$, since endogenous aspirations might play an important role concerning the shape of the latter. The next proposition establishes conditions under which the saving rate is increasing in the accumulation of capital.

Proposition 2: In the competitive economy, the saving rate is increasing in the capital stock if and only if

$$(1-\theta)\gamma(k_t)a_t^\sigma\left(-\frac{\gamma'(k_t)k_t}{\gamma(k_t)}+\alpha\right) > \frac{\alpha\theta\gamma(k_{t+1})d_t^\sigma}{R_{t+1}}\left(-\frac{\gamma'(k_{t+1})k_{t+1}}{\gamma(k_{t+1})}+\alpha-\sigma\right). \quad (5)$$

Proof. The saving rate of our model is increasing in the capital stock provided that $\partial x_t/\partial k_t > 0$. Since our production function is of the Cobb-Douglas form, we know that this is equivalent to

$$\frac{1}{Ak_t^\alpha}\left(\frac{\partial k_{t+1}}{\partial k_t} - \alpha\frac{k_{t+1}}{k_t}\right) > 0.$$

We define ϵ_k as the elasticity of savings with respect to the capital stock k_t which must be larger than α for the saving rate to be increasing. By using the implicit function theorem we are able to compute ϵ_k and obtain

$$\epsilon_k = \frac{(1-\theta)[\alpha(1-\alpha)Ak_t^\alpha - \gamma'(k_t)k_t a_t^\sigma] + \theta(\alpha A)^{\sigma-1}\sigma\alpha k_t^{\sigma\alpha}\gamma(k_{t+1})k_{t+1}^{1-\alpha}}{k_{t+1} - \theta(\alpha A)^{\sigma-1}k_t^{\sigma\alpha}[\gamma'(k_{t+1})k_{t+1}^{2-\alpha} + (1-\alpha)\gamma(k_{t+1})k_{t+1}^{1-\alpha}]}. \quad (6)$$

The saving rate is then increasing if and only if

$$\begin{gathered}(1-\theta)[\alpha(1-\alpha)Ak_t^\alpha - \gamma'(k_t)a_t^\sigma] - \alpha k_{t+1} \\ > \\ -\alpha\theta(\alpha A)^{\sigma-1}k_t^{\sigma\alpha}[\gamma'(k_{t+1})k_{t+1}^{2-\alpha} + (1+\sigma-\alpha)\gamma(k_{t+1})k_{t+1}^{1-\alpha}].\end{gathered}$$

Using expression (3) to substitute for k_{t+1} on the left hand side of the latter inequality we obtain

$$\begin{gathered}(1-\theta)a_t^\sigma(-\gamma'(k_t)k_t + \alpha\gamma(k_t)) \\ > \\ -\alpha\theta(\alpha A)^{\sigma-1}k_t^{\sigma\alpha}[\gamma'(k_{t+1})k_{t+1}^{2-\alpha} + (\sigma-\alpha)\gamma(k_{t+1})k_{t+1}^{1-\alpha}].\end{gathered}$$

Using the fact that $d_t = \alpha Ak_t^\alpha$, $R_{t+1} = \alpha Ak_t^{\alpha-1}$ and rearranging, we obtain condition (5) stated in the proposition. □

Proposition 2 shows that a decreasing saving rate is only possible if the elasticity of the aspiration function is sufficiently large. Indeed, a necessary condition for a decreasing saving rate is

$$-\frac{\gamma'(k_{t+1})k_{t+1}}{\gamma(k_{t+1})} > \sigma - \alpha.$$

Since $\sigma > 1$ for second-order conditions to be satisfied, we can conclude that the elasticity of the aspiration function must be at least larger than the share of labor in the production function $1-\alpha$ in order to obtain a saving rate that is decreasing in capital. The presence of σ in the latter expression is due to the fact that an increase in the capital stock has a direct impact on the aspiration in old age d_t^σ implying that the individual has an incentive to increase savings. From expression (5), it can be concluded that the dynamics of the saving rate are mostly driven by the impact of aspirations in both ages (the two terms outside brackets) and by the elasticity of the aspiration function. In the standard case of exogenous aspirations where the elasticity is zero and $\sigma = 1$, the saving rate is thus always increasing with the capital stock. In the present case, if the elasticity of the aspiration function in old age weighted by the impact of aspirations at that age is sufficiently large compared with a similar term in young age, then it is possible for the saving rate to decrease with the capital stock. Even though we cannot derive a formal proof, it seems reasonable to think that such an outcome requires an increasing elasticity of the aspiration function. In that case, the impact of one unit of savings on old age aspirations is increasing during the transition and subsequent generations will have less incentives to increase the future capital stock. It is then possible to obtain a decreasing saving rate at some point in time during the transition. While this is not the focus of the present paper, our specification could be useful in explaining the hump-shaped pattern of the saving rate observed in OECD countries between 1950 and 1990 approximately (see, for example, Christiano, 1989; Antras, 2001; Alvarez-Cuadrado, 2008).

We focus next on the existence of a steady-state equilibrium. By setting $k_{t+1} = k_t = k$ and $a_{t+1} = a_t = a = c$ as well as using the difference equations (3) and (4) we obtain:

$$k = (1-\theta)[(1-\alpha)Ak^{\alpha} - \gamma(k)(w-k)^{\sigma}] + \theta(\alpha A)^{\sigma-1}\gamma(k)k^{1+\alpha(\sigma-1)}, \quad (7)$$

$$c = w - k = (1-\alpha)Ak^{\alpha} - k. \quad (8)$$

It is straightforward to notice that there is a steady-state at $(k, c) = (0, 0)$ with levels of capital and consumption equal to zero. In the following we will focus on the existence and stability of a unique positive steady-state. To begin with, it is useful to define $k_{max} > 0$ which is the capital stock that solves $w = k > 0$. Given our production function, we obtain $k_{max} = [(1-\alpha)A]^{1/1-\alpha}$. In the following study of the steady-state equilibrium, we can then focus on the interval $k \in (0, k_{max})$. We also rewrite expression (7) as $k = \phi(k)$ since we will use the latter in the following. The next proposition presents the conditions guaranteeing the existence of a unique positive steady-state and derives the ones under which the system is asymptotically stable.

Proposition 3: In the competitive economy,

(i) There is a unique positive steady-state equilibrium $(k, c) > (0, 0)$ if and only if

$$(\alpha A)^{\sigma-1}[(1-\alpha)A]^{\alpha(\sigma-1)/1-\alpha} < 1/\gamma(k_{max}),$$

and when $k = \phi(k)$, $\phi'(k) < 1$ $\forall k$.

(ii) The steady-state (k, c) is asymptotically stable if and only if

$$\begin{aligned} (1-\theta)(1-\alpha)\gamma(k)\sigma c^{\sigma-1}\alpha A k^{\alpha-1} &< 1, & (9)\\ J_{11} + (1-\theta)\gamma(k)\sigma c^{\sigma-1}[1-(1-\alpha)\alpha A k^{\alpha-1}] &< 1, & (10)\end{aligned}$$

where

$$J_{11} = \frac{(1-\theta)[(1-\alpha)\alpha A k^{\alpha-1} - \gamma'(k)c^{\sigma}] + \theta\sigma\alpha(\alpha A)^{\sigma-1}\gamma(k)k^{\alpha(\sigma-1)}}{1-\theta(\alpha A)^{\sigma-1}k^{\alpha(\sigma-1)}[\gamma'(k)k + (1-\alpha)\gamma(k)]}.$$

(iii) The convergence to the steady-state (k, c) is oscillatory if and only if conditions (9), (10) and the following one are satisfied:

$$[J_{11} + (1-\theta)\gamma(k)\sigma c^{\sigma-1}]^2 < 4(1-\theta)(1-\alpha)\alpha A k^{\alpha-1}\gamma(k)\sigma c^{\sigma-1}. \quad (11)$$

(iv) There is a specific value of σ implying the existence of a Neimark-Sacker bifurcation. Let $(k_{\sigma_0}, c_{\sigma_0})$ define the steady-state for which $\sigma = \sigma_0$ and where:

$$(1-\theta)(1-\alpha)\gamma(k_{\sigma_0})\sigma_0 c^{\sigma_0 - 1}\alpha A k_{\sigma_0}^{\alpha-1} = 1. \quad (12)$$

There is a neighborhood of U of σ_0 for which there is, either for the case $\sigma_0 < \sigma$ or for the case $\sigma_0 > \sigma$, a closed invariant curve $\Gamma(\sigma)$ which encircles $(k_{\sigma_0}, c_{\sigma_0})$ with $\Gamma(\sigma_0) = (k_{\sigma_0}, c_{\sigma_0})$.

Proof. We derive first the conditions ensuring the existence of a unique positive steady-state and in order to do so we study the function $g(k) = \phi(k) - k$. It is straightforward to check that $\lim_{k\to 0} g(k) = 0$ and $\lim_{k\to k_{max}} g(k) = z$ where z is a constant. Moreover

$$\begin{aligned}\phi'(k) = &(1-\theta)(1-\alpha)\alpha A k^{\alpha-1}[1-\gamma(k)\sigma(w-k)^{\sigma-1}]\\ &+(1-\theta)(w-k)^{\sigma-1}[-\gamma'(k)(w-k) + \sigma\gamma(k)]\\ &+\theta(\alpha A)^{\sigma-1}k^{\alpha(\sigma-1)}\{\gamma'(k)k + [1+\alpha(\sigma-1)]\gamma(k)\}.\end{aligned}$$

implying that $\lim_{k\to 0} g'(k) = +\infty$. In order to obtain a unique steady-state, it is necessary that $z < 0$ which is only possible if

$$\begin{aligned} k_{max} &> (1-\theta)k_{max} + \theta(\alpha A)^{\sigma-1}\gamma(k_{max})k_{max}^{1+\alpha(\sigma-1)},\\ \frac{1}{\gamma(k_{max})} &> (\alpha A)^{\sigma-1}[(1-\alpha)A]^{\alpha(\sigma-1)/1-\alpha}.\end{aligned}$$

Moreover, when $g(k) = 0$ it is as well necessary that $g'(k) < 0$ $\forall k$ which is equivalent to $\phi'(k) < 1$. These two conditions ensure the existence of a

unique steady-state.
We now linearize the system around the steady-state in order to assess local stability (see, for example, Azariadis, 1993). We start by computing the Jacobian matrix around the steady-state (k, c):

$$J = \begin{bmatrix} J_{11} & -(1-\theta)\gamma(k)\sigma c^{\sigma-1} \\ (1-\alpha)\alpha A k^{\alpha-1} - J_{11} & (1-\theta)\gamma(k)\sigma c^{\sigma-1} \end{bmatrix}.$$

where

$$J_{11} = \frac{(1-\theta)[(1-\alpha)\alpha A k^{\alpha-1} - \gamma'(k)c^{\sigma}] + \theta\sigma\alpha(\alpha A)^{\sigma-1}\gamma(k)k^{\alpha(\sigma-1)}}{1-\theta(\alpha A)^{\sigma-1}k^{\alpha(\sigma-1)}[\gamma'(k)k + (1-\alpha)\gamma(k)]}. \quad (13)$$

We know that $\phi'(k) < 1$ at the steady-state implying that $J_{11} < 1$ as well. The characteristic function $P(\lambda)$ is given by:

$$P(\lambda) = \lambda^2 - Tr(J)\lambda + Det(J) = 0,$$

where

$$\begin{aligned} Tr(J) &= J_{11} + (1-\theta)\gamma(k)\sigma c^{\sigma-1} > 0, \quad (14) \\ Det(J) &= (1-\theta)(1-\alpha)\alpha A k^{\alpha-1}\gamma(k)\sigma c^{\sigma-1} > 0. \quad (15) \end{aligned}$$

In planar maps, the steady-state is asymptotically stable if $1 + Tr(J) + Det(J) > 0$, $Det(J) < 1$ and $1 - Tr(J) + Det(J) > 0$. In our case, the first condition is always satisfied since $Tr(J) > 0$ and $Det(J) > 0$. Using the definitions of the determinant, the second condition is satisfied provided that expression (9) is satisfied. Finally, using the definitions of the trace and the determinant, the third condition is satisfied provided that (10) is satisfied.
Convergence to the steady-state displays damped oscillations provided that the steady-state is asymptotically stable and additionally $Tr(J)^2 < 4Det(J)$, which is equivalent to condition (11).
The last part of the proposition focuses on the existence of Neimark-Sacker bifurcation. A necessary condition for the existence of such a bifurcation is that the eigenvalues of the Jacobian matrix are complex conjugates with modulus equal to one. This condition being equivalent to $Det(J) = 1$ and $Tr(J) \in (-2, 2)$. Given that the trace is always positive in our model, we can focus on the restriction $Tr(J) < 2$. Choosing σ as our bifurcation parameter, we then look for values of σ such that condition (9) is satisfied with equality. This is the case for a sufficiently large value of σ that we denote σ_0 obtaining expression (12). We also know that when $Det(J) = 1$, condition (10) reduces to $J_{11} + (1-\theta)\gamma(k)\sigma c^{\sigma-1} < 2$, which is always satisfied since $J_{11} < 1$ and $(1-\theta)\gamma(k)\sigma c^{\sigma-1} < 1$. The latter being required for utility to be increasing

in young age consumption at the steady-state.
It can then be checked that the two eigenvalues cross the unit circle at non-zero speed when σ changes around σ_0 and that none of them may be of the first four roots of unity. These conditions are sufficient for the existence of a Neimark-Sacker bifurcation. $\square$

The first necessary condition guaranteeing the existence of a unique steady-state requires $\gamma(k_{max})$ to be sufficiently small. This implies that the impact of capital on the aspiration function must be sufficiently large. The second necessary condition requires that the steady-state equation for capital (7) is not increasing too much at the steady-state.
Concerning asymptotic stability, it is interesting to notice that larger values of σ are associated with potential instability since the left hand side of (9) and (10) are both increasing in σ. In the standard aspiration model of de la Croix (1996), asymptotic stability is ensured provided that the constant γ is not too large and in our case σ plays a similar role by increasing the impact of aspirations. While σ must be larger than one in order to ensure the strict concavity of the utility function, it cannot be too large in order to avoid instability of the dynamical system. When $\sigma = \sigma_0$, the system undergoes a Neimark-Sacker bifurcation implying the existence of a limit-cycle around $(k_{\sigma_0}, c_{\sigma_0})$. The issue of the stability of the cycle requires additional restrictions on third-order derivatives and we should only state that in the supercritical case, the system permanently oscillates around the steady-state.
It is also interesting to notice that if the steady-state is dynamically inefficient $(\alpha A k^{\alpha-1} < 1)$, condition (9) is always satisfied since $(1-\theta)\gamma(k)\sigma c^{\sigma-1} < 1$ for utility to be increasing in young age consumption at the steady-state. This implies that the system cannot exhibit fluctuations through a limit-cycle. Moreover, oscillatory convergence to the steady-state is more difficult to observe in this case since the right hand side of expression (11) is increasing in the marginal productivity of capital. In the case where the steady-state is dynamically efficient $(\alpha A k^{\alpha-1} > 1)$, condition (9) might not be satisfied while (11) is more likely to be implying that the system might exhibit endogenous fluctuations under the form of damped oscillations or through a limit cycle.
The possibility to observe cyclical behavior in the present model deserves some further comments concerning the economic mechanism leading to this outcome. In a situation where capital grows, the agent tends to consume more in young age given that the aspiration effect is larger at that age. However, the decreasing marginal productivity of labor does not generate enough income to satisfy the young age aspirations of future generations. At some point in time, the economy suffers from a recession due to low capital ac-

cumulation. In our framework, this produces a change in the intensity of aspirations which become larger in old age, generating in turn an incentive to increase savings in order to satisfy old age aspirations. Endogenous aspirations act as a stabilization device by increasing young age consumption in expansion periods and savings (or old age consumption) in recession periods. Damped oscillations seem thus to be a natural outcome in our competitive framework with endogenous aspirations, a fact that will be confirmed when we will numerically simulate the model. However, when $\sigma = \sigma_0$, it is possible that this mechanism generates everlasting fluctuations under the form of a limit-cycle. The fact that endogenous fluctuations are more probable when the steady-state is dynamically efficient is intuitive since in this case the economy exhibits a relatively low accumulation of capital which is compatible with endogenous fluctuations. An additional interesting observation is that the increase in savings during recessions generated by endogenous aspirations is in line with empirical evidence provided by Adema and Pozzi (2015), Chakrabarti et al. (2013) or Crossley et al. (2013). Our model thus provides a complementary explanation for this increase in savings beyond the standard precautionary motive.

4 Optimality

4.1 Optimal allocation

In this section, we consider the case of a central planner who chooses the allocation of resources in order to maximize the discounted sum of utilities of present and future generations. Contrary to the representative generation in the competitive case, the planner takes into account the impact of aspirations and the fact that the latter decrease with the average capital stock. The social discount factor is given by $\beta \in (0,1)$ and the planner will maximise the discounted sum of utilities subject to the feasibility constraint and the evolution of aspirations. The optimization problem is the following:

$$\max_{\{c_t, d_{t+1}, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [\theta \ln(c_t - \gamma(k_t) c_{t-1}^{\sigma}) + (1-\theta) \ln(d_{t+1} - \gamma(k_{t+1}) d_t^{\sigma})]$$

subject to:

$$\begin{cases} Ak_t^{\alpha} = c_t + d_t + k_{t+1}, \\ c_t, d_{t+1}, k_{t+1} \geq 0, \end{cases}$$

given initial conditions $\{k_0, c_{-1}, d_{-1}\}$.
The Lagrangian function is the following:

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t [\theta \ln(c_t - \gamma(k_t) c_{t-1}^{\sigma}) + (1-\theta)\ln(d_{t+1} - \gamma(k_{t+1}) d_t^{\sigma})] \\ &\quad + \sum_{t=0}^{\infty} \beta^t \lambda_t (A k_t^{\alpha} - c_t - d_t - k_{t+1}).\end{aligned}$$

The first-order conditions of the maximization problem are

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_t} &= \frac{\beta^t \theta}{c_t - \gamma(k_t) c_{t-1}^{\sigma}} - \frac{\beta^{t+1}\theta\sigma\gamma(k_{t+1}) c_t^{\sigma-1}}{c_{t+1} - \gamma(k_{t+1}) c_t^{\sigma}} - \beta^t \lambda_t = 0, \\ \frac{\partial \mathcal{L}}{\partial d_{t+1}} &= \frac{\beta^t \delta}{d_{t+1} - \gamma(k_{t+1}) d_t^{\sigma}} - \frac{\beta^{t+1}\delta\sigma\gamma(k_{t+2}) d_{t+1}^{\sigma-1}}{d_{t+2} - \gamma(k_{t+2}) d_{t+1}^{\sigma}} - \beta^{t+1}\lambda_{t+1} = 0, \\ \frac{\partial \mathcal{L}}{\partial k_{t+1}} &= -\frac{\beta^{t+1}\theta\gamma'(k_{t+1}) c_t^{\sigma}}{c_{t+1} - \gamma(k_{t+1}) c_t^{\sigma}} - \frac{\beta^t \delta\gamma'(k_{t+1}) d_t^{\sigma}}{d_{t+1} - \gamma(k_{t+1}) d_t^{\sigma}} - \beta^t \lambda_t \\ &\quad + \beta^{t+1}\lambda_{t+1}\alpha A k_{t+1}^{\alpha-1} = 0, \\ \lim_{t\to\infty} \beta^t \lambda_t k_t &= 0.\end{aligned}$$

Since our objective function is strictly concave, the first-order conditions are also sufficient for optimality and we obtain:

$$\frac{\theta}{c_t^* - \gamma(k_t^*) c_{t-1}^{*\sigma}} - \frac{\beta\theta\sigma\gamma(k_{t+1}^*) c_t^{*\sigma-1}}{c_{t+1}^* - \gamma(k_{t+1}^*) c_t^{*\sigma}} = \frac{1-\theta}{\beta(d_t^* - \gamma(k_t^*) d_{t-1}^{*\sigma})} - \frac{(1-\theta)\sigma\gamma(k_{t+1}^*) d_t^{*\sigma-1}}{d_{t+1}^* - \gamma(k_{t+1}^*) d_t^{*\sigma}}, \tag{16}$$

$$-\left(\frac{\beta\theta\gamma'(k_{t+1}^*) c_t^{*\sigma}}{c_{t+1}^* - \gamma(k_{t+1}^*) c_t^{*\sigma}} + \frac{(1-\theta)\gamma'(k_{t+1}^*) d_t^{*\sigma}}{d_{t+1}^* - \gamma(k_{t+1}^*) d_t^{*\sigma}} \right) = \lambda_t - \beta\lambda_{t+1}\alpha A k_{t+1}^{*\alpha-1}, \tag{17}$$

$$\frac{\theta}{c_t^* - \gamma(k_t^*) c_{t-1}^{*\sigma}} - \frac{\beta\theta\sigma\gamma(k_{t+1}^*) c_t^{*\sigma-1}}{c_{t+1}^* - \gamma(k_{t+1}^*) c_t^{*\sigma}} = \lambda_t, \tag{18}$$

$$\frac{1-\theta}{d_{t+1}^* - \gamma(k_{t+1}^*) d_t^{*\sigma}} - \frac{\beta(1-\theta)\sigma\gamma(k_{t+2}^*) d_{t+1}^{*\sigma-1}}{d_{t+2}^* - \gamma(k_{t+2}^*) d_{t+1}^{*\sigma}} = \beta\lambda_{t+1}. \tag{19}$$

where starred variables denote optimal outcomes. Expression (16) describes the allocation of consumption between generations alive at the same time. The marginal utilities of consumption are equalized between the young and the old generation living at the same period. These marginal utilities are adjusted in order to internalize the impact of consumption on future aspirations. Expressions (17), (18) and (19) describe the intertemporal allocation of consumption. It is easy to see that the latter will be different from the standard Diamond model where the left hand side of expression (17) is equal to zero. In the present case, this term is positive since $\gamma'(k^*_{t+1}) < 0$.

Definition 2: An intertemporal optimal allocation of this economy is a sequence $\{c^*_t, d^*_t, k^*_t\}_0^\infty$ with initial conditions $\{k_0, c_{-1}, d_{-1}\}$ that satisfies the difference equations (16), (17), (18) and (19) as well as the following constraint:

$$k^*_{t+1} = Ak^{*\alpha}_t - c^*_t - d^*_t.$$

The steady-state of this optimal allocation is defined by the following set of equations:

$$\begin{aligned}
\frac{\theta(1-\beta\sigma\gamma(k^*)c^{*\sigma-1})}{c^*-\gamma(k^*)c^{*\sigma}} &= \frac{1-\theta}{\beta}\left(\frac{1-\beta\sigma\gamma(k^*)d^{\sigma-1}}{d^*-\gamma(k^*)d^{*\sigma}}\right), \\
-\left(\frac{\beta\theta\gamma'(k^*)c^{*\sigma}}{c-\gamma(k^*)c^{*\sigma}} + \frac{(1-\theta)\gamma'(k^*)d^{*\sigma}}{d^*-\gamma(k^*)d^{*\sigma}}\right) &= \lambda(1-\beta\alpha Ak^{*\alpha-1}), \qquad (20) \\
\frac{\theta(1-\beta\sigma\gamma(k^*)c^{*\sigma-1})}{c^*-\gamma(k^*)c^{*\sigma}} &= \lambda, \qquad (21) \\
k^* &= Ak^{*\alpha} - c^* - d^*.
\end{aligned}$$

Expressions (20) and (21) characterize the modified golden rule defining the optimal steady-state capital stock. In a standard OLG model, the modified golden rule reduces to $1/\beta = \alpha Ak^{*\alpha-1}$. Since $\gamma'(k^*) < 0$ in the present model, $1/\beta > \alpha Ak^{*\alpha-1}$ and the optimal steady-state capital stock is larger in our economy with endogenous aspirations. If $\gamma'(k) = 0$, we recover the standard modified golden rule. The next proposition summarizes the result.

Proposition 4: Since $\gamma'(k^*) < 0$, the optimal steady-state capital stock of our economy with endogenous aspirations (k^*) is always larger than the optimal steady-state capital stock of the standard Diamond model.

It is worth noticing that in models with exogenous aspirations, the optimal capital stock converges to the standard modified golden rule implying that our framework introduces a major difference concerning the optimal accumulation of capital. The reason why the planner decides to accumulate more capital in the present framework is that this allows him to reduce the magnitude of the externality linked to endogenous aspirations. Contrary to the representative agent, the planner knows that aspirations are a decreasing function of the average capital stock and a faster accumulation of the latter

allows to mitigate the impact of the externality during both the transition and at the steady-state.
We will now compare the competitive and optimal allocations at the steady-state in terms of consumption by using the first-order conditions in both cases. We proceed by comparing the effective consumption ratio $(c-\gamma(k)c^{\sigma}/d-\gamma(k)d^{\sigma})$ in both economies.

Proposition 5: The optimal steady-state effective consumption ratio $c^*-\gamma(k^*)c^{*\sigma}/d^*-\gamma(k^*)d^{*\sigma}$ is larger than its competitive counterpart $c-\gamma(k)c^{\sigma}/d-\gamma(k)d^{\sigma}$ if and only if

$$\alpha A k^{\alpha-1} > \frac{1}{\beta}\left(\frac{1-\beta\gamma(k^*)\sigma d^{*\sigma-1}}{1-\beta\gamma(k^*)\sigma c^{*\sigma-1}}\right). \tag{22}$$

Proof. By using the first-order conditions in both economies, we obtain the following ratios at the steady-state:

$$\begin{aligned}\frac{c-\gamma(k)c^{\sigma}}{d-\gamma(k)d^{\sigma}} &= \frac{\theta}{(1-\theta)\alpha A k^{\alpha-1}},\\ \frac{c^*-\gamma(k^*)c^{*\sigma}}{d^*-\gamma(k^*)d^{*\sigma}} &= \frac{\beta\theta(1-\beta\gamma(k^*)\sigma c^{*\sigma-1})}{(1-\theta)(1-\beta\gamma(k^*)\sigma d^{*\sigma-1})}.\end{aligned}$$

By comparing both expressions, we obtain inequality (22). □

The effective consumption ratio is larger in the optimal case provided that the steady-state marginal productivity of capital of the competitive economy is sufficiently large (or equivalently the capital stock is sufficiently small). In a model with exogenous aspirations, where $\gamma(k)=\gamma$ and $\sigma=1$, the condition reduces to $\alpha A k^{\alpha-1} > 1/\beta$ such that the planner implements a steady-state allocation with a larger (smaller) effective consumption ratio if the competitive allocation is on the left (right) of the modified golden rule. In the present case, the comparison between both ratios depends directly on the optimal levels of consumption c^* and d^*. When $d^* > c^*$, the right hand side of expression (22) is smaller than in a model with exogenous aspirations and a planner that would implement an optimal allocation with more young age effective consumption in a model with exogenous aspirations will also do so in the present model. However, if the planner implements an allocation with more old age consumption in a model with exogenous aspirations, he will not necessarily do so in a model with endogenous ones. The result is reversed when $c^* > d^*$, since then the right hand side of expression (22) is larger than in a model with exogenous aspirations. The presence of endogenous aspirations is thus particularly important when comparing the allocation of consumption in the competitive and the optimal economies.

4.2 Decentralizing the first best allocation

In order to decentralize the first best allocation, a policy intervention should focus on two objectives: adjusting the consumption of both generations correcting for the future impact of aspirations and adjusting savings in order to reach the optimal capital stock. As we will see shortly, these objectives can be reached by subsidizing or taxing savings and using appropriate lump sum transfers. Let's denote the investment subsidy by i_t and define lump-sum transfers to the young g_t, and to the old n_t.
The maximization problem of the individual becomes

$$\max_{c_t,d_{t+1},s_t} \theta \ln(c_t - \gamma(\overline{k}_t)a_t^\sigma) + (1-\theta)\ln(d_{t+1} - \gamma(\overline{k}_{t+1})d_t^\sigma)$$

subject to

$$\begin{cases} w_t + g_t = c_t + s_t, \\ d_{t+1} = R_{t+1}(1+i_{t+1})s_t + n_{t+1}, \\ c_t, d_{t+1}, s_t \geq 0, \end{cases}$$

given a_t, d_t, $\overline{k}_t$, $\overline{k}_{t+1}$, w_t and R_{t+1}.
We then obtain the following first-order condition:

$$\frac{(1-\theta)R_{t+1}}{d_{t+1} - \gamma(\overline{k}_{t+1})d_t^\sigma}(1+i_{t+1}) \quad = \quad \frac{\theta}{c_t - \gamma(\overline{k}_t)a_t^\sigma}. \tag{23}$$

Proposition 6: In order to decentralize the optimal allocation, the investment subsidy satisfies

$$\begin{aligned} \left[\frac{(1-\theta)\alpha A k_{t+1}^{*\alpha-1}}{d_{t+1}^* - \gamma(k_{t+1}^*)d_t^{*\sigma}}\right] i_{t+1} \quad = \quad & \frac{\beta\theta(\gamma(k_{t+1}^*)\sigma c_t^{*\sigma-1} - \gamma'(k_{t+1}^*))}{c_{t+1}^* - \gamma(k_{t+1}^*)c_t^{*\sigma}} \\ & - \frac{(1-\theta)\gamma'(k_{t+1}^*)d_t^{*\sigma}}{d_{t+1}^* - \gamma(k_{t+1}^*)d_t^{*\sigma}} \\ & - \left[\frac{\beta(1-\theta)\sigma\gamma(k_{t+2}^*)d_{t+1}^{*\sigma-1}}{d_{t+2}^* - \gamma(k_{t+2}^*)d_{t+1}^{*\sigma}}\right]\alpha A k_{t+1}^{*\alpha-1}. \end{aligned} \tag{24}$$

Proof. Combining the first-order conditions of the planner's problem with expression (23) it is straightforward to compute the value of the tax rate. □

The decentralization of the first best allocation is completed by the following lump-sum transfers:

$$g_t \quad = \quad k_{t+1}^* + c_t^* - w_t^*, \tag{25}$$
$$n_t + g_t \quad = \quad -i_t R_t^* k_t^*. \tag{26}$$

Expression (25) ensures that the capital stock is set at the level of the modified golden rule while expression (26) is the planner's budget constraint.
As can be seen from expression (24), the planner will not necessarily subsidize investment since the right hand side of the latter expression can take both positive and negative values. The elements in favor of an investment subsidy are the first two terms on the right hand side of expression (24). The first term integrates the impact of young age consumption on future young age aspirations as well as the positive role of investment in decreasing the young age aspiration effect in the future. An additional unit of consumption is a missed opportunity to decrease the intergenerational externality generated by aspirations. The second term integrates the similar impact of capital on the intensity of old age aspirations. These three factors tend to increase the investment subsidy. However, the last term is negative and integrates the impact of old age consumption on future old age aspirations. This element decreases the investment subsidy since a lower capital stock when old generates less future aspirations. It is useful to notice that the introduction of endogenous aspirations tends to increase the investment subsidy since both terms depending on $\gamma'(k^*_{t+1})$ are positive.

5 Numerical example

In this section, we will illustrate our results with a numerical example for both the competitive and the optimal allocations. We consider that one period of the model is equivalent to 30 years which is a standard assumption in this type of models. In order to proceed, we first need to choose a functional form for our aspiration function $\gamma(k_t)$. This function should satisfy the restriction obtained in Proposition 1 to ensure the strict concavity of the objective function in the planner's case.

Assumption 2: The aspiration function takes the following functional form:

$$\gamma(k_t) = \frac{\overline{\gamma}}{1+k_t},$$

with $0 < \overline{\gamma} < 1$.

With such a function, the restriction ensuring the strict concavity of the utility function reduces to $\sigma > 2$. The advantage of such a specification is that the restriction is satisfied for all values of the capital stock. This function will also allow us to obtain a unique steady-state in our numerical example. The parameter $\overline{\gamma}$ represents the maximum value of the aspiration effect which is set at 0.65. This is the value chosen by de la Croix and Michel (1999) in their paper with exogenous aspirations in young age and it seems reasonable to think of this value has an upper bound for the aspiration parameter. In order to satisfy second-order conditions, σ is set at 2.1. We assume a quarterly psychological discount factor of 0.99 implying that $\theta = 0.77$. The elasticity of capital in the production function α is set at 0.33 which is in accordance with empirical evidence while the productivity parameter A is set at 3. Concerning the optimal allocation, we focus on the quasi golden rule case with a social discount factor β set at 0.99. Table 1 summarizes the values for the different parameters.

Table 1: Values for the parameters

Parameter	Value
A	3
α	0.33
β	0.99
θ	0.77
$\overline{\gamma}$	0.65
σ	2.1

Figure 1 presents the dynamic paths for young age consumption, old age consumption and the capital stock in the competitive economy. The large intensity of aspirations at the beginning of the transition generates an increase in young age consumption which can only be temporary given the corresponding low accumulation of capital. At some point in time, capital decreases followed by young and old age consumption. As explained before, this generates a change in the intensity of aspirations which become larger in old age and tend to increase savings. Capital accumulation then allows to increase consumption in both ages once again. This behavior is confirmed by the fact that old age consumption appears to be less volatile than young age one. Indeed, while in expansion periods, old age consumption increases less due to larger aspirations in young age, in recessions, old age consumption decreases less due to larger aspirations at that age. The cyclical behavior of the economy repeats itself with damped oscillations converging to the unique steady-state after 20 periods. Figure 2 presents the life cycle utility of successive generations in the competitive case and shows that the latter exhibits damped oscillations as well due to the behavior of endogenous variables. The first generations enjoy relatively large levels of utility at the expenses of future ones due to large consumption levels and low capital accumulation.

Insert Figure 1 and 2

We proceed in the same way concerning the optimal allocation and obtain Figure 3 which displays the dynamic paths of the same variables in the optimal case. At the beginning of the transition, young age consumption is larger than capital due to the intensity of aspirations inherited from the past. However, the planner will progressively increase capital in order to avoid fluctuations as in the competitive case and to ensure a monotonic convergence to the steady-state. While steady-state young age consumption and capital are larger in the optimal economy, this is not the case for old age consumption which is larger in the competitive economy. In order to maximize lifetime utility, the planner decides to increase the effective consumption ratio in favor of young generations. Figure 4 presents the life cycle utility of successive generations in the optimal case and show that the latter is monotonically increasing along the transition. It can also be noticed that the planner will implement a smaller utility for the first generations in order to accumulate more capital and obtain larger utility levels for the forthcoming generations.

Insert Figure 3 and 4

Figure 5 presents the evolution of the capital subsidy as well as the one of transfers to the young and the old generations. The capital subsidy is positive and increasing along the transition in order to obtain a capital stock equivalent to the one of the optimal allocation. Concerning transfers, the planner implements positive transfers to young generations and finances the latter together with investment subsidies through a lump-sum tax on old generations. Given that transfers to the young and the investment subsidy are increasing along the transition, the lump-sum tax on the old must also increase in order to satisfy the planner's budget constraint. This policy is intuitive since it allocates relatively more resources to the young generation which needs to increase savings.

Insert Figure 5

6 Conclusion

In this paper, we have extended the overlapping generations literature by introducing endogenous aspirations into an otherwise standard model. The representative agent has aspirations in both periods of life and the latter are a decreasing function of the average capital stock. Moreover, effective consumption in both periods does not depend linearly on the level of aspirations. In order to ensure that the utility function is jointly strictly concave, we have derived restrictions guaranteeing that the Hessian matrix is negative definite.

These are a sufficiently convex aspiration function as well as a relatively large parameter governing the impact of aspirations on utility.
In the competitive case, we derived a condition characterizing the behavior of the saving rate along the transition. The latter can only decrease if the elasticity of the aspiration function in old age is sufficiently large. We also derived conditions guaranteeing both the existence of a unique positive steady-state and the asymptotic stability of the dynamical system. When the steady-state is dynamically efficient, the system might undergo both short and long run fluctuations given that endogenous aspirations tend to increase young age consumption during expansion periods and old age consumption during recessions.
We then focused on the optimal allocation which generates a steady-state capital stock that is larger than the one implied by the standard modified golden rule and is clearly different from the competitive equilibrium in terms of effective consumption allocation. In order to decentralize the first best allocation, we have implemented an optimal tax policy by using an investment tax and appropriate lump-sum transfers. The policy is characterized by a subsidy or a tax on investment depending on the direct effect of aspirations as well as on the indirect one related to the endogeneity of aspirations.
We finally presented a numerical illustration where the competitive equilibrium displays cyclical behavior until convergence to the steady-state while the optimal allocation displays a monotonic transition associated to a larger capital stock in the long run. In this specific case, the planner implements an investment subsidy and positive transfers to young generations which are financed by a lump-sum tax on old generations.
Further research could of course focus on the introduction of endogenous habits in a similar framework. A different extension to consider is to introduce habits or aspirations in other types of goods for which average capital increases the comparison effect. We can think about environmental quality or other types of public goods.

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Figure 1: Dynamic paths in the competitive economy

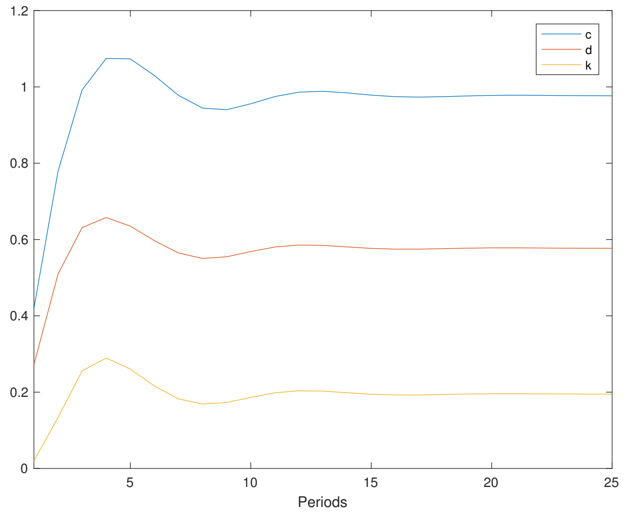

Figure 2: Utility in the competitive economy

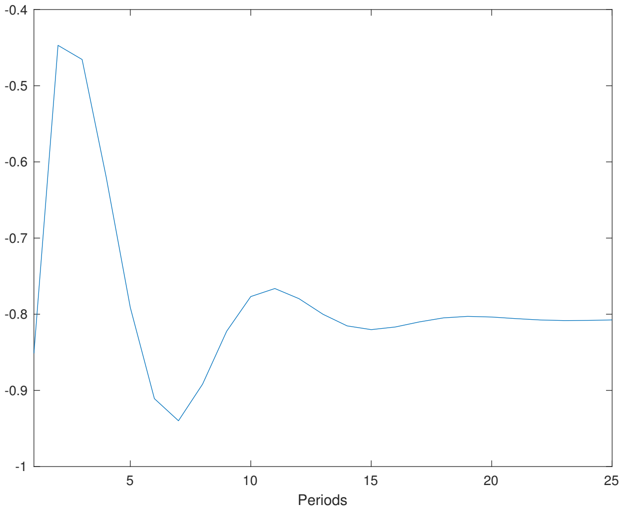

Figure 3: Dynamic paths in the optimal economy

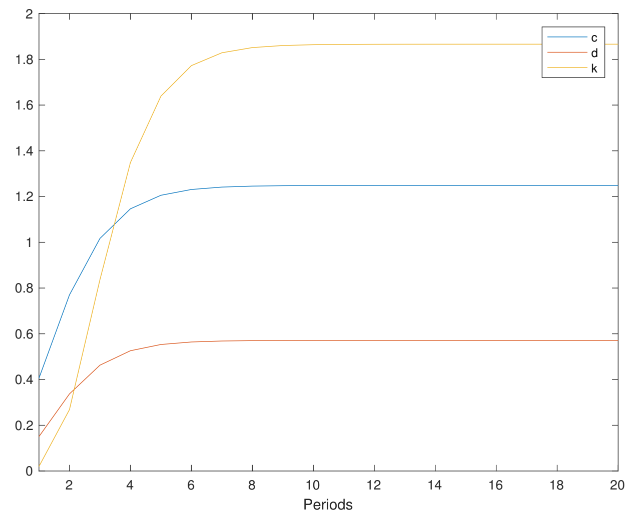

Figure 4: Utility in the optimal economy

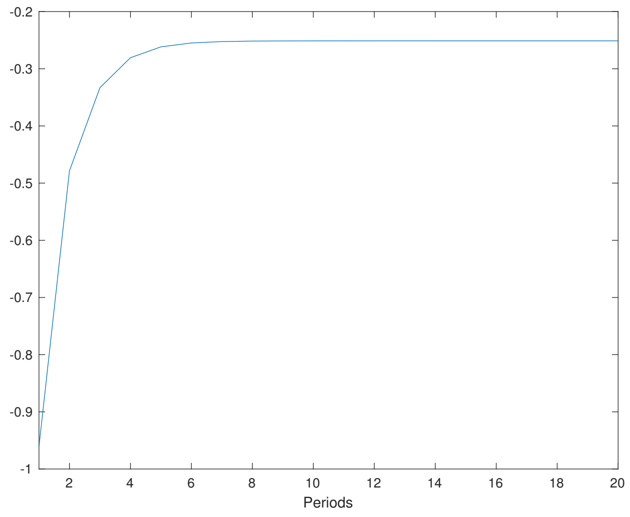

Figure 5: Investment subsidy and transfers