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Labor mobility, structural change and economic growth*

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Abstract

This paper develops a two-sector growth model in which the process of structural change in the sectoral composition of employment and GDP is jointly determined by income effects, derived from non-homothetic preferences, and by a substitution mechanism derived from a labor mobility cost. This cost, paid by workers moving to another sector, generates a sectoral wage gap that limits structural change. Our model can explain the following patterns of development of the US economy throughout the period 1880-2000: (i) balanced growth of the aggregate variables in the second half of the last century; (ii) structural change in the sectoral composition of employment between agricultural and non-agricultural sectors; (iii) structural change process in the sectoral composition of GDP between these sectors; and (iv) wage convergence between the two sectors. We outline that in the absence of wage gaps the model is not able to jointly explain the process of structural change in the sectoral composition of both GDP and employment. This cost reduces the Possibilities Production Frontier (PPF) by constraining the sectoral allocation of production factors. This implies a loss of GDP which amounts to over 30% of the GDP throughout initial periods according to the calibrated model. During the transition, the loss of GDP decreases and eventually vanishes. Thus, the elimination of this technological constraint explains part of the increase in the GDP. Additionally, this study points out that the aforementioned technological constraint introduces a mechanism through which cross-country differences in sectoral composition may account for cross-country income differences.

JEL classification codes: O41, O47.

Keywords: structural change, non-homothetic preferences, labor mobility.

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1 Introduction

Recent multisector growth literature has built models aimed at explaining together the balanced growth of aggregate variables as well as the process of structural change observed in most developed economies (see Acemoglu and Guerrieri, 2008; Boppart, 2014; Dennis and Iscan, 2009; Melck, 2002; Foellmi and Zweimuller, 2008; Kongsamut et al., 2001; or Ngai and Pissarides, 2007). On the one hand, the balanced growth of aggregate variables consists of an almost constant ratio of capital to GDP and an almost constant interest rate. On the other hand, the process of structural change consists in a large shift of both employment and aggregate production from agriculture to other sectors. This process, common to most economies, is illustrated in the first two columns of Table 1 for the US economy over the period 1880 to 2000. This table shows that the shares of both employment and GDP in the agricultural sector decline during the entire period and attained almost the same value at the end of the period. However, at the beginning of the period, the employment share almost doubles the GDP share. Obviously, this implies that the employment share in the agricultural sector declines much faster than the GDP share.

[Insert Table 1]

The aforementioned literature explains both balanced growth of aggregate variables and the process of structural change in the sectoral composition of employment. This literature can be split into two different groups. One set of studies outlines that demand factors are the driving force of structural change (see, e.g., Kongsamut et al., 2001). These demand factors comprise income effects generated by non-homothetic preferences that drive structural change as the economy develops. The other set argues that supply factors are the driving force of structural change (see, e.g., Acemoglu and Guerrieri, 2008; or Ngai and Pissarides, 2007). These factors encompass variations in relative prices that cause structural change through a substitution effect. More recently, the literature combines demand and supply factors to explain structural change (see, e.g., Boppart, 2014; Comin et al., 2015; Dennis and Iscan, 2009; or Herrendorf et al., 2013 and 2014). While these papers explain the process of structural change in the sectoral composition of employment, none of them explains the magnitudes of the two patterns of structural change: the shifts in employment and aggregate production from agriculture to other sectors. Buera and Kaboski (2009) argue that quantitative models require some other ingredients as, for instance, sector specific factor distortions, to be consistent with structural change in both value added terms and in terms of factors allocation. We outline in this paper that explaining both patterns of structural change is fundamental to determine the effect of structural change on economic growth, as the differences in both sectoral value added and factors allocation imply sectoral differences in productivity. Therefore, the mechanisms that explain the different patterns of structural change may also contribute to explain cross-country income differences.

In this paper, we show that the two features of structural change can be explained when factor distortions cause sectoral wages differentials. In order to motivate this conclusion, we use the definition of the labor income share (LIS) at the sectoral level, and we decompose the ratio between the LIS in the agricultural sector and the LIS in the non-agricultural sector as the product of the following three other ratios: the ratio between wages in the agricultural and non-agricultural sectors; the ratio between the employment shares in agricultural and in the non-agricultural sector; and the ratio between the GDP shares in the non-agricultural and agricultural sectors.[1] We can use the US data for the sectoral composition of employment and GDP shown in Table 1 to compute the value of the ratio between the two sectoral LIS that is compatible with the process of structural change in both employment and GDP. The fifth column of Table 1 shows that the value of this ratio should be equal to 2.15 in the year 1880 and it should decrease to 1.05 in the year 2000 in the hypothetical case of equal wages across sectors. These values are problematic as they are completely different from actual estimates of this ratio, which set its value at approximately equal to 0.68.[2]

The previous analysis suggests that the two features of structural change cannot be explained if we assume that wages are equal across sectors. Furthermore, empirical evidence clearly demonstrates that wages are different across sectors, especially when we compare the agricultural and non-agricultural sectors (see Helwege, 1992; Caselli and Coleman, 2001; and Herrendorf and Schoellman, 2018). Table 1 shows the relative wage between agricultural and non-agricultural sectors. According to the table, wages are lower in the agricultural sector and they have clearly exhibited convergence during the last century.[3] However, wage differentials across sectors currently continue to be large. Using this observed data on relative wages, we compute the ratio between the sectoral LIS consistent with the two features of the process of structural change when wages are unequal across sectors. The last column of Table 1 shows this ratio. Note that after 1920 the value of this ratio is close to its empirical estimates and does not exhibit a trend.[4] This numerical analysis suggests that it might be convenient to introduce sectoral differences in wages to spell out the two features of structural change. In Section 2, we discuss in detail the theoretical foundations behind the role of wage gaps in explaining the observed patterns of structural change.

A first contribution of this paper is to show that a simple multisector growth model can illustrate the two aforementioned features of structural change when wages do not equalize across sectors. To this end, we develop an exogenous two-sector growth model with two main features. First, preferences are non-homothetic owing to the introduction of minimum consumption requirements, as in Kongsamut et al. (2001) or

[1] The LIS in sector i is defined as $LIS_i = w_i L_i / P_i Y_i$ where w_i is the wage in sector i, L_i is the number of employed workers in this sector, P_i is the relative price and Y_i is the production in this sector. Using this definition, it is straightforward to obtain that the ratio between the LIS in sectors a and n is $LIS_a/LIS_n = (w_a/w_n)(u_a/u_n)(\kappa_n/\kappa a)$, where u_i and κ_i are the employment and GDP shares in sector i, respectively.

[2] This value is obtained from Valentinyi and Herrendorf (2008) that use data for the US in the period 1990-2000. From using US KLEMS 2013 release, we obtain a ratio of the sectoral LIS between 0.8 and 1 during the period 1947-2010. These values are larger than the ones obtained by Valentinyi and Herrendorf (2008), but they are still substantially lower than the value of the ratio of sectoral LIS compatible with the process of structural change when wages are equal.

[3] The relative wages in Table 1 are obtained from Caselli and Coleman (2001) and they cover the period 1880-2000. Table 2 provides relative wages obtained from US KLEMS release 2013 that cover the period 1947-2010. These relative wage show the same pattern: wages are lower in the agricultural sector and they show convergence across sectors.

[4] Before 1920 data on relative wages are controversial as has been explained by Caselli and Coleman (2001). Therefore, measurement errors in the value of relative wages may explain the low values of the ratio between sectoral LIS before 1920.

Alonso-Carrera and Raurich (2015). Second we introduce differences in wages across sectors by considering that labor mobility is costly. There is a large debate in the literature regarding the source of wage differences between the agricultural and non-agricultural sectors. Literature explains sectoral differences in wages as the result of: (i) differences in human capital across sectors (Caselli and Coleman, 2001; Herrendorf and Schoellman, 2015); (ii) barriers to mobility (Hayashi and Prescott, 2008); (iii) differences in the number of hours worked (Gollin et al., 2014); (iv) differences in the unemployment rate (Hatton and Williamson, 1992); (v) measurement error in agricultural value added data (Herrendorf and Schoellman, 2018); or (vi) labor mobility cost (Lee and Wolpin, 2006; Dennis and İscan, 2007; Raurich et al., 2015). Gollin et al. (2014) show that labor productivity is lower in the agricultural sector even though we control for human capital, the number of hours employed and different measures of sector output constructed from households survey data. This signals that labor mobility cost may explain part of the wage differences.

The introduction of the labor mobility cost segments the labor market into two sector specific labor markets. The existing number of workers in each sector determines the labor supply of the corresponding market. Thus this supply is determined by the sectoral employment share. The labor demand in each sectoral market rests on the demand for consumption goods in every sector that depends on economic development in a model with non-homothetic preferences. In every period, market clearing determines the wages paid in each sector. Therefore, sectoral wage differences exist because the labor mobility cost prevents workers from instantaneously moving to the higher wage sector. However, as the economy develops, the labor mobility cost, as a fraction of the GDP, declines. This triggers wage convergence across sectors.

The process of structural change is thus driven by demand and supply factors. On the one hand, due to the non-homotheticity of preferences, the sectoral composition of consumption expenditures changes as the economy develops. Obviously, this is the classical demand factor explained in Kongsamut et al. (2001). Economic development reduces the effect of the minimum consumption requirement on the sectoral composition and this effect eventually vanishes. As a consequence, preferences are homothetic in the long run, so that the equilibrium converges to a balanced growth path (BGP). On the other hand, the supply factor is primarily based on wage convergence, rather than on the standard mechanism in the literature which is based on changes in the relative prices of goods. In fact, this process of wage convergence gives rise to two supply mechanisms driving structural change. First, wage convergence implies faster-growing wages in the agricultural sector than in the non-agricultural sector. As a consequence, firms in the agricultural sector substitute capital for labor. This makes the production in the agricultural sector more capital intensive and pushes workers out of this sector.[5] Second, the increase in wages, together with an exogenous process of TFP growth, affect relative prices that, through a substitution effect, cause an additional impact in the sectoral composition.

We calibrate the proposed model to explain the process of structural change in the US for the period 1880-2000. From numerical simulations, we show that the model explains: (i) the balanced growth of aggregate magnitudes over time with structural

[5] Cheremukhin et al. (2013) consider a related mechanism based on intersectoral distortions on wages, consumption and production.

change; (ii) the process of structural change in the sectoral composition of employment; (iii) the process of structural change in the sectoral composition of GDP; and (iv) the convergence of wages across sectors. We outline that in the absence of sectoral wage gaps the model is not able to jointly explain the process of structural change in the sectoral composition of GDP and employment.

The model in this paper is similar to the models in Caselli and Coleman (2001) and Dennis and İscan (2007) that also introduce a mechanism of structural change based on labor mobility costs. However, these authors only consider the effect of this mechanism on the path of the employment share. In contrast, we show that this mechanism contributes to explain the path of both the employment share and the value added share. This is important as then the model introduces differences in productivity across sectors, which is obviously a key element to study the effects of structural change on GDP. The second contribution of this paper is to show that models that only explain the time path of the employment share are not able to explain the impact of structural change on GDP, as they do not consider the differences in productivity across sectors. This contribution is illustrated in Section 5, where we display several economies exhibiting the same patterns of structural change in the sectoral composition of employment, large differences in the value added shares and, as a consequence, important differences in GDP levels.

The differences in sectoral wages are a consequence of the technological constraint that the mobility cost imposes on the sectoral allocation of production factors: the sector with larger wages has too large capital intensity. This technological constraint causes a loss of GDP. This loss is not due to inefficiencies arising from barriers as, for instance, in Restuccia et al. (2008). Instead, the GDP loss in this paper must be interpreted as the reduction in GDP with respect to the level that would be attained in the absence of the labor mobility cost. Intuitively, moving a worker from a low to a high wage sector implies moving a worker to a sector with larger productivity, which increases the GDP. Therefore, GDP loss will depend on the wage gap between the two sectors and on the size of the low wage sector (the agricultural sector). Both the wage gap and the size of the low wage sector were large in the US in the XIX century, which implies a large GDP loss. We use numerical simulation to quantify the GDP loss in our calibrated model. It turns out that this cost was about 30% of GDP in the last twenty years of the XIX century, it declined during the transition and eventually vanished. Consequently, part of the increase in the GDP during the transition, especially in the initial periods, is explained by the reduction of the relative importance of mobility cost in terms of GDP.

GDP loss then introduces a mechanism through which cross-country differences in the sectoral composition of employment cause cross-country differences in income per capita. This mechanism indicates that those countries specialized in the sector with the lowest wage (the agricultural sector) will have a lower GDP. This conclusion is also obtained in Gollin et al. (2004, 2007). In these papers, the specialization in the low wage sector is explained by the presence of home production or minimum consumption requirements. By contrast, this paper explains this specialization as the result of a larger labor mobility cost, which can be justified by labor market regulations or larger reallocation expenses.

The rest of the paper is organized as follows. Section 2 shows the relationship between the value added shares and the employment shares. Section 3 introduces the model and Section 4 characterizes the equilibrium. Section 5 solves the model numerically and obtains the main results. Finally, Section 6 includes some concluding remarks.

2 Value added shares vs. employment shares

In this section, we show that the observed patterns of structural change in the US economy cannot be explained if we assume that wages are equal across sectors. To this end, we consider an economy with two sectors producing agricultural and non-agricultural goods. These sectors use the following general constant returns to scale technologies:

$$Y_n = F\left(sk, A_n u\right) = A_n u f\left(z_n\right), \tag{1}$$

and

$$Y_a = G\left(\left(1-s\right)k, A_a\left(1-u\right)\right) = A_a\left(1-u\right) g\left(z_a\right), \tag{2}$$

where Y_a and Y_n are the output of the agricultural and non-agricultural sectors, respectively, s is the fraction of the stock of capital, k, employed in the non-agricultural sector, u is the fraction of workers employed in the non-agricultural sector, A_a and A_n are efficiency units of labor, and $z_a = \left(1-s\right)k/A_a\left(1-u\right)$ and $z_n = sk/A_n u$ measure the capital per efficient unit of labor employed in the agricultural and non-agricultural sectors, respectively.

We assume perfect competition, so that

$$w_n = A_n\left[f\left(z_n\right) - z_n f'\left(z_n\right)\right],$$

and

$$w_a = pA_a\left[g\left(z_a\right) - z_a g'\left(z_a\right)\right],$$

where p is the relative price of agricultural goods in units of non-agricultural goods, and w_a and w_n are the wage in the agricultural and non-agricultural sectors, respectively. Let $\lambda = w_a/w_n$ be the relative wage between the two sectors. From using the expression of the wages, we obtain

$$p = \frac{\lambda A_n\left[f\left(z_n\right) - z_n f'\left(z_n\right)\right]}{A_a\left[g\left(z_a\right) - z_a g'\left(z_a\right)\right]}. \tag{3}$$

Let $Q = Y_n + pY_a$ be the GDP. We use (1), (2) and (3) to rewrite the GDP as

$$Q = A_n f\left(z_n\right)\left[u + \lambda\left(\frac{1-\alpha_n}{1-\alpha_a}\right)\left(1-u\right)\right], \tag{4}$$

where $\alpha_n = z_n f'\left(z_n\right)/f\left(z_n\right)$ and $\alpha_a = z_a g'\left(z_a\right)/g\left(z_a\right)$ are the capital output elasticities, which also correspond to the capital income shares. Finally, from using (1) and (4), we obtain

$$\frac{Y_n}{Q} = \frac{u}{\Omega}. \tag{5}$$

The variable Ω determines the relationship between the GDP share of the non-agricultural sector and the sectoral share of employment. This variable Ω can be rewritten as

$$\Omega = 1 + (1 - u)\left(\frac{\alpha_a - \alpha_n}{1 - \alpha_a}\right) + (1 - u)\left(\frac{1 - \alpha_n}{1 - \alpha_a}\right)(\lambda - 1). \quad (6)$$

Note that if the wages are equal across sectors (i.e., $\lambda = 1$) and there are no technological differences among sectors (i.e., $\alpha_a = \alpha_n$), then $\Omega = 1$. In this case, the relation between the sectoral shares of employment and GDP will be constant and these two shares will, in fact, be equal. However, as follows from Tables 1 and 2, this is not consistent with actual data for the US economy. According to the data, the GDP share is larger than the sectoral employment share in the non-agricultural sector. This implies that the value of Ω should be smaller than one. As follows from the previous expression of Ω, a low value of Ω can be explained by either the wage gap between the agricultural and non-agricultural sectors ($\lambda < 1$) or by sectoral differences in capital output elasticities, which imply sectoral differences in LIS. However, the observed sectoral differences in LIS do not explain the low value of Ω. To illustrate this point, we compute in Table 2 the ratio from employment to GDP shares in US non-agricultural sectors. We use the data from US KLEMS 2013 release because it provides data on sectoral labor income shares in US economy during the period 1947-2010. The forth column shows the ratio from employment to GDP shares in the non-agricultural sectors observed in data. Alternatively, the last two columns give the value of this ratio from simulating Equation (6) by assuming that the sectors exhibit the same labor income share ($\alpha_a = \alpha_n$) and by considering that the wage rates are the same across sectors ($\lambda = 1$), respectively. Observe that the simulated value of Ω by using the observed labor income shares and by considering $\lambda = 1$ is always substantially larger than the ratio from employment to GDP shares observed in data.[6] In contrast, the differences in wages across sectors explain the sectoral differences between value added shares and employment shares. Effectively, the simulated value of Ω by using the observe relative wage rate and by assuming $\alpha_a = \alpha_n$ is almost identical to the actual values of the ratio from employment to GDP shares.

[Insert Table 2]

From the previous analysis, we conclude that wage gaps must be introduced to explain the two dimensions of structural change. An important remark is that this conclusion is quite general as it neither depends on preferences nor on technologies. In the rest of the paper, we use a dynamic general equilibrium model to study how wage gaps affect the patterns of structural change and economic development.

[6] The sectoral LIS of the US economy during the period 1990-2000 are also provided by Valentinyi and Herrendorf (2008). These authors obtain $\alpha_n = 0.33$ and $\alpha_a = 0.54$, which implies a value of the ratio between the LIS in the agricultural sector and the LIS in the non-agricultural sector of 0.69 which is even lower than the values obtained from US KLEMS. Hence, the simulated values of Ω by considering this alternative LISs are even much larger than those in data.

3 Model

We consider an exogenous growth model with the production structure used in the previous section. We assume that the non-agricultural sector is the numeraire and produces a single good that can either be consumed or invested. The agricultural sector produces a good that can only be devoted to consumption.

3.1 Household

The economy is populated by an infinitely lived representative household, formed by a continuum of members distributed on the interval $[0, 1]$. Every member inelastically supplies one unit of time so that the aggregate labor supply is inelastic and equal to unity. The household obtains income from renting capital and labor to firms. This income is devoted to consuming, investing or paying the cost of moving to another sector. Therefore, the budget constraint of the household is

$$rk + w_a (1 - u) + uw_n = pc_a + c_n + \dot{k} + \pi \dot{u}, \quad (7)$$

where r is the rental price of capital, c_a is the consumed units of the good produced in the agricultural sector, c_n is the consumed units of the good produced in the non-agricultural sector, $\pi = \{\pi_1 > 0 \text{ if } \dot{u} \geq 0 \text{ and } \pi_2 < 0 \text{ if } \dot{u} < 0\}$ is the constant unitary labor mobility cost that every worker moving to another sector pays, and $\dot{u}$ is the fraction of workers that move every period.[7]

The labor mobility cost accounts for any cost that workers moving to another sector must pay. This may include reallocation expenses (transport and housing costs), formal training to acquire the skills used in another sector or an opportunity cost (the time spent looking for a job in a different sector). As moving out of the agricultural sector generally entails moving from a rural to an urban area, we consider that the relevant labor mobility cost is associated to reallocation expenses. As the expenses are not proportional to the wage, we assume that the unitary labor mobility cost is constant. Artuc et al. (2015) estimate labor mobility cost for both developed and developing economies. They show that this cost, as a fraction of annual wage, is larger in developing economies. This means that the labor mobility cost as a fraction of GDP declines along the development process. Note that this pattern is consistent with the assumption of a constant unitary labor mobility cost.[8]

The representative household's utility function is

$$U = \int_0^\infty e^{-\rho t} \left[\theta \ln (c_a - \widetilde{c}_a) + (1 - \theta) \ln c_n\right] dt, \quad (8)$$

[7] The sign of the mobility cost must change so that $\pi \dot{u}$ is positive both when $\dot{u} > 0$ and when $\dot{u} < 0$. The latter pattern of structural change implies that workers move to the agricultural sector, which is in fact a pattern that never occurs in our simulations.

[8] To best of our knowledge, there is no evidence on mobility cost for the US economy during the XIX century and the first half of the XX century. Therefore, we use the evidence provided by Artuc et al. (2015), based on cross-country data. This evidence suggests that mobility cost decline along the development process. This finding is consistent with either a constant or declining unitary labor mobility cost. At this point, it is important to clarify that the results obtained in this paper would also hold if we had assumed a non-constant mobility cost.

where $\widetilde{c}_a > 0$ is the minimum consumption requirement of the agricultural good; $\rho > 0$ is the subjective discount rate; and $\theta \in (0,1)$ measures the weight of the agricultural good in the utility function. Note that this utility function is non-homothetic when $\widetilde{c}_a \neq 0$.

The representative household chooses the amount of consumption expenditure, the sectoral composition of consumption expenditure and the number of members that move their supply to the non-agricultural sector every period in order to maximize the utility function (8) subject to the budget constraint (7). By using a standard procedure, we find the first order conditions in Appendix A, and finally we rearrange expressions to obtain the necessary conditions for optimality as follows:

$$v = \theta + \frac{\widetilde{E}}{E}\left(1-\theta\right), \tag{9}$$

$$\frac{\dot{E}}{E} = \left(\frac{E-\widetilde{E}}{E}\right)(r-\rho) + \left(\frac{\widetilde{E}}{E}\right)\frac{\dot{p}}{p}, \tag{10}$$

and

$$w_n - w_a = r\pi, \tag{11}$$

where $E = pc_a + c_n$ is the value of consumption expenditure, $v = pc_a/E$ is the expenditure share in the agricultural good and $\widetilde{E} = p\widetilde{c}_a$ is the value at market prices of the minimum consumption requirement. Equation (9) determines the intratemporal allocation of consumption expenditure across sectors. Note that the expenditure shares would be constant and equal to θ if $\widetilde{c}_a = 0$. In contrast, if $\widetilde{c}_a > 0$, preferences are non-homothetic and the fraction of expenditures devoted to the agricultural good decreases as the economy develops and consumption expenditure increases. This mechanism is the classical demand factor driving structural change. Equation (10) is the Euler condition governing the intertemporal decision between consumption expenditure and savings. Finally, equation (11) is a non-arbitrage condition between two investment decisions: investment in capital goods and investment in moving out of the agricultural sector. The left-hand side is the return from investing π units of numeraire in moving a worker to another sector. The right-hand side is the return from investing these π units in capital. This non-arbitrage condition implicitly determines the number of workers moving out of the agricultural sector in every period and thus determines the relative labor supplies in both sectors.

3.2 Firms

The supply side is as described in Section 2. However, we introduce two simplifying assumptions. First, we will assume that both sectors produce with the following constant returns to scale Cobb-Douglas technologies:

$$Y_a = \left[(1-s)\,k\right]^{\alpha_a}\left[A_a\,(1-u)\right]^{1-\alpha_a} = A_a\,(1-u)\,z_a^{\alpha_a}, \tag{12}$$

and

$$Y_n = (sk)^{\alpha_n}\,(A_n u)^{1-\alpha_n} = A_n u z_n^{\alpha_n}, \tag{13}$$

where the capital output elasticities are now given by the parameters $\alpha_a \in (0,1)$ and $\alpha_n \in (0,1)$.[9] Second, we assume that efficiency units of labor, A_a and A_n, respectively grow at the exogenous growth rates γ_a and γ_n. This implies that technological progress is sectoral biased and the long-run growth rate of GDP is γ_n. From the analysis of Section 2, we know that these two assumptions do not affect the relationship between the employment shares and the value added shares.

Perfect competition implies that each production factor is paid according to its marginal product, so that

$$w_i = A_i p_i (1-\alpha_i) z_i^{\alpha_i}, \tag{14}$$

and

$$r = p_i \alpha_i z_i^{\alpha_i - 1} - \delta, \tag{15}$$

with $i = a, n$; and where $\delta \in [0,1]$ is the depreciation rate. Capital can freely move across sectors, so the marginal product of capital is identical across sectors. By contrast, the introduction of the labor mobility cost implies that wages may be different across sectors. As in Section 2, we define the relative wage between the two sectors by $\lambda = w_a/w_n$. Using (14) and (15), we obtain that

$$z_a = \left(\frac{\lambda \psi A_n}{A_a}\right) z_n, \tag{16}$$

and

$$p = \left(\frac{\alpha_n}{\alpha_a}\right)\left(\frac{\lambda \psi A_n}{A_a}\right)^{1-\alpha_a} z_n^{\alpha_n - \alpha_a}, \tag{17}$$

where

$$\psi = \left(\frac{\alpha_a}{\alpha_n}\right)\left(\frac{1-\alpha_n}{1-\alpha_a}\right).$$

Equations (16) and (17) characterize the two supply-based mechanisms driving structural change in this economy. The first mechanism is illustrated in equation (16) that shows how the relationship between the sectoral capital intensities depends on the relative wage. To be consistent with the evidence, we focus on the case where the relative wage λ is smaller than unity, and it increases as the economy develops. This, in turn, causes an increase in the capital intensity of the agricultural sector relative to the capital intensity of the other sector. The intuition is as follows. An increase in the relative wage implies that wages in the agricultural sector increase relative to wages in the non-agricultural sector. As a consequence, firms in the agricultural sector substitute labor for capital. This describes a new supply mechanism driving structural change. Note that this differs from the supply mechanism usually proposed by the literature, which is based on changes in relative price caused by either biased technological change (Ngai and Pissarides, 2007) or capital deepening jointly with sectoral differences in capital output elasticities (Acemoglu and Guerrieri, 2008).

Note that wage convergence implies that the agricultural sector becomes a more capital intensive sector as the economy develops. This helps to explain cross-country differences in sectoral capital intensities that clearly indicate the agricultural sector is

[9] As shown in Table 2, the sectoral labor income shares are not constant. However, for the sake of simplicity, we consider a Cobb-Douglas technology that implies constant labor income shares.

more relatively capital intensive in developed economies (see Alvarez-Cuadrado et al., 2017). It should be noted here that the aforementioned classical supply mechanisms of structural change would not explain this evidence. Using (16) and the definitions of z_n and z_a, it follows to say that neither sectoral differences in capital-output elasticities nor biased technological change can explain cross-country differences in relative capital intensities when the production function is Cobb-Douglas. To the best of our knowledge, cross-country differences in sectoral capital intensity have only been explained by Alvarez-Cuadrado et al. (2017). Using CES production functions, they claim these differences result from different sectoral elasticities of substitution between capital and employment. This paper therefore offers a complementary explanation based on wage gaps across sectors. Wage convergence contributes to explain differences in sectoral capital intensities even if the production function is Cobb-Douglas.

Secondly, our economy also contains the classical mechanism based on changes in relative prices. This second supply mechanism is illustrated in equation (17). This equation shows that the relative price depends on: (i) the relative wage; (ii) the ratio between the efficiency units of labor in the non-agricultural sector and the efficiency units of labor in the agricultural sector; and (iii) capital deepening. The proposed supply mechanism, based on wage convergence across sectors, implies an increase in the relative price of agriculture. The marginal cost of producing agricultural products increases as the relative wage increases, which explains the increase in the relative price. Yet, sectoral biased technological change and capital deepening may cause a reduction in this price. On the one hand, biased technological change implies an increase in this relative price before 1946 and a decrease after this year, as empirical evidence shows that productivity growth is smaller in the agricultural sector before 1946 and larger after this year.[10] On the other hand, capital deepening implies a reduction in the relative price because the estimates of the sectoral capital output elasticities suggest that this magnitude is larger in the agricultural sector (See Valentinyi and Herrendorf, 2008). As a consequence, this sector is the most benefited from capital deepening, which causes the reduction in the relative price. As in the model we combine three different supply mechanisms, the relative price can either increase or decrease along the development process. Interestingly, this is consistent with the observed differences in the patterns of relative prices along the development process.[11]

4 Equilibrium

The non-agricultural sector produces a commodity that can be devoted to consuming, investing and covering the cost of moving to a different sector. Therefore, the market-

[10] Alvarez-Cuadrado and Poschke (2011) use existing estimates of sectoral productivities in United States to show that productivity growth is almost 1% larger in the non-agricultural sector before 1946 and 0.5% smaller in the period after 1946.

[11] Dennis and Iscan (2009) provide evidence that relative prices of agriculture in the US increase during the XIX century and decrease after 1920. Alvarez-Cuadrado and Poschke (2011) find large disparities in the behavior of agriculture prices across both countries and time. Throughout the period 1920-1959 these prices grow for some countries (e.g., Canada, UK or Japan), whereas they decrease for others (e.g., Belgium, France or Netherlands). During the period 1960-2000 these prices decrease for the whole sample of the aforementioned study.

clearing condition in this sector is

$$Y_n = c_n + \dot{k} + \delta k + \dot{u}\pi.$$

By contrast, the agricultural sector only produces a consumption good so that the market clearing condition in this sector is $c_a = Y_a$, which can be rewritten by using (12) as

$$1 - u = \frac{c_a}{A_a z_a^{\alpha_a}}. \tag{18}$$

Let $z = k/A_n$ be the stock of aggregate capital per efficiency units of labor in the economy. Thus, z measures the capital intensity of the economy. Using the definition of z, we derive that

$$z_n = \left(\frac{s}{u}\right) z, \tag{19}$$

and

$$z_a = \left[\frac{(1-s)\, A_n}{(1-u)\, A_a}\right] z.$$

From the last equation and (16), we get that

$$\lambda\psi\,(1-u)\, z_n = (1-s)\, z. \tag{20}$$

From using the equilibrium condition in the capital market and equations (19) and (20), we obtain

$$\frac{z}{z_n} = \lambda\psi\,(1-u) + u \equiv \phi, \tag{21}$$

where ϕ measures the capital intensity of the economy relative to the capital intensity of the non-agricultural sector.

Note that wage differentials between sectors would not emerge without mobility cost, so that $\lambda = 1$ and $\phi = \psi\,(1-u) + u \equiv \widetilde{\phi}$ in this case. However, the labor mobility cost implies that during the transition $\lambda < 1$ and, therefore, $\phi < \widetilde{\phi}$. The introduction of the labor mobility cost, by increasing the wages of the non-agricultural sector, increases the capital intensity of this sector relative to the capital intensity of the whole economy. Thus, the labor mobility cost introduces a technological constraint to the sectoral allocation of production factors. This constraint is measured by the gap between ϕ and $\widetilde{\phi}$, which is given by

$$\widetilde{\phi} - \phi = (1-u)\,\psi\,(1-\lambda)\,.$$

The aforementioned technological constraint will cause a GDP loss. Using (16), (17) and (21), GDP, as is defined in Section 2, can be rewritten as

$$Q = A_n^{1-\alpha_n}\Omega\phi^{-\alpha_n}k^{\alpha_n}, \tag{22}$$

where

$$\Omega = \left(\frac{\alpha_n}{\alpha_a}\right)\phi + u\left(\frac{\alpha_a - \alpha_n}{\alpha_a}\right), \tag{23}$$

and $\Phi = \Omega\phi^{-\alpha_n}$ measures the sectoral composition component of the total factor productivity (TFP), which is given by $A_n^{1-\alpha_n}\Omega\phi^{-\alpha_n}$. From using (21), it is immediate to show that Ω in (6) coincides with the expression of Ω in (23). Therefore, the variable Ω determines both the sectoral composition component of the total factor productivity and, as shown in (5), the relationship between the GDP share of the agricultural sector and the sectoral share of employment.

By defining $\widetilde{Q}$ as the GDP level that would be attained if $\lambda = 1$ (i.e., if $\pi = 0$), we measure the GDP loss as a fraction of GDP by

$$\frac{\widetilde{Q} - Q}{Q} = \left(\frac{\widetilde{\Omega}}{\Omega}\right)\left(\frac{\widetilde{\phi}}{\phi}\right)^{-\alpha_n} - 1, \tag{24}$$

where $\widetilde{\Omega}$ is the value of Ω when $\lambda = 1$. Note that the loss of GDP depends on λ and on the employment share in agriculture $1 - u$. In the numerical simulations of Section 5, we show that the GDP loss has declined in the US during the last century as a result of wage convergence and the fall of the employment share in the agricultural sector.

An important remark that follows from the expression of the TFP is that differences in the sectoral composition of employment cause differences in the TFP when there are either differences in capital output elasticities or differences in wages across sectors.[12] If we had assumed both $\alpha_a = \alpha_n$ and $\lambda = 1$, then disparities in the sectoral composition would not have implied differences in TFP levels since $\Omega\phi^{-\alpha_n} = 1$ in this case. In other words, TFP increases when economies specialize in sectors with larger capital output elasticities or in sectors with larger wages. In the numerical analysis performed in the next section, we will compare economies with different sectoral compositions and we will decompose the fraction of income differences explained by differences in sectoral wages and the fraction explained by differences in capital output elasticities. From this numerical analysis, we will show that the main mechanism explaining income differences through TFP is based on differences in sectoral wages.

4.1 Sectoral Composition

In this subsection, we obtain the sectoral composition of consumption expenditures, the sectoral employment shares and the relative wage, λ, as a function of: the expenditure to GDP ratio, $e = E/Q$; the capital intensity, $z = k/A_n$; the intensity of the minimum consumption requirement, measured by the ratio $\widetilde{e} = \widetilde{E}/Q$; and the intensity of the labor mobility cost, measured by $m = \pi/A_n$. Note that as the economy develops, the intensity of the minimum consumption requirement and of the labor mobility cost both decline and eventually converge to zero.

We first use (9) and the definitions of e and $\widetilde{e}$ to directly obtain the sectoral share of expenditure as

$$v = \theta + \frac{\widetilde{e}}{e}(1 - \theta). \tag{25}$$

The sectoral composition of expenditures determines the sectoral composition of GDP. In fact, from manipulating the market clearing condition in agriculture it can be shown that the GDP share in agriculture equals ve. In contrast, the sectoral share

[12] Observe that TFP is endogenously determined when either $\alpha_a \neq \alpha_n$ or $\lambda < 1$. As a consequence, in our economy aggregate output cannot be represented by a Cobb-Douglas production function that uses capital and labor as inputs.

of employment u and the relative wage λ are jointly determined by the market clearing conditions for the agricultural sector, which is given by (18), and for the labor market. Observe that the labor supply is determined by the non-arbitrage condition (11), whereas the labor demand is given by the condition (14). From the manipulation of these two market-clearing conditions, we derive in Appendix B the following result characterizing the equilibrium value of the relative wage and the sectoral share of employment.

Proposition 1 *The relative wage is determined by the function*

$$\lambda = \widehat{\lambda}\left(e, z, m\right), \tag{26}$$

and the sectoral share of employment satisfies

$$u = \frac{\lambda \psi \left(\frac{\alpha_n}{\alpha_a}\right)(1 - ve)}{ve + \lambda \psi \left(\frac{\alpha_n}{\alpha_a}\right)(1 - ve)}. \tag{27}$$

Furthermore, $\partial \lambda / \partial m < 0$.

Equation (27) confirms that structural change in the sectoral composition of employment is driven by demand factors, measured by ve, and supply factors, measured by λ. In particular, as follows from Proposition 1, the relative wage λ is a decreasing function of the intensity of the labor mobility cost m, and, as follows from (27), the employment share u is an increasing function of λ. The latter relationship is obviously explained by the reduction in the demand for workers from the agricultural sector due to the increase in the relative wage. Therefore, a large mobility cost implies that the relative wage will be smaller and, thus, the employment share of agriculture will be larger. Both effects imply that the GDP loss increases with labor mobility cost.

4.2 Equilibrium Dynamics

Given initial conditions $\widetilde{e}_0$, m_0 and z_0, an equilibrium is a path of $\{e, \widetilde{e}, z, m, \lambda, v, u, \phi\}$ that solves the consumers' optimization conditions, the firms optimization conditions, the market clearing conditions and the transversality condition $\lim_{t \to \infty} \frac{k}{c_n} e^{-\rho t} = 0$. Furthermore, we define a balanced growth path (BGP) as an equilibrium along which both the ratio of capital to GDP and the interest rate remain constant. We obtain in Appendix C the full system of differential equations characterizing the equilibrium path of the transformed variables z, e, m and $\widetilde{e}$. By using Appendix D, the next result characterizes the existence and local stability of the BGP

Proposition 2 *There is a unique BGP along which the variables* $\{e, \widetilde{e}, z, m, \lambda, v, u, \phi\}$ *remain constant and their long-run values are* $\widetilde{e}^* = 0$, $m^* = 0$, $\lambda^* = 1$, $v^* = \theta$,

$$e^* = \frac{1 - \alpha_n \Delta}{1 + \Delta \left(\alpha_a - \alpha_n\right)\theta},$$

$$u^* = \frac{\psi \alpha_n \left(1 - \theta e^*\right)}{\alpha_a \theta e^* + \psi \alpha_n \left(1 - \theta e^*\right)},$$

$$\phi^* = \psi(1 - u^*) + u^*,$$

and

$$z^* = \left(\frac{\gamma_n + \delta + \rho}{\alpha_n}\right)^{\frac{1}{\alpha_n - 1}} \phi^*,$$

where $\Delta = (\delta + \gamma_n) / (\delta + \rho + \gamma_n)$. *Furthermore, this BGP is saddle-path stable.*

Note that the BGP is attained asymptotically, as $\widetilde{e}$ and m converge to zero. Labor mobility cost vanishes and, therefore, wages converge across sectors and the GDP loss disappears as the economy approaches the BGP. Moreover, there is no structural change along the BGP. Thus, the economy asymptotically converges to an equilibrium along which the interest rate and the ratio of capital to GDP remain constant and there is no structural change. As this only happens asymptotically, it is particularly significant to analyze the transitional dynamics. In the following section, we numerically analyze the transition and we demonstrate that aggregate variables exhibit a period of unbalanced growth followed by a long period in which they exhibit an almost constant time path of the interest rate and the ratio of capital to GDP. We also show that there is structural change over this period. We then conclude that (an almost) balanced growth of aggregate variables and structural change can simultaneously be observed in this economy.

The equilibrium is characterized by three state variables: capital intensity, z, intensity of minimum consumption requirements, $\widetilde{e}$, and intensity of the labor mobility cost, m. Saddle-path stability implies that given initial conditions on these three state variables, a unique equilibrium path converges to the BGP. In the following section, the uniqueness of the equilibrium path is used to calibrate and simulate the economy.

5 Transitional Dynamic Analysis: Structural Change

In this section we numerically simulate the economy in order to show how mobility cost affects the process of structural change. To this end, we first calibrate the parameters of the economy as follows. We define a period as a year to fit our model with data and we set: (i) the initial value of the sectoral efficiency unit of labor in the non-agricultural sector as $A_n(0) = 1$ because this parameter only affects the units of measurement of commodities Y_a and Y_n; (ii) $A_a(0) = 1.37$ to obtain the relative sectoral productivity in 1880; (iii) $\gamma_n = 0.02$ to obtain a long-run GDP growth rate equal to 2%, which is in the range used by the literature and corresponds with the growth rate of US GDP per capita between 1960 and 2000; (iv) $\gamma_a = 0.09$ before 1946 and $\gamma_a = 0.0273$ after 1946 to match the evolution of relative sectoral productivities in the period 1880-2000;[13] (v) $\alpha_a = 0.54$ and $\alpha_n = 0.33$ as estimated by Valentinyi and Herrendorf (2008); and (vi) $\theta = 0.01$ to fit the long-run expenditure share in agriculture obtained in Herrendorf

[13] To calibrate the efficiency parameters, $A_a(0)$, $A_n(0)$, and γ_a, we cannot use data on value added prices, as it is available only for the period 1947-2000. We instead use direct estimates of sectoral TFP that for the US economy are available for the whole period. Alvarez-Cuadrado and Poschke (2011) report the ratio between non-agriculture to agriculture productivities. They show that this ratio equals 0.73 in 1880, it increases until 1.49 in 1945 and then it declines until 1 in 2000. Our calibration matches these facts.

et al. (2013).[14] The parameters ρ and δ are respectively set to 0.032 and 0.056 by imposing the BGP to satisfy: (i) the interest rate equals to 5.2%; and (ii) the ratio of investment to capital is 7.6% (see, e.g., Cooley and Prescott, 1995). Table 3 reports the targets and the implied parameter values.

Secondly, we assume in all of the simulations $z_0 = 0.75z^*$. The initial value of this state variable mainly determines the length of the transition of aggregate variables. We choose an initial value that is consistent with an almost constant time path of both the interest rate and the ratio of capital to GDP over the last 50 years of the simulation.[15] Finally, we simulate two benchmark models (labeled *Model 1* and *Model 2*) that differ according to whether the labor mobility across sectors is a costless activity. In the simulation of Model 1 we assume that there is no mobility cost (i.e., $\pi = 0$ and, therefore, $m_0 = 0$) and we set the initial condition on the other state variable, $\widetilde{e}_0 = 0.588$, to match the employment share in the US in the initial year 1880. In the simulation of Model 2, we assume that there is labor mobility cost (i.e., $\pi \neq 0$) and we set the initial conditions on the two state variables, $\widetilde{e}_0 = 0.279$ and $m_0 = 13.27$, to match the values of the shares of employment and GDP in the US agricultural sector in the year 1880.[16] Note that, given z_0, when we set $\widetilde{e}_0$ and m_0, we are implicitly setting the values of π and $\widetilde{c}_a$. Table 4 summarizes the initial conditions of the two simulations.

[Insert Tables 3 and 4]

Figure 1 shows the first numerical simulation in which we assume that there is no mobility cost. In this case, wages equalize across sectors. This implies that the relative wage is equal to one and, thus, Model 1 does not explain wage convergence. This means that there is not a technological constraint to the sectoral reallocation of inputs and thus there is no GDP loss. Panel (i) shows that this simulation reproduces practically the entire decline of the employment share in the agricultural sector. However, the model does not provide a reasonable explanation for the process of structural change in the sectoral composition of GDP. In order to measure this different performance of the model, we use the relocation index introduced by Swiecki (2017). The relocation index of a variable x measures the fraction of the change in a variable explained by the model. This measure is defined as

$$RI = 1 - \frac{\left|\Delta x^m - \Delta x^d\right|}{\left|\Delta x^d\right|}, \tag{28}$$

where $\Delta x^m = \left[x^m\left(t_f\right) - x^m\left(t_i\right)\right] / \left(t_f - t_i\right)$ measures the average annual change of the variable x between time periods t_i and t_f according to the simulation and Δx^d is the

[14] Valentinyi and Herrendorf (2008) obtain the labor income shares using data for the US in the period 1990-2000. Obviously, the labor income shares have been very different during the period considered in this paper. However, as shown in Section 2, the introduction of large wage gaps is necessary to explain the two patterns of structural change even if we assume other values of the labor income shares. We can then safely conclude that the main insights that we obtain from the numerical analysis would hold if we had considered other values of the labor income shares. For simplicity, we consider the values of the labor income shares provided by Valentinyi and Herrendorf (2008).

[15] In any case, the main results of our numerical analysis still hold under different initial values of z.

[16] An alternative target of calibration in Model 2 could have been the relative wage in the initial year 1880. However, as explained by Caselli and Coleman (2001) and also Dennis and İscan (2007), data on relative wages is problematic before 1920.

corresponding change in the variable according to the data. If $RI = 1$ the model explains the entire change in the variable and if $RI = 0$ the model does not explain the change in the variable. In Model 1, the value of this index is 0.9494 when the variable considered is the employment share and it is only 0.0635 when the variable considered is the sectoral composition of GDP. This analysis clearly shows that the model fails to explain structural change in the sectoral composition of GDP.[17]

This model without mobility cost overestimates the share of agricultural output in GDP along the whole transition. To provide intuition of this result, we can rewrite (5) as $\Omega = u\,(Q/Y_n)$. From Table 2, we observe that $u < Y_n/Q$ in actual data implying that Ω should be substantially lower than one in order to explain the two dimensions of structural change as described in Section 2. However, since $\alpha_a > \alpha_n$ in our benchmark calibration, then Ω is larger than one when $\lambda = 1$ as follows from (6). Therefore, the model fails to explain simultaneously the two dimensions of structural change in the absence of mobility cost.

[Insert Figure 1]

Figure 2 displays the second simulation, where a mobility cost is introduced. This cost, as a fraction of GDP, declines from 8% of GDP in the initial year to zero in the long-run. This ratio declines because GDP increases and the process of sectoral structural change declines in the long-run. The cost implied by the calibration of Model 2 is consistent with the labor mobility costs estimated in Artuc et al. (2015). These authors estimate a labor mobility cost for the US during the period 1986-2007 of 2.21 times the annual wage. In a similar time period, 1980-2000, the simulation of Model 2 implies an average mobility cost of 2.16 times the annual wage, which is extremely close to the estimates obtained by Artuc et al. (2015). Using data from both developed and developing countries, these authors also show that the mobility cost substantially increases as GDP decreases.[18] Model 2 is also consistent with this pattern.

The simulation of Model 2 replicates the declining path of the employment share in the agricultural sector, the declining path of the share of GDP produced in the agricultural sector and the process of wage convergence. In particular, this simulation explains practically the entire decline in employment and GDP shares of the agricultural sector. In Model 2, the relocation index RI given by (28) equals 0.9678 when the variable considered is the employment share and it is equal to 0.8887 when the variable considered is the sectoral composition of GDP. Regarding wage convergence, the simulation depicts the convergence of the relative wage, as the model almost matches

[17] We could have calibrated Model 1 to match the initial GDP share. In this case, the model would explain structural change in the sectoral composition of GDP, but it would fail to explain structural change in the sectoral composition of employment. Thus, Model 1 cannot simultaneously explain both processes of structural change.

[18] Artuc, et al. (2015) show that the average mobility cost in developed economies is 2.76 times the annual wage, whereas the mobility cost in developing countries is substantially larger, 3.71 times the annual wage. These mobility costs are in line with those obtained by Dix-Carneiro (2014) in the Brazilian economy. This author estimates a mobility cost of 2.15 times the annual wage when a worker moves to the non-tradeable sector, of 1.5 when he moves to the low-tech manufacturing sector and of 3.25 when he moves to a high-tech manufacturing sector.

the annual growth rate of wages in the data.[19] However, it is not able to explain the level of the relative wage, as it is obvious from Panel (ii) in Figure 2. We interpret this as partial evidence of other relevant explanations concerning the sectoral wage differences apart from the mobility cost.[20] However, it must be outline that the patterns of structural change in the sectoral composition of GDP depend on the growth of the relative wage and not in the level of this variable. This explains that the performance of this simulation in explaining the process of structural change in the sectoral composition of GDP is decidedly better than the previous simulation of Model 1.

[Insert Figure 2]

Table 5 provides three measures of performance for comparing the simulations of Models 1 and 2. Following these measures, both simulations are equally accurate in explaining the process of structural change in the sectoral composition of employment. For instance, the coefficient of determination is 87% in Model 1 and 88% in Model 2. Thus, the performance is very similar and only slightly better in Model 2. A comparable conclusion is attained if we compute the fraction of the reduction in the employment share of the agricultural sector over the period 1880-2000 explained by both simulations. Model 1 explains 95% of the reduction, whereas Model 2 explains 97%. The conclusion is completely different when we consider the performance of both simulations in explaining the process of structural change in the sectoral composition of GDP. Model 1, based on the absence of labor mobility cost, performs poorly. In the simulation of this Model, the coefficient of determination is negative and the reduction in the GDP share over the period 1880-2000 almost doubles the reduction in actual data. However, Model 2 based on the introduction of the labor mobility cost performs very well. The coefficient of determination is 85.5% and the fraction of the reduction explained by this simulation is 89%. We can then safely conclude that the model with mobility cost explains the process of structural change substantially better.

[Insert Table 5]

As depicted in the previous section, the mobility cost introduces a technological constraint to the sectoral reallocation of production factors that causes a loss of GDP. Panel (iv) in Figure 2 provides a measure of this loss as a percentage of GDP, based on the definition of GDP loss in equation (24). This loss initially amounts to 35% of GDP, and it declines and converges to zero as the sectoral wage differences vanish and the labor share in the agricultural sector declines. Therefore, the elimination of this technological constraint to the sectoral mobility of labor explains part of the increase in GDP during the transition.

[19] The average annual growth rate of the relative wage in the data is 1.04% and the implied growth rate of the relative wage in the model is 0.98%.

[20] Candidates for explaining low relative wages are, among others, metapreferences associated to working in one sector or different skills across sectors. Other authors argue that wage differences can be explained by differences in the cost of living between urban and rural areas (see Esteban-Pretel and Sawada, 2014). If we assume that workers in urban areas are employed in the non-agriculture sector while workers in rural areas can be employed in the agriculture sector, permanent sectoral wage differences intend compensate for the differences in the cost of living.

The labor mobility cost and the subsistence consumption also modify the time path of the growth rate of GDP. Following Panel (vi) in Figure 2 we may see that the time path of the growth rate is hump-shaped. Interestingly, this finding is consistent with the observed development patterns.[21] Christiano (1989) and, more recently, Steger (2000, 2001) explain this hump-shaped pattern in models with minimum consumption requirements.[22] In these models, a sufficiently intensive minimum consumption requirement initially deters investment, which explains the initial low growth. As the economy develops, the intensity of the minimum consumption requirement declines and investment and growth both initially increase. Eventually, the interest rate goes down due to diminishing returns to capital and, therefore, capital accumulation and the growth rate decline until they converge to its long-run value. This mechanism is present in our paper. However, we add another mechanism based on the interaction between capital accumulation and labor mobility that can also explain the hump-shaped growth pattern. In this model, a large intensity of the labor mobility cost explains the initial low labor mobility in addition to a low initial capital accumulation. As this intensity declines, capital accumulation increases and the GDP loss declines because of the increase in the number of workers leaving the agricultural sector. These two changes point to an increase in the growth rate of GDP. Finally, diminishing returns to capital and labor imply that capital accumulation and labor mobility decline. This explains the final reduction in the growth rate of GDP. Note that both mechanisms (i.e., minimum consumption requirements and labor mobility cost) introduce complementary explanations for the hump-shaped time path of the GDP growth rate. Interestingly, the calibrated economy represented by Model 1, in which labor mobility cost is absent, cannot explain the hump-shaped time path of the GDP growth rate even when the minimum consumption is strictly positive. This stresses the relevance of the complementarity between the two mechanisms in determining the time path of the GDP growth rate.

An important stylized fact of the patterns of development in the US economy since the second half of the last century is the balanced growth of the aggregate variables. Over this period, the interest rate and the ratio of capital to GDP remained almost constant, while the sectoral composition of employment and GDP changed. To illustrate that our simulations are consistent with this pattern, we follow Acemoglu and Guerrieri (2008) and Alonso-Carrera and Raurich (2015) and we compute the average annual growth rate of: the capital to GDP ratio, the interest rate, the employment share in the agricultural sector, and the agricultural share in GDP over the last 50 years of the simulations. Results are displayed in Table 6. According to this table, the annual growth rates of the interest rate and the capital to GDP ratio are both almost null in both simulations. This is consistent with the balanced growth of the aggregate variables observed in the data. Moreover, the annual growth rates of the employment share and the GDP share are close to 2% and consistent with actual data. Thus, the calibrated

[21] Papageorgiou and Perez-Sebastian (2005) illustrate that some fast growing economies exhibit a hump-shaped transition of the GDP growth rate. Using data from the Bureau of Economic Analysis, it can be shown that the growth rate in the US exhibits a hump-shaped pattern. The pick is in the period 1930-1950, much later than in our simulated model.

[22] Ngai (2004) also shows that the model of Hansen and Prescott (2002) can generate a hump-shaped pattern of the growth rate.

model is consistent with balanced growth and structural change.

[Insert Table 6]

Figure 3 shows the simulated time path of the employment share in agricultural when demand factors and the different supply factors considered in this paper drive structural change (dashed line) and when wage convergence does not drive structural change (continuous line).[23] The former employment share is directly obtained from simulation of Model 2, whereas the latter is obtained from an alternative simulation of Model 2 in which the relative wage does not increase (i.e., in this simulation we maintain the initial value of relative wage corresponding to Model 2 constant along the equilibrium path). If we compare the two cases, it seems reasonable to say that demand factors explain most of the reduction in the employment share over the period 1880-2000. In fact, wage convergence only explains 14.5% of the reduction in the employment share over the whole period, while wage convergence explains a much larger fraction of the reduction during the first part of the transition. As an example, wage convergence explains 50% of the fall in that share over the period 1880-1920.

[Insert Figure 3]

5.1 Sensitivity Analysis

This subsection intends to shed light on our understanding concerning the dynamic effects of the minimum consumption requirement and the labor mobility cost. To this end, we consider three comparative dynamic exercises in which we modify the value of the parameters in the calibrated economy labeled Model 2 (see Tables 3 and 4).

The first exercise is displayed in Figure 4. This figure shows the effects of changing the initial intensity of the minimum consumption requirement by comparing three economies which differ only in the initial value of $\widetilde{e}_0$. The continuous line shows the calibrated economy named Model 2. In this benchmark economy, $\widetilde{e}_0 = 0.279$. The dashed line is an economy with a lower value for the initial intensity of the minimum consumption requirement, $\widetilde{e}_0 = 0.15$, and the dotted line is an economy with zero initial intensity, $\widetilde{e}_0 = 0$. As follows from Panels (i) and (iii) of Figure 4, a larger minimum consumption requirement implies that the employment share and the GDP share of the agricultural sector are both larger. The larger demand of labor in the agricultural sector causes an initially larger relative wage, as shown in Panel (ii). However, wage convergence is slower in this economy. This happens because a larger initial intensity of the minimum consumption requirement reduces the willingness of agents to substitute consumption intertemporally. Thus agents in these economies are less willing to reduce current consumption and invest either in capital or in moving to a different sector. As a consequence, the reduction in the employment share of the agricultural sector is at a lower rate. Obviously, this explains slower wage convergence.

[23] We have shown before that the GDP share in agriculture equals ve and, hence, the relative wage does not affect directly the GDP share. Thus, supply factors do not directly drive structural change in the GDP share when preferences are given by (8). It follows that the decomposition performed in Figure 3 cannot be done with the GDP share.

Figure 5 shows the effects of changing the labor mobility cost by comparing three economies with different unitary mobility costs π. The continuous line displays the benchmark economy labeled Model 2. The dashed line displays an economy with a labor mobility cost that is 25% smaller than that of the benchmark economy, whereas the dotted line displays an economy with a mobility cost that is 75% smaller than that of the benchmark economy. From the comparison between these economies, it then follows to argue that a lower mobility cost causes: a lower amount of workers in the agricultural sector; a larger relative wage; a smaller GDP loss; and a lower mobility cost as a percentage of GDP. Note also that the GDP share almost does not change when the mobility cost changes. This occurs because this share is determined by the sectoral composition of consumption expenditures and, hence, the effect of the mobility cost is indirect. Finally, the hump-shaped pattern of the growth rate shown in Panel (vi) disappears when the mobility cost is initially small. Similarly, Figure 4 shows that this also occurs when the intensity of the subsistence consumption is sufficiently small. This clearly illustrates that the hump-shaped growth pattern in Model 2 occurs as a consequence of both the labor mobility cost and the subsistence consumption.

[Insert Figures 4, 5 and 6]

Figure 6 compares three economies that are distinct in terms of the initial intensity in both labor mobility cost and minimum consumption requirement, whereas they initially exhibit identical sectoral composition of employment. The continuous line displays the benchmark economy labeled Model 2. The dotted line displays an economy without labor mobility cost, whereas the dashed line displays an intermediate situation with a positive but small labor mobility cost. In these economies, the initial intensity of the minimum consumption requirement has been calibrated so that the three economies have the same initial employment share. In fact, they exhibit a similar time path of the employment share in agriculture, as can be seen in Panel (i). However, the transitional dynamics of the other variables differ significantly given that these economies have a different labor mobility cost. A larger mobility cost implies a smaller relative wage and, therefore, a larger GDP loss. Note that economies revealing a similar process of structural change in employment will exhibit different levels of GDP due to the differences in the GDP loss generated by the technological constraint to sectoral mobility of labor. The constrained allocation of production factors can be observed from the dynamic comparison between the employment share and the GDP share of agricultural sector. Those economies with a larger GDP loss are economies with a lower GDP share in agriculture. In these economies, workers employed in the agricultural sector are much more unproductive according to the comparison between the employment share and the GDP share. This explains the lower level of GDP. We conclude from this analysis that understanding the effects of sectoral composition on GDP requires a prior explanation of the sectoral composition of employment and GDP through multisector growth models. Clearly, multisector growth models explaining only the time path of the employment share do not suffice to analyze the effects of structural change on GDP, as they neglect the differences in productivity across sectors.

5.2 Implications for Development

The conclusions in the previous subsection indicate that structural change derived from the presence of labor mobility cost may be an important mechanism driving the observed differences in GDP levels across countries. The purpose of this subsection is to analyze how differences in the level of technology generate differences in sectoral structure that result in differences in the level of GDP.[24] Equation (22) illustrates that GDP decomposes in: (a) the direct contribution of technology factor, $A_n^{1-\alpha_n}$; (b) the contribution of sectoral composition, $\Phi = \Omega\phi^{-\alpha_n}$; and (c) the contribution of productive factors, k^{α_n}. Changes in the level of technology propagate to the level of GDP by means of these three channels because these changes alter the capital accumulation and the sectoral structure. We are especially interested in quantifying the relative importance of the structural change as a propagation mechanism. As mentioned in this paper, sectoral composition affects GDP through two different mechanisms: technological constraint to sectoral mobility of labor and sectoral differences in capital output elasticities. According to the technological constraint, a larger employment share in the agricultural sector reduces GDP per capita, given that this sector has a lower wage. In contrast, according to the second mechanism, a larger employment share in agriculture increases GDP per capita, as capital output elasticity is larger in the agricultural sector.[25]

Figures 7 and 8 compare two economies, say Rich and Poor, that differ only in terms of their initial level of technologies $A_a(0)$ and $A_n(0)$. The poor economy is the benchmark economy labeled Model 2 (see Tables 3 and 4), whereas the rich economy is built considering the values of $A_a(0)$ and $A_n(0)$ as twice the size of the respective levels in the benchmark economy. Figure 7 compares these two economies by displaying the time path of several variables. In consonance with Panel (i), the poor economy devotes a larger fraction of employment to the agricultural sector. Due to this larger labor demand in the agricultural sector, the relative wage is initially larger in the poor economy. In the more advanced technological economy, labor mobility is larger because it is a richer economy. This implies that the labor mobility cost is initially larger in the rich economy and the reduction of the employment share in the agricultural sector is faster. As a consequence, the relative wage converges more rapidly in the rich economy, which implies that the relative wage will eventually become larger in the rich economy. As may be seen in Panel (iv), the GDP loss is initially much the same for both economies. This happens because the initially larger relative wage compensates the effect of a larger employment share on the GDP loss in the poor economy. Nonetheless, differences in the GDP loss increase during the transition because the rich economy experiments a faster reduction in the GDP loss. This is driven by the faster reduction in employment share and the faster wage convergence. Finally, the differences in the time path of the GDP loss explain the differences in GDP growth rates. This once again stresses the importance of the technological constraint to labor mobility across sectors in explaining GDP growth patterns.

[Insert Figures 7 and 8]

[24] As mentioned in the previous subsection, our model also explains different levels of development as the result of different minimum consumption requirements or different labor mobility costs.

[25] In Appendix E, we obtain the relative contribution of each mechanism in explaining cross-country differences in levels of GDP.

Figure 8 shows the differences in terms of GDP levels. Panel (i) displays the ratio of GDP between the rich and the poor economy. The initial GDP differences are explained only by technological differences. In effect, as is shown in Panel (ii), the direct contribution of technology in explaining GDP differences is initially 100%. During the transition, a period of divergence is followed by a period of convergence in the levels of GDP. The impact of technological differences on both capital accumulation and sectoral composition explains this transition. On the one hand, the larger technological level entails a faster capital accumulation in the rich economy, which in turn drives a permanent divergence in GDP levels. As shown in Panel (iii), the contribution of capital permanently increases. On the other hand, the larger technological level also involves a faster structural change in the rich economy. This faster structural change drives an initial period of divergence that is followed by a period of convergence. The hump-shaped time path of the contribution of sectoral composition in Panel (iv) explains this. This hump-shaped contribution is obviously explained by the fact that both economies eventually converge into the same sectoral composition. Thus, technological differences only have temporary effects on the sectoral composition (see Panel (i) in Figure 7). Note that the contribution of sectoral composition is sizeable. It explains up to 15% of the GDP differences between the two economies. Moreover, this contribution explains the period of convergence between the two economies. In fact, in the absence of the effect of sectoral composition, there would not be a period of convergence in the levels of GDP. As previously mentioned, the contribution of sectoral composition on GDP differences is governed by two different mechanisms: a technological constraint to sectoral mobility of labor and sectoral differences in capital output elasticities. Panel (v) shows that the channel based on the technological constraint to labor mobility explains slightly more than 100% of the contribution of sectoral composition. This means that the other mechanism slightly reduces the contribution of sectoral composition. This is because capital output elasticity is larger in the agricultural sector and it reduces the GDP gap between the two economies, given that the poor economy specializes in the agricultural sector.

6 Concluding Remarks

We have developed a two-sector growth model in which structural change is driven by both demand and supply factors. The demand factor is an income effect generated by non-homothetic preferences. The supply factor is a substitution effect jointly generated by the change in relative wage between the two sectors and a process of sectoral biased technological change. In order to calibrate the economy, we have identified the two sectors as the agricultural and non-agricultural sectors. We have shown that this model can explain the following patterns of development: (i) balanced growth of the aggregate variables; (ii) structural change in the sectoral composition of employment; (iii) structural change in the sectoral composition of GDP; and (iv) wage convergence across sectors. We have also illustrated that in the absence of sectoral wage gaps the model fails to jointly explain structural change in the sectoral composition of both employment and GDP. We have then concluded that any model of structural change should also include a theory of sectoral differentials in wages.

As sectoral mobility of production factors is constrained, wages are not equal across sectors: the agricultural sector has smaller wages and lower capital intensity, whereas the non-agricultural sector has larger wages and larger capital intensity. Obviously, this technological constraint causes a loss of GDP. We measure this loss and obtain that it initially amounts to over 30% of the GDP. During the transition, the loss declines and finally vanishes. Therefore, the elimination of the constraint to sectoral mobility of labor explains part of the GDP growth, especially throughout the initial years of the transition.

GDP loss introduces a relevant insight on cross-country income differences: part of these differences can be explained by differences in the sectoral composition of employment when wages are different across sectors. In this paper, wage differences are explained by an exogenous labor mobility cost. Future research should try to contribute to a better understanding of determinants of wage differences across sectors. Among others, this could include the study of labor market regulations, fiscal policy, or geographical characteristics.

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Appendix

A Solution of the Representative Household Problem

The representative consumer maximizes the utility function (8) subject to the budget constraint (7). The Hamiltonian function is

$$\begin{aligned} H &= \theta \ln (c_a - \widetilde{c}_a) + (1-\theta) \ln c_n + \\ & \quad \eta \{rk + [w_a (1-u) + uw_n] - pc_a - c_n - \varphi\pi\} + \mu\varphi \end{aligned}$$

where $\varphi = \dot{u}$. The first order conditions with respect to c_a, c_n, φ, k and u are, respectively,

$$\frac{\theta}{c_a - \widetilde{c}_a} = \eta p, \tag{A.1}$$

$$\frac{1-\theta}{c_n} = \eta, \tag{A.2}$$

$$\mu = \eta\pi, \tag{A.3}$$

$$r - \rho = -\frac{\dot{\eta}}{\eta}, \tag{A.4}$$

$$\frac{\eta}{\mu}(w_a - w_n) = \frac{\dot{\mu}}{\mu} - \rho. \tag{A.5}$$

From combining (A.1) and (A.2), we obtain (9) in the main text. We log-differentiate (A.1) to obtain

$$\frac{\dot{c}_a}{c_a - \widetilde{c}_a} + \frac{\dot{p}}{p} = -\frac{\dot{\eta}}{\eta} = \frac{\dot{c}_n}{c_n}$$

and from (A.3) we obtain

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{\eta}}{\eta}.$$

Using these two equations, we rewrite (A.4) as

$$r - \rho = \frac{\dot{c}_a}{c_a - \widetilde{c}_a} + \frac{\dot{p}}{p}, \tag{A.6}$$

and (A.5) as

$$\frac{w_a - w_n}{\pi} = \frac{\dot{\eta}}{\eta} - \rho. \tag{A.7}$$

Using the definition of E, (A.6) can be rewritten as (10) in the main text. Finally, (A.7) can be rewritten as (11) in the main text.

B Proof of Proposition 1

The employment shares are obtained from combining (18), (16), (17) and (25) as follows

$$1 - u = v\left(\frac{\alpha_a}{\alpha_n}\right)\left(\frac{E}{\lambda\psi A_n z_n^{\alpha_n}}\right).$$

Using (21) and (22), the previous equation simplifies as follows

$$1 - u = ve\left(\frac{\alpha_a}{\alpha_n}\right)\left(\frac{\Omega}{\lambda\psi}\right). \tag{B.1}$$

By manipulating the definition of GDP, $Q = pY_a + Y_n$, with the market clearing condition $Y_a = c_a$, the definition of v and (22), we derive after some algebra

$$\Omega = \frac{u}{1 - ve}. \tag{B.2}$$

By introducing (B.2) into (B.1), we directly obtain (27).

We rewrite the labor supply, (11), as follows

$$w_n - \lambda w_n = \pi r.$$

Rearranging terms and using the labor demand, (14), and equations (15) and (21), we obtain

$$\lambda = 1 - m\left(\frac{\alpha_n\left(\frac{z}{\phi}\right)^{\alpha_n - 1} - \delta}{(1 - \alpha_n)\left(\frac{z}{\phi}\right)^{\alpha_n}}\right). \tag{B.3}$$

We finally substitute (21) and (27) into (B.3) and we use the definition of m to obtain

$$\left(\frac{1-\lambda}{\lambda\psi}\right) z\left(\frac{1-\alpha_n}{m}\right)\left(\frac{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}\lambda\psi}{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}}\right) = \alpha_n - \delta\left(\frac{z}{\lambda\psi}\right)^{1-\alpha_n}\left(\frac{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}\lambda\psi}{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}}\right)^{1-\alpha_n}. \tag{B.4}$$

This equation implicitly defines $\lambda = \widehat{\lambda}(e, z, m)$. From using the implicit function theorem, we obtain

$$\frac{\partial\lambda}{\partial m} = -\frac{\left(\frac{1-\lambda}{m^2}\right)\left(\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}\lambda\psi\right)}{\left(\frac{1}{m}\right)\left[\left(\frac{1}{\lambda}\right)\frac{ve}{1-ve} + \lambda\frac{\alpha_n}{\alpha_a}\psi\right] + (1-\alpha_n)\delta\left(\frac{z}{\lambda\psi}\right)^{-\alpha_n}\left(\frac{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}\lambda\psi}{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}}\right)^{-\alpha_n}\left(\frac{1}{\lambda}\right)\frac{ve}{1-ve}} < 0.$$

C System of differential equations

We proceed to obtain the system of differential equations characterizing the time path of the transformed variables: z, e, $\widetilde{e}$, and m. First, the resource constraint is

$$Q = E + \dot{k} + \delta k + \dot{u}\pi. \tag{C.1}$$

We use the definition of e, (C.1) and (22) to obtain

$$\frac{\dot{k}}{k} = \Omega z^{\alpha_n - 1}\phi^{-\alpha_n}(1 - e) - \delta - \frac{\dot{u}\pi}{k}.$$

We log-differentiate the definition of z and we use the previous equation to obtain a differential equation governing the path of z

$$\frac{\dot{z}}{z} = \Omega z^{\alpha_n - 1}\phi^{-\alpha_n}(1 - e) - \delta - \dot{u}\frac{m}{z} - \gamma_n. \tag{C.2}$$

We use the definition of e and (17) to rewrite (10) as follows

$$\begin{aligned}\frac{\dot{E}}{E} &= \left(\frac{e-\widetilde{e}}{e}\right)\left(\alpha_n z^{\alpha_n-1}\phi^{1-\alpha_n}-\delta-\rho\right)\\ &+\frac{\widetilde{e}}{e}\left[(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda}+\gamma_n-\gamma_a\right)+(\alpha_n-\alpha_a)\left(\frac{\dot{z}}{z}-\frac{\dot{\phi}}{\phi}\right)\right].\end{aligned}$$

We log-differentiate the definition of e to obtain the differential equation governing the time path of e

$$\begin{aligned}\frac{\dot{e}}{e} &= \left(\frac{e-\widetilde{e}}{e}\right)\left(\alpha_n z^{\alpha_n-1}\phi^{1-\alpha_n}-\delta-\rho\right)+ \qquad \text{(C.3)}\\ &\frac{\widetilde{e}}{e}\left[(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda}+\gamma_n-\gamma_a\right)+(\alpha_n-\alpha_a)\left(\frac{\dot{z}}{z}-\frac{\dot{\phi}}{\phi}\right)\right]-\frac{\dot{\Omega}}{\Omega}-\gamma_n-\alpha_n\left(\frac{\dot{z}}{z}-\frac{\dot{\phi}}{\phi}\right).\end{aligned}$$

From the definition of $\widetilde{e}$, we obtain

$$\frac{\dot{\widetilde{e}}}{\widetilde{e}}=(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda}+\gamma_n-\gamma_a\right)-\frac{\dot{\Omega}}{\Omega}-\gamma_n-\alpha_a\left(\frac{\dot{z}}{z}-\frac{\dot{\phi}}{\phi}\right). \qquad \text{(C.4)}$$

We next obtain the differential equations driving the path of ϕ, λ, u and Ω. First, we use (B.2) to obtain

$$\frac{\dot{\Omega}}{\Omega}=\frac{\dot{u}}{u}+\left(\frac{ve}{1-ve}\right)\left(\frac{\dot{v}}{v}+\frac{\dot{e}}{e}\right). \qquad \text{(C.5)}$$

Second, from the definition of ϕ and Ω we obtain

$$\dot{\phi}=\psi\dot{\lambda}\,(1-u)+\dot{u}\,(1-\lambda\psi) \qquad \text{(C.6)}$$

and

$$\dot{\Omega}=\left(\frac{\alpha_n}{\alpha_a}\right)\psi\,(1-u)\,\dot{\lambda}+\left(1-\frac{\alpha_n}{\alpha_a}\psi\lambda\right)\dot{u}. \qquad \text{(C.7)}$$

Next, from (25) we obtain

$$\dot{v}=(1-\theta)\,\frac{\widetilde{e}}{e}\left(\frac{\dot{\widetilde{e}}}{\widetilde{e}}-\frac{\dot{e}}{e}\right). \qquad \text{(C.8)}$$

Finally, from (B.3) we obtain

$$-\frac{\dot{\lambda}}{1-\lambda}=-\gamma_n-h\,(z,\phi)\left(\frac{\dot{z}}{z}-\frac{\dot{\phi}}{\phi}\right), \qquad \text{(C.9)}$$

where

$$h\,(z,\phi)=\alpha_n\left(\frac{\left(\frac{z}{\phi}\right)^{\alpha_n-1}-\delta}{\alpha_n\left(\frac{z}{\phi}\right)^{\alpha_n}-\delta}\right).$$

We proceed to solve the system of equations (C.2)-(C.9). First, from (C.8), we obtain

$$\frac{\dot{v}}{v}+\frac{\dot{e}}{e}=\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)\frac{\dot{\widetilde{e}}}{\widetilde{e}}+\left(1-\frac{1-\theta}{v}\frac{\widetilde{e}}{e}\right)\frac{\dot{e}}{e}$$
$$=\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)\left[(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda}+\gamma_n-\gamma_a\right)-\frac{\dot{\Omega}}{\Omega}-\gamma_n-\alpha_a\left(\frac{\dot{z}}{z}-\frac{\dot{\phi}}{\phi}\right)\right]$$
$$+\left(1-\frac{\widetilde{e}}{e}\frac{1-\theta}{v}\right)\frac{\dot{e}}{e}.$$

We substitute into (C.5) to obtain

$$\frac{\dot{\Omega}}{\Omega}=\frac{\frac{\dot{u}}{u}+\left(\frac{ve}{1-ve}\right)\left\{\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)\left[(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda}+\gamma_n-\gamma_a\right)-\gamma_n-\alpha_a\left(\frac{\dot{z}}{z}-\frac{\dot{\phi}}{\phi}\right)\right]+\left(1-\frac{\widetilde{e}}{e}\frac{1-\theta}{v}\right)\frac{\dot{e}}{e}\right\}}{1+\left(\frac{ve}{1-ve}\right)\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)}. \tag{C.10}$$

Next, we use (C.6) to obtain

$$\dot{\phi}=\psi\dot{\lambda}\left(1-u\right)+\left(1-\psi\lambda\right)\dot{u}. \tag{C.11}$$

Combining the previous two equations, we obtain

$$\frac{\dot{\Omega}}{\Omega}=\frac{\frac{\dot{u}}{u}+\left(\frac{ve}{1-ve}\right)\left\{\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)\left[(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda}+\gamma_n-\gamma_a\right)-\gamma_n-\alpha_a\left(\frac{\dot{z}}{z}-\frac{\psi\dot{\lambda}(1-u)+(1-\psi\lambda)\dot{u}}{\phi}\right)\right]+\left(1-\frac{\widetilde{e}}{e}\frac{1-\theta}{v}\right)\frac{\dot{e}}{e}\right\}}{1+\left(\frac{ve}{1-ve}\right)\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)}.$$

We use (C.7) to obtain

$$\dot{\Omega}=\left(\frac{\alpha_n}{\alpha_a}\right)\dot{\lambda}\psi\left(1-u\right)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\lambda\right)\dot{u}. \tag{C.12}$$

We next combine the previous two equations, to obtain

$$\frac{\left(\frac{\alpha_n}{\alpha_a}\right)\dot{\lambda}\psi\left(1-u\right)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\lambda\right)\dot{u}}{\Omega}$$
$$=\frac{\frac{\dot{u}}{u}+\left(\frac{ve}{1-ve}\right)\left\{\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)\left[(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda}+\gamma_n-\gamma_a\right)-\gamma_n-\alpha_a\left(\frac{\dot{z}}{z}-\frac{\psi\dot{\lambda}(1-u)+(1-\psi\lambda)\dot{u}}{\phi}\right)\right]+\left(1-\frac{\widetilde{e}}{e}\frac{1-\theta}{v}\right)\frac{\dot{e}}{e}\right\}}{1+\left(\frac{ve}{1-ve}\right)\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)} \tag{C.13}$$

We combine (C.6) and (C.9) to obtain

$$-\frac{\dot{\lambda}}{1-\lambda}=-\gamma_n-h\left(z,\phi\right)\left(\frac{\dot{z}}{z}-\frac{\psi\dot{\lambda}\left(1-u\right)+\left(1-\psi\lambda\right)\dot{u}}{\phi}\right),$$

which can be rewritten as

$$\frac{\dot{\lambda}}{\lambda}=\frac{\gamma_n+h\left(z,\phi\right)\left(\frac{\dot{z}}{z}-\frac{(1-\psi\lambda)\dot{u}}{\phi}\right)}{\frac{\lambda}{1-\lambda}+h\left(z,\phi\right)\frac{\psi\lambda(1-u)}{\phi}}. \tag{C.14}$$

We proceed by using (C.13) and (C.14) to obtain

$$\frac{\left(\frac{\alpha_n}{\alpha_a}\right)\lambda\left(\frac{\gamma_n+h(z,\phi)\left(\frac{\dot{z}}{z}-\frac{(1-\psi\lambda)\dot{u}}{\phi}\right)}{\frac{\lambda}{1-\lambda}+h(z,\phi)\frac{\psi\lambda(1-u)}{\phi}}\right)\psi(1-u)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\lambda\right)\dot{u}}{\Omega}=$$

$$\frac{\left(\frac{ve}{1-ve}\right)\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)\left[\left[(1-\alpha_a)+\alpha_a\frac{\psi}{\phi}\lambda(1-u)\right]\left(\frac{\gamma_n+h(z,\phi)\left(\frac{\dot{z}}{z}-\frac{(1-\psi\lambda)\dot{u}}{\phi}\right)}{\frac{\lambda}{1-\lambda}+h(z,\phi)\frac{\psi\lambda(1-u)}{\phi}}\right)+(1-\alpha_a)(\gamma_n-\gamma_a)-\gamma_n-\alpha_a\left(\frac{\dot{z}}{z}-\frac{(1-\psi\lambda)\dot{u}}{\phi}\right)\right]}{1+\left(\frac{ve}{1-ve}\right)\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)}$$

$$+\frac{\frac{\dot{u}}{u}+\left(\frac{ve}{1-ve}\right)\left(1-\frac{\widetilde{e}}{e}\frac{1-\theta}{v}\right)\frac{\dot{e}}{e}}{1+\left(\frac{ve}{1-ve}\right)\left(\frac{1-\theta}{v}\right)\left(\frac{\widetilde{e}}{e}\right)},$$

which can be rewritten as follows

$$\frac{\dot{u}}{u}\left[\begin{array}{c}\left(\frac{\alpha_n}{\alpha_a}\right)\lambda\left(\frac{h(z,\phi)\left(-\frac{(1-\psi\lambda)u}{\phi}\right)}{\frac{\lambda}{1-\lambda}+h(z,\phi)\frac{\psi\lambda(1-u)}{\phi}}\right)\psi\left(1-u\right)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\lambda\right)u\\-\Omega\left(\frac{1-ve+\widetilde{e}(1-\theta)\left(\frac{(1-\psi\lambda)u}{\phi}\right)\left[\frac{-(1-\alpha_a)h(z,\phi)+\alpha_a\frac{\lambda}{1-\lambda}}{\frac{\lambda}{1-\lambda}+h(z,\phi)\frac{\psi\lambda(1-u)}{\phi}}\right]}{1-ve+(1-\theta)\widetilde{e}}\right)\end{array}\right]$$

$$=\frac{\Omega\left\{(1-\theta)\widetilde{e}\left[\left[(1-\alpha_a)+\alpha_a\frac{\psi}{\phi}\lambda(1-u)\right]\left(\frac{\gamma_n+h(z,\phi)\frac{\dot{z}}{z}}{\frac{\lambda}{1-\lambda}+h(z,\phi)\frac{\psi\lambda(1-u)}{\phi}}\right)+(1-\alpha_a)(\gamma_n-\gamma_a)-\gamma_n-\alpha_a\frac{\dot{z}}{z}\right]+[ve-\widetilde{e}(1-\theta)]\frac{\dot{e}}{e}\right\}}{1-ve+(1-\theta)\widetilde{e}}$$

$$-\left(\frac{\alpha_n}{\alpha_a}\right)\lambda\left(\frac{\gamma_n+h\left(z,\phi\right)\frac{\dot{z}}{z}}{\frac{\lambda}{1-\lambda}+h\left(z,\phi\right)\frac{\psi\lambda(1-u)}{\phi}}\right)\psi\left(1-u\right).$$

Let

$$\kappa_2=\frac{\Omega}{(1-ve)+\widetilde{e}\left(1-\theta\right)},$$

$$\kappa_3=\frac{1-\lambda}{\lambda+h\left(z,\phi\right)\frac{\psi\lambda(1-\lambda)(1-u)}{\phi}},$$

and

$$\begin{aligned}\kappa_1 &= -\left(\frac{\alpha_n}{\alpha_a}\right)\lambda\kappa_3 h\left(z,\phi\right)\left(\frac{\left(1-\psi\lambda\right)u}{\phi}\right)\psi\left(1-u\right)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\lambda\right)u\\ &\quad -\kappa_2\left\{1-ve+\frac{\widetilde{e}\left(1-\theta\right)\left(\frac{(1-\psi\lambda)u}{\phi}\right)}{\lambda+h\left(z,\phi\right)\frac{\psi\lambda(1-\lambda)(1-u)}{\phi}}\left[-\left(1-\alpha_a\right)\left(1-\lambda\right)h\left(z,\phi\right)+\alpha_a\lambda\right]\right\}.\end{aligned}$$

Then, the previous equation can be rewritten as

$$\begin{aligned}&\kappa_1\frac{\dot{u}}{u}\\ &= \left(1-\theta\right)\widetilde{e}\kappa_2\left\{\begin{array}{c}\left[\left(1-\alpha_a\right)+\alpha_a\frac{\psi}{\phi}\lambda\left(1-u\right)\right]\left(\gamma_n+h\left(z,\phi\right)\frac{\dot{z}}{z}\right)\kappa_3\\+\left(1-\alpha_a\right)\left(\gamma_n-\gamma_a\right)-\gamma_n-\alpha_a\frac{\dot{z}}{z}\end{array}\right\}\\ &\quad+\kappa_2\left[ve-\widetilde{e}\left(1-\theta\right)\right]\frac{\dot{e}}{e}-\left(\frac{\alpha_n}{\alpha_a}\right)\lambda\left(\gamma_n+h\left(z,\phi\right)\frac{\dot{z}}{z}\right)\kappa_3\psi\left(1-u\right),\end{aligned}$$

which simplifies as follows

$$\frac{\dot{u}}{u}=\eta_1\frac{\dot{z}}{z}+\eta_2+\eta_3\frac{\dot{e}}{e},$$

where

$$\eta_1=\frac{\kappa_2(1-\theta)\widetilde{e}\left\{\left[(1-\alpha_a)+\alpha_a\frac{\psi}{\phi}\lambda(1-u)\right]\kappa_3 h(z,\phi)-\alpha_a\right\}-\left(\frac{\alpha_n}{\alpha_a}\right)\lambda\kappa_3 h(z,\phi)\psi(1-u)}{\kappa_1},$$

$$\eta_2=\frac{\kappa_2(1-\theta)\widetilde{e}\left\{\left[(1-\alpha_a)+\alpha_a\frac{\psi}{\phi}\lambda(1-u)\right]\gamma_n\kappa_3+(1-\alpha_a)(\gamma_n-\gamma_a)-\gamma_n\right\}-\left(\frac{\alpha_n}{\alpha_a}\right)\lambda\kappa_3\psi(1-u)}{\kappa_1},$$

and

$$\eta_3=\frac{\kappa_2\left[ve-\widetilde{e}\left(1-\theta\right)\right]}{\kappa_1}.$$

We substitute into (C.14) to obtain

$$\begin{aligned}\frac{\dot{\lambda}}{\lambda} &= \kappa_3\left[\gamma_n-h\left(z,\phi\right)\frac{\left(1-\psi\lambda\right)}{\phi}u\eta_2\right]+\kappa_3 h\left(z,\phi\right)\left(1-\frac{\left(1-\psi\lambda\right)u\eta_1}{\phi}\right)\frac{\dot{z}}{z}\\ &\quad -\kappa_3 h\left(z,\phi\right)\frac{\left(1-\psi\lambda\right)u\eta_3}{\phi}\frac{\dot{e}}{e},\end{aligned}$$

and we substitute into (C.11) to obtain

$$\begin{aligned}\frac{\dot{\phi}}{\phi} &= \frac{\psi\lambda\left(1-u\right)}{\phi}\left\{\begin{array}{c}\kappa_3\left[\gamma_n-h\left(z,\phi\right)\frac{(1-\psi\lambda)}{\phi}u\eta_2\right]+\kappa_3 h\left(z,\phi\right)\left(1-\frac{(1-\psi\lambda)u\eta_1}{\phi}\right)\frac{\dot{z}}{z}\\ -\kappa_3 h\left(z,\phi\right)\frac{(1-\psi\lambda)u\eta_3}{\phi}\frac{\dot{e}}{e}\end{array}\right\}\\ &\quad +\frac{\left(1-\psi\lambda\right)}{\phi}\left[\eta_1\frac{\dot{z}}{z}+\eta_2+\eta_3\frac{\dot{e}}{e}\right]u.\end{aligned}$$

This equation can be rewritten as

$$\frac{\dot{\phi}}{\phi}=\psi_1\frac{\dot{z}}{z}+\psi_2+\psi_3\frac{\dot{e}}{e},$$

where

$$\psi_1=\frac{\psi\lambda\left(1-u\right)}{\phi}\kappa_3 h\left(z,\phi\right)\left(1-\frac{\left(1-\psi\lambda\right)u\eta_1}{\phi}\right)+\frac{\left(1-\psi\lambda\right)}{\phi}\eta_1 u,$$

$$\psi_2=\frac{\psi\lambda\left(1-u\right)}{\phi}\kappa_3\left[\gamma_n-h\left(z,\phi\right)\frac{\left(1-\psi\lambda\right)}{\phi}u\eta_2\right]+\frac{\left(1-\psi\lambda\right)}{\phi}\eta_2 u,$$

and

$$\psi_3=-\frac{\psi\lambda\left(1-u\right)}{\phi}\kappa_3 h\left(z,\phi\right)\frac{\left(1-\psi\lambda\right)u\eta_3}{\phi}+\frac{\left(1-\psi\lambda\right)}{\phi}\eta_3 u.$$

We next substitute the expressions of $\dot{\lambda}$ and $\dot{u}$ into (C.12) to obtain

$$\begin{aligned}\dot{\Omega} &= \left(\frac{\alpha_n}{\alpha_a}\right)\lambda\psi\left(1-u\right)\left\{\begin{array}{c}\kappa_3\left[\gamma_n-h\left(z,\phi\right)\frac{(1-\psi\lambda)}{\phi}u\eta_2\right]+\kappa_3 h\left(z,\phi\right)\left(1-\frac{(1-\psi\lambda)u\eta_1}{\phi}\right)\frac{\dot{z}}{z}\\ -\kappa_3 h\left(z,\phi\right)\frac{(1-\psi\lambda)u\eta_3}{\phi}\frac{\dot{e}}{e}\end{array}\right\}\\ &\quad +\left(1-\frac{\alpha_n}{\alpha_a}\psi\lambda\right)u\left[\eta_1\frac{\dot{z}}{z}+\eta_2+\eta_3\frac{\dot{e}}{e}\right].\end{aligned}$$

This equation can be rewritten as

$$\dot{\Omega} = \varphi_1 \frac{\dot{z}}{z} + \varphi_2 + \varphi_3 \frac{\dot{e}}{e},$$

where

$$\varphi_1 = \left(\frac{\alpha_n}{\alpha_a}\right) \lambda\psi (1-u) \kappa_3 h(z,\phi) \left(1 - \frac{(1-\psi\lambda) u\eta_1}{\phi}\right) + \left(1 - \frac{\alpha_n}{\alpha_a}\psi\lambda\right) u\eta_1,$$

$$\varphi_2 = \left(\frac{\alpha_n}{\alpha_a}\right) \lambda\psi (1-u) \kappa_3 \left[\gamma_n - h(z,\phi) \frac{(1-\psi\lambda)}{\phi} u\eta_2\right] + \left(1 - \frac{\alpha_n}{\alpha_a}\psi\lambda\right) u\eta_2,$$

and

$$\varphi_3 = -\left(\frac{\alpha_n}{\alpha_a}\right) \lambda\psi (1-u) \kappa_3 h(z,\phi) \frac{(1-\psi\lambda) u\eta_3}{\phi} + \left(1 - \frac{\alpha_n}{\alpha_a}\psi\lambda\right) u\eta_3.$$

We substitute into (C.2) to obtain

$$\frac{\dot{z}}{z} = \Omega z^{\alpha_n - 1} \phi^{-\alpha_n} (1-e) - \delta - u\frac{m}{z}\left[\eta_1 \frac{\dot{z}}{z} + \eta_2 + \eta_3 \frac{\dot{e}}{e}\right] - \gamma_n,$$

which can be rewritten as

$$\frac{\dot{z}}{z} = \omega_1 - \omega_2 \frac{\dot{e}}{e}, \tag{C.15}$$

where

$$\omega_1 = \frac{\Omega z^{\alpha_n - 1} \phi^{-\alpha_n} (1-e) - \delta - \left(u\frac{m}{z}\eta_2 + \gamma_n\right)}{1 + \frac{um}{z}\eta_1}, \tag{C.16}$$

and

$$\omega_2 = \frac{\frac{\eta_3 um}{z}}{1 + \frac{um}{z}\eta_1}. \tag{C.17}$$

We use (C.3) to obtain

$$\begin{aligned} \frac{\dot{e}}{e} &= \left(\frac{e - \widetilde{e}}{e}\right) \left[\alpha_n z^{\alpha_n - 1} \phi^{1-\alpha_n} - \delta - \rho\right] \\ &\quad + \left(\frac{\widetilde{e}}{e}\right) \left[(1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda} + \gamma_n - \gamma_a\right) + (\alpha_n - \alpha_a)\left(\frac{\dot{z}}{z} - \frac{\dot{\phi}}{\phi}\right)\right] \\ &\quad - \frac{\dot{\Omega}}{\Omega} - \alpha_n \left(\frac{\dot{z}}{z} - \frac{\dot{\phi}}{\phi}\right) - \gamma_n, \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \frac{\dot{e}}{e} &= \left(\frac{e - \widetilde{e}}{e}\right) \left[\alpha_n z^{\alpha_n - 1} \phi^{1-\alpha_n} - \delta - \rho\right] - \gamma_n + \\ &\left(\frac{\widetilde{e}}{e}\right)(1-\alpha_a) \left\{ \begin{array}{c} \kappa_3 \left[\gamma_n - h(z,\phi) \frac{(1-\psi\lambda)}{\phi} u\eta_2\right] + (\gamma_n - \gamma_a) + \\ \kappa_3 h(z,\phi) \left(1 - \frac{(1-\psi\lambda)u\eta_1}{\phi}\right) \left[\omega_1 - \omega_2 \frac{\dot{e}}{e}\right] - \kappa_3 h(z,\phi) \frac{(1-\psi\lambda)u\eta_3}{\phi} \frac{\dot{e}}{e} \\ + \frac{(\alpha_n - \alpha_a)}{(1-\alpha_a)} \left[\omega_1 - \omega_2 \frac{\dot{e}}{e} - \psi_1 \left(\omega_1 - \omega_2 \frac{\dot{e}}{e}\right) - \psi_2 - \psi_3 \frac{\dot{e}}{e}\right] \end{array} \right\} \\ &- \left[\varphi_1 \left(\omega_1 - \omega_2 \frac{\dot{e}}{e}\right) + \varphi_2 + \varphi_3 \frac{\dot{e}}{e}\right] - \alpha_n \left(\omega_1 - \omega_2 \frac{\dot{e}}{e} - \psi_1 \left(\omega_1 - \omega_2 \frac{\dot{e}}{e}\right) - \psi_2 - \psi_3 \frac{\dot{e}}{e}\right). \end{aligned}$$

This equation simplifies as follows

$$\dot{e} = e\omega_3, \tag{C.18}$$

where

$$\omega_3 = \tfrac{\omega_{31}}{\omega_{32}}, \tag{C.19}$$

$$\begin{aligned}\omega_{31} &= \left(\frac{e-\widetilde{e}}{e}\right)\left[\alpha_n z^{\alpha_n - 1}\phi^{1-\alpha_n} - \delta - \rho\right] - (\varphi_1\omega_1 + \varphi_2) - \alpha_n(\omega_1 - \psi_1\omega_1 - \psi_2) - \gamma_n \\ &+ \left(\frac{\widetilde{e}}{e}\right)\left(\begin{array}{c} (1-\alpha_a)\left\{ \begin{array}{c} \kappa_3\left[\gamma_n - h(z,\phi)\frac{(1-\psi\lambda)}{\phi}u\eta_2\right] + \\ (\gamma_n - \gamma_a) + \kappa_3 h(z,\phi)\left(1 - \frac{(1-\psi\lambda)u\eta_1}{\phi}\right)\omega_1 \end{array}\right\} \\ +(\alpha_n - \alpha_a)(\omega_1 - \psi_1\omega_1 - \psi_2) \end{array}\right),\end{aligned}$$

and

$$\begin{aligned}\omega_{32} &= 1 - \varphi_1\omega_2 + \varphi_3 + \alpha_n(-\omega_2 + \psi_1\omega_2 - \psi_3) \\ &- \left(\frac{\widetilde{e}}{e}\right)\left\{ \begin{array}{c} -\kappa_3 h(z,\phi)(1-\alpha_a)\left[\left(1 - \frac{(1-\psi\lambda)u\eta_1}{\phi}\right)\omega_2 + \frac{(1-\psi\lambda)u\eta_3}{\phi}\right] \\ +(\alpha_n - \alpha_a)(-\omega_2 + \psi_1\omega_2 - \psi_3) \end{array}\right\}.\end{aligned}$$

We proceed by using the definition of $\widetilde{e}$ to obtain

$$\frac{\dot{\widetilde{e}}}{\widetilde{e}} = (1-\alpha_a)\left(\frac{\dot{\lambda}}{\lambda} + \gamma_n - \gamma_a\right) - \gamma_n - \alpha_a\left(\frac{\dot{z}}{z} - \frac{\dot{\phi}}{\phi}\right) - \frac{\dot{\Omega}}{\Omega} = \omega_4, \tag{C.20}$$

where

$$\begin{aligned}\omega_4 &= (1-\alpha_a)\left\{ \begin{array}{c} \kappa_3\left[\gamma_n - h(z,\phi)\frac{(1-\psi\lambda)}{\phi}u\eta_2\right] + \gamma_n - \gamma_a \\ \kappa_3 h(z,\phi)\left(1 - \frac{(1-\psi\lambda)u\eta_1}{\phi}\right)(\omega_1 - \omega_2\omega_3) - \kappa_3 h(z,\phi)\frac{(1-\psi\lambda)u\eta_3}{\phi}\omega_3 \end{array}\right\} \\ &-\gamma_n - \alpha_a\left[(\omega_1 - \omega_2\omega_3) - \psi_1(\omega_1 - \omega_2\omega_3) - \psi_2 - \psi_3\omega_3\right] \\ &-\frac{\varphi_1(\omega_1 - \omega_2\omega_3) + \varphi_2 + \varphi_3\omega_3}{\Omega}.\end{aligned} \tag{C.21}$$

Finally, from log-differentiating $m = \pi/A_m$, we obtain the differential equation $\dot{m} = -\gamma_n m$. This differential equation together with (C.15), (C.18) and (C.20) form a system of differential equations characterizing the equilibrium.

Definition 1 *An equilibrium is a path of* $\{z, m, \widetilde{e}, e, \lambda, v, u, \phi\}$ *that given initial conditions* z_0, m_0 *and* $\widetilde{e}_0$ *satisfies equations (25), (26), (27), (21) and the transversality condition* $\lim_{t\to\infty} kc_n^{-1}e^{-\rho t} = 0$ *and solves the system of differential equations*

$$\dot{z} = (\omega_1 - \omega_2\omega_3)\, z,$$

$$\dot{e} = \omega_3 e,$$

$$\dot{m} = -\gamma_n m,$$

and

$$\dot{\widetilde{e}} = \omega_4\widetilde{e},$$

where the functions $\{\omega_i\}_{i=1}^4$ *are defined in equations (C.16), (C.17), (C.19) and (C.21).*

D Proof of Proposition 2

It is straightforward to show that in a BGP $\omega_1^* = \omega_2^* = \omega_3^* = 0$ and $\omega_4^* = -\gamma_n$. Using these relationships, it can be shown that the Jacobian matrix satisfies

$$J = \begin{pmatrix} \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial e} & \frac{\partial \dot{z}}{\partial m} & \frac{\partial \dot{z}}{\partial \widetilde{e}} \\ \frac{\partial \dot{e}}{\partial z} & \frac{\partial \dot{e}}{\partial e} & \frac{\partial \dot{e}}{\partial m} & \frac{\partial \dot{e}}{\partial \widetilde{e}} \\ 0 & 0 & \frac{\partial \dot{m}}{\partial m} & 0 \\ 0 & 0 & 0 & \frac{\partial \dot{\widetilde{e}}}{\partial \widetilde{e}} \end{pmatrix} = \begin{pmatrix} \frac{\partial \omega_1}{\partial z} z & \frac{\partial \omega_1}{\partial e} z & \frac{\partial \dot{z}}{\partial m} & \frac{\partial \dot{z}}{\partial \widetilde{e}} \\ \frac{\partial \omega_3}{\partial z} e & \frac{\partial \omega_3}{\partial e} e & \frac{\partial \dot{e}}{\partial m} & \frac{\partial \dot{e}}{\partial \widetilde{e}} \\ 0 & 0 & -\gamma_n & 0 \\ 0 & 0 & 0 & -\gamma_n \end{pmatrix}.$$

It follows that the eigenvalues are $\lambda_1 = \lambda_2 = \gamma_n$, and the other two, λ_3 and λ_4, are the solution of the following polynomial

$$P(\lambda) = \lambda^2 - \lambda \left(\frac{\partial \omega_1}{\partial z} z + \frac{\partial \omega_3}{\partial e} e \right) + D,$$

where

$$D = \frac{\partial \omega_1}{\partial z} z \frac{\partial \omega_3}{\partial e} e - \frac{\partial \omega_1}{\partial e} z \frac{\partial \omega_3}{\partial z} e.$$

Using the definitions of ω_1 and ω_3, and taking into account that $\eta_2^* = \varphi_1^* = \psi_1^* = 0$, we obtain

$$\begin{aligned}
\frac{\partial \omega_1}{\partial z} &= \left[\frac{\Omega_z}{\Omega} + \frac{\alpha_n - 1}{z} - \alpha_n \frac{\phi_z}{\phi} \right] (\delta + \gamma_n), \\
\frac{\partial \omega_1}{\partial e} &= \left[\frac{\Omega_e}{\Omega} - \frac{1}{1-e} - \alpha_n \frac{\phi_e}{\phi} \right] (\delta + \gamma_n), \\
\frac{\partial \omega_3}{\partial z} &= \frac{\alpha_n (\alpha_n - 1) z^{\alpha_n - 1} \phi^{1-\alpha_n} \left(\frac{1}{z} - \frac{\phi_z}{\phi} \right) - \frac{\partial \varphi_2}{\partial z} - \alpha_n \left(\frac{\partial \omega_1}{\partial z} - \frac{\partial \psi_2}{\partial z} \right)}{1 + \varphi_3 - \alpha_n \psi_3}, \\
\frac{\partial \omega_3}{\partial e} &= \frac{\alpha_n (1 - \alpha_n) z^{\alpha_n - 1} \phi^{1-\alpha_n} \left(\frac{\phi_e}{\phi} \right) - \frac{\partial \varphi_2}{\partial e} - \alpha_n \left(\frac{\partial \omega_1}{\partial e} - \frac{\partial \psi_2}{\partial e} \right)}{1 + \varphi_3 - \alpha_n \psi_3}.
\end{aligned}$$

From using (26), we obtain that in a BGP $\frac{\partial \lambda}{\partial z} = 0$ and $\frac{\partial \lambda}{\partial e} = 0$. Using this result and (27), we obtain $\frac{\partial \phi}{\partial z} = 0$, $\frac{\partial \Omega}{\partial z} = 0$, $\frac{\partial \phi}{\partial e} = -\left(\frac{\alpha_a - \alpha_n}{(1-\alpha_a)\alpha_n} \right) \frac{\partial u}{\partial e}$ and $\frac{\partial \Omega}{\partial e} = -\left(\frac{\alpha_a - \alpha_n}{1-\alpha_a} \right) \frac{\partial u}{\partial e}$ in a BGP. Using the expressions of ψ_2 and φ_2, we also obtain that

$$\varphi_2 = \left(\frac{\alpha_n}{\alpha_a} \right) \lambda \psi (1-u) \kappa_3 \left[\gamma_n - h(z, \phi) \frac{(1 - \psi\lambda)}{\phi} u \eta_2 \right] + \left(1 - \frac{\alpha_n}{\alpha_a} \psi \lambda \right) u \eta_2,$$

$$\begin{aligned}
\frac{\partial \psi_2}{\partial z} &= \psi \frac{(1-u)}{\phi} \gamma_n \frac{\partial \kappa_3}{\partial z} + \frac{(1-\psi)}{\phi} u \frac{\partial \eta_2}{\partial z} = 0, \\
\frac{\partial \psi_2}{\partial e} &= \psi \frac{(1-u)}{\phi} \gamma_n \frac{\partial \kappa_3}{\partial e} + \frac{(1-\psi)}{\phi} u \frac{\partial \eta_2}{\partial e} = 0, \\
\frac{\partial \varphi_2}{\partial z} &= \frac{\alpha_n}{\alpha_a} \lambda \psi (1-u) \gamma_n \frac{\partial \kappa_3}{\partial z} + \left(1 - \frac{\alpha_n}{\alpha_a} \psi \right) u \frac{\partial \eta_2}{\partial z} = 0, \\
\frac{\partial \varphi_2}{\partial e} &= \frac{\alpha_n}{\alpha_a} \lambda \psi (1-u) \gamma_n \frac{\partial \kappa_3}{\partial e} + \left(1 - \frac{\alpha_n}{\alpha_a} \psi \right) u \frac{\partial \eta_2}{\partial e} = 0.
\end{aligned}$$

Therefore, we obtain that

$$\frac{\partial \omega_1}{\partial z} = \left(\frac{\alpha_n - 1}{z}\right)(\delta + \gamma_n),$$
$$\frac{\partial \omega_1}{\partial e} = \left[\frac{\Omega_e}{\Omega} - \frac{1}{1-e} - \alpha_n \frac{\phi_e}{\phi}\right](\delta + \gamma_n),$$
$$\frac{\partial \omega_3}{\partial z} = \frac{\alpha_n(\alpha_n - 1) z^{\alpha_n - 1}\phi^{1-\alpha_n}\frac{1}{z} - \alpha_n \frac{\partial \omega_1}{\partial z}}{1 + \varphi_3 - \alpha_n \psi_3},$$
$$\frac{\partial \omega_3}{\partial e} = \frac{\alpha_n(1 - \alpha_n) z^{\alpha_n - 1}\phi^{1-\alpha_n}\left(\frac{\phi_e}{\phi}\right) - \alpha_n \frac{\partial \omega_1}{\partial e}}{1 + \varphi_3 - \alpha_n \psi_3}.$$

Finally, we obtain that

$$\begin{aligned}
D &= \left(\frac{ze\alpha_n(1-\alpha_n) z^{\alpha_n-1}\phi^{1-\alpha_n}}{1+\varphi_3 - \alpha_n\psi_3}\right)\left[\left(\frac{\phi_e}{\phi}\right)\frac{\partial \omega_1}{\partial z} + \frac{1}{z}\frac{\partial \omega_1}{\partial e}\right] \\
&= (\delta+\gamma_n)\left(\frac{e\alpha_n(1-\alpha_n) z^{\alpha_n-1}\phi^{1-\alpha_n}}{1+\varphi_3 - \alpha_n\psi_3}\right)\left[\frac{\Omega_e}{\Omega} - \frac{1}{1-e} - \frac{\phi_e}{\phi}\right] \\
&= (\delta+\gamma_n)\left(\frac{e\alpha_n(1-\alpha_n) z^{\alpha_n-1}\phi^{1-\alpha_n}}{1+\varphi_3 - \alpha_n\psi_3}\right)\left[\left(\frac{\alpha_a - \alpha_n}{1-\alpha_a}\right)\frac{\partial u}{\partial e}\left(\frac{-1}{\Omega} + \frac{1}{\phi\alpha_n}\right) - \frac{1}{1-e}\right] \\
&= (\delta+\gamma_n)\left(\frac{e\alpha_n(1-\alpha_n) z^{\alpha_n-1}\phi^{1-\alpha_n}}{1+\varphi_3 - \alpha_n\psi_3}\right)\left[\left(\frac{\alpha_a - \alpha_n}{1-\alpha_a}\right)\frac{\partial u}{\partial e}\left(\frac{\Omega - \phi\alpha_n}{\Omega\phi\alpha_n}\right) - \frac{1}{1-e}\right] \\
&= (\delta+\gamma_n)\left(\frac{e\alpha_n(1-\alpha_n) z^{\alpha_n-1}\phi^{1-\alpha_n}}{1+\varphi_3 - \alpha_n\psi_3}\right)\left[-\theta\Omega^2\left(\frac{\alpha_a - \alpha_n}{1-\alpha_n}\right)\left(\frac{\Omega - \phi\alpha_n}{\Omega\phi\alpha_n}\right) - \frac{1}{1-e}\right] = \\
&= (\delta+\gamma_n)\left(\frac{e\alpha_n(1-\alpha_n) z^{\alpha_n-1}\phi^{1-\alpha_n}}{1+\varphi_3 - \alpha_n\psi_3}\right)\left[-\theta\Omega\left(\frac{\alpha_a - \alpha_n}{1-\alpha_n}\right)\left(\frac{1 - \alpha_n}{\phi\alpha_n}\right) - \frac{1}{1-e}\right] = \\
&= -(\delta+\gamma_n)\left(\frac{e\alpha_n(1-\alpha_n) z^{\alpha_n-1}\phi^{1-\alpha_n}}{1+\varphi_3 - \alpha_n\psi_3}\right)\left[\frac{\theta\Omega(\alpha_a - \alpha_n)}{\phi\alpha_n} + \frac{1}{1-e}\right] < 0.
\end{aligned}$$

This inequality follows because first

$$\begin{aligned}
\frac{\theta\Omega(\alpha_a - \alpha_n)}{\phi\alpha_n} + \frac{1}{1-e} &= \frac{\theta(\alpha_a - \alpha_n)}{\phi\alpha_n}\frac{u}{1 - vq} + \frac{1}{1-e} > \\
\frac{\theta(\alpha_a - \alpha_n)u + \phi\alpha_n}{\phi\alpha_n(1-e)} &= \frac{\theta\alpha_a u + [\psi(1-u) + u(1-\theta)]\alpha_n}{\phi\alpha_n(1-e)} > 0,
\end{aligned}$$

and, second,

$$
\begin{aligned}
& 1+\varphi_3-\alpha_n\psi_3 \\
= \; & 1+\left[\left(1-\frac{\alpha_n}{\alpha_a}\psi\right)-\alpha_n\frac{(1-\psi)}{\phi}\right]\frac{\Omega^2 ve}{\kappa_1} \\
= \; & 1+\left[\frac{\left(1-\frac{\alpha_n}{\alpha_a}\psi\right)\psi(1-u)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\right)u-\alpha_n(1-\psi)}{\psi(1-u)+u}\right]\frac{\Omega^2 ve}{\kappa_1} \\
= \; & 1+\left[\frac{\left(1-\frac{\alpha_n}{\alpha_a}\psi\right)\psi(1-u)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\right)(u-1)+\left(1-\frac{\alpha_n}{\alpha_a}\psi\right)-\alpha_n(1-\psi)}{\psi(1-u)+u}\right]\frac{\Omega^2 ve}{\kappa_1} \\
= \; & 1+\left[\frac{\left(1-\frac{\alpha_n}{\alpha_a}\psi\right)(\psi-1)(1-u)}{\psi(1-u)+u}\right]\frac{\Omega^2 ve}{\kappa_1}= \\
= \; & 1-\left(\frac{\alpha_n-\alpha_a}{1-\alpha_a}\right)^2\left[\frac{(1-u)}{(\psi(1-u)+u)\alpha_n}\right]\frac{\Omega^2 ve}{\kappa_1}>0.
\end{aligned}
$$

The last inequality follows because

$$
\begin{aligned}
\kappa_1 \; = \; & \left(1-\frac{\alpha_n}{\alpha_a}\psi\right)u-\Omega. \\
& \left(\frac{\alpha_n-\alpha_a}{1-\alpha_a}\right)u-\frac{u}{1-\theta q}= \\
& \left[\left(\frac{(\alpha_n-\alpha_a)(1+\Delta(\alpha_a)\theta-\theta)-(1-\alpha_a)(1+\Delta(\alpha_a-\alpha_n)\theta)}{(1-\alpha_a)(1+\Delta(\alpha_a)\theta-\theta)}\right)\right]u \\
= \; & \left[\left(\frac{(\alpha_n-1)(1+\Delta(\alpha_a)\theta-\theta)-(1-\alpha_a)\theta(1-\alpha_n\Delta)}{(1-\alpha_a)(1+\Delta(\alpha_a)\theta-\theta)}\right)\right]u<0
\end{aligned}
$$

This implies that either λ_3 or λ_4 is negative and the other eigenvalue is positive.

E Decomposition of GDP

The purpose of this appendix is to obtain the expression of the measures used in the comparative dynamic exercise of Figure 8. To this end, we use (22) to decompose GDP. Remember that $A_n^{1-\alpha_n}\Phi$ amounts for the TFP and $\Phi=\Omega\phi^{-\alpha_n}$ measures the contribution of sectoral composition to the TFP. This contribution goes through two different mechanisms: (i) sectoral differences in technologies $(\alpha_a\neq\alpha_n)$; and (ii) constrained allocation of inputs across sectors due to mobility cost, which generates sectoral differences in wages $(\lambda\neq 1)$.

In what follows we explain the differences in GDP per capita between two economies (Rich and Poor) as the result of differences in technology, capital and sectoral composition. In a second step, we measure the relevance of those two mechanisms driving the contribution of sectoral composition. In order to decompose the differences in GDP levels, we compute the logarithm of relative GDP as follows[26]

$$\log\left(\frac{Q^R}{Q^P}\right) = (1-\alpha_n)\log\left(\frac{A_n^R}{A_n^P}\right) + \log\left(\frac{\Phi^R}{\Phi^P}\right) + \alpha_n \log\left(\frac{k^R}{k^P}\right).$$

From this expression, we obtain the contribution to GDP of technology, capital and sectoral composition, that are, respectively,

$$C_A = \frac{(1-\alpha_n)\log\left(\frac{A_n^R}{A_n^P}\right)}{\log\left(\frac{Q^R}{Q^P}\right)} * 100,$$

$$C_\Phi = \frac{\log\left(\frac{\Phi^R}{\Phi^P}\right)}{\log\left(\frac{Q^R}{Q^P}\right)} * 100,$$

and

$$C_k = \frac{\alpha_n \log\left(\frac{k^R}{k^P}\right)}{\log\left(\frac{Q^R}{Q^P}\right)} * 100.$$

These magnitudes are displayed in Panels (ii), (iii), and (iv) of Figure 8.

We next measure the relevance of the two mechanisms determining the contribution of the sectoral composition. However, this decomposition cannot be done directly as these two mechanisms generate complementaries. For our purpose, we follow the following steps:

1. First, note that if $\alpha_a = \alpha_n$ and $\lambda = 1$ then $\Phi = 1$. This implies that we can decompose Φ as $\Phi = 1 + \Phi_\alpha + \Phi_\lambda$ where Φ_α measures the contribution of sectoral composition to GDP through sectoral different technologies and Φ_λ measures the contribution of sectoral composition to GDP through the technological constraint to labor mobility.

2. We obtain Φ_α from measuring the value of Φ with $\lambda = 1$ but taking u equal to the sectoral composition obtained when wages are different across sectors $(\lambda < 1)$. We, therefore, obtain Φ_α as follows:

$$\Phi_\alpha = \left(\Omega\phi^{-\alpha_n}\right)\big|_{\lambda=1} - 1 = \left[\left(\frac{\alpha_n}{\alpha_a}\right)\psi(1-u) + u\right]\left[\psi(1-u)+u\right]^{-\alpha_n}.$$

3. We compute the contribution of sectoral composition to GDP through the technological constraint to labor mobility (Φ_λ) by using Φ_α and Φ as follows

$$\Phi_\lambda = \Phi - \Phi_\alpha - 1.$$

[26] The superindex R amounts for the rich economy and the superindex P amounts for the poor economy

4. We next compute the weight of the allocation mechanism as the following ratio:

$$\varepsilon = \frac{\left(\frac{1+\Phi_{\lambda}^{R}}{1+\Phi_{\lambda}^{P}}\right)}{\left(\frac{\Phi^{R}}{\Phi^{P}}\right)}.$$

The numerator of this ratio measures the relative contribution of sectoral composition between the two countries due to the allocation mechanism. Therefore, the ratio ε measures the fraction of the differences between the two countries in the contribution of the sectoral composition explained by the allocation mechanism. We label this measure as the weight of the allocation mechanism, and we display it in Panel (v) of Figure 8.

5. Finally, we compute the contribution of the allocation mechanism to GDP as $C_{\Phi_{\lambda}} = \varepsilon C_{\Phi}$. This magnitude is displayed in Panel (vi) of Figure 8.

Figures and Tables

Table 1. Structural change in the US economy. Period 1880-2000.

Period	GDP share in Agriculture	Agriculture Employment share	Relative Wage	$\frac{LIS_a}{LIS_n}^{(a)}$	$\frac{LIS_a}{LIS_n}^{(b)}$
1880-1900	0.251	0.412	0.203	2.151	0.438
1900-1920	0.174	0.304	0.257	2.082	0.535
1920-1940	0.117	0.222	0.333	2.169	0.723
1940-1960	0.071	0.135	0.413	2.021	0.834
1960-1980	0.041	0.049	0.602	1.202	0.723
1980-2000	0.021	0.022	0.697	1.054	0.735

Source: Historical statistics of the U.S; Caselli and Coleman (2001); Bureau of labor Statistic.

Notes: (a) This column shows the ratio of LIS obtained when wages are equal across sectors.
(b) This column shows the ratio of LIS obtained when wages are not equal across sectors.

Table 2. Structural change in the US economy. Period 1947-2010.

Period	Relative Wage (λ)	$\frac{LIS_a}{LIS_n}$	Ratio from employment to GDP shares in Non-Agriculture		
			Data	If $LIS_a = LIS_n$ $^{(a)}$	If $w_a = w_n$ $^{(b)}$
1947-1960	0.608	1.064	0.955	0.958	0.993
1960-1980	0.673	0.934	0.986	0.985	1.003
1980-2000	0.668	0.822	0.996	0.993	1.005
2000-2010	0.728	0.939	0.997	0.996	1.001

Source: US KLEMS 2013.

Notes: (a) The simulated value of Ω when sectors exhibit the same labor income share.
(b) The simulated value of Ω when wages are equal across sectors $(\lambda = 1)$.

Table 3. Parameter values.

Parameters	Values	Targets
ρ	0.032	Long-run interest rate is 5.2%
θ	0.010	Long-run expenditure share in agriculture[1]
δ	0.056	Long-run ratio of investment to capital to GDP is 7.6%
α_a	0.540	Labor-income share in agriculture[2]
α_n	0.330	Labor-income share in non-agriculture[2]
γ_n	0.020	Long-run growth rate of GDP is 2%
$A_n(0)$	1	Normalization
$A_a(0)$	1.385	Ratio of sectoral productivities in 1880[3]
γ_a	0.009 before 1945	Growth of sectoral productivity ratio 1880-1945[3]
	0.0273 after 1945	Growth of sectoral productivity ratio1945-2000[3]

Notes: (1) Herrendorf, et al. (2013).
(2) Valentinyi and Herrendorf (2008).
(3) Alvarez-Cuadrado and Poschke (2011).

Table 4. Initial conditions for simulations.

	Values			Targets: Year 1880	
	z_0	$\widetilde{e}_0$	m_0	$u = 0.52$	$Y_n/Q = 0.73$
Model 1	$0.75z^*$	0.588	0	$\surd$	$\times$
Model 2	$0.75z^*$	0.279	13.27	$\surd$	$\surd$

Table 5. Performance of the simulations.

	Employment share in Agriculture			GDP share in Agriculture		
	SSR	U-Theil	R^2	SSR	U-Theil	R^2
Model 1	0.3206	0.0476	0.8699	1.6529	0.3002	−1.0885
Model 2	0.2876	0.0449	0.8833	0.1154	0.1061	0.8542

Table 6. Average annual growth rate in the last 50 years.

	r	k/Q	$1-u$	pY_a/Q
Model 1	−0.12%	−0.06%	−2.15%	−2.08%
Model 2	−0.11%	−0.03%	−2.20%	−1.96%

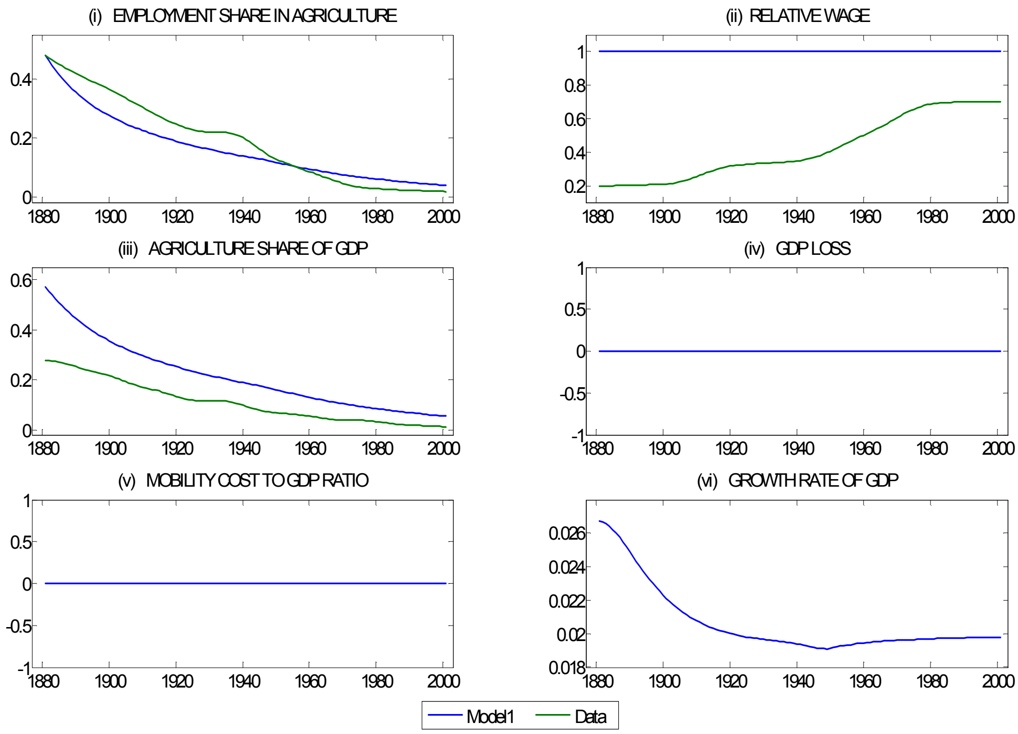

Figure 1. Numerical simulation without labor mobility cost

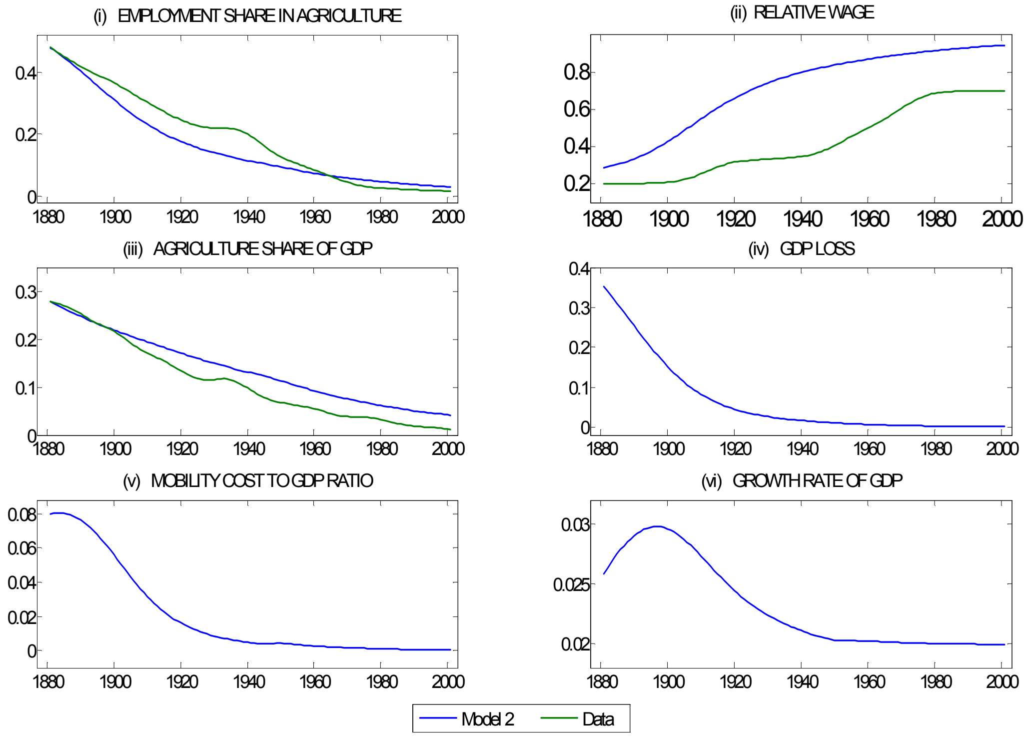

Figure 2. Numerical simulation with labor mobility cost

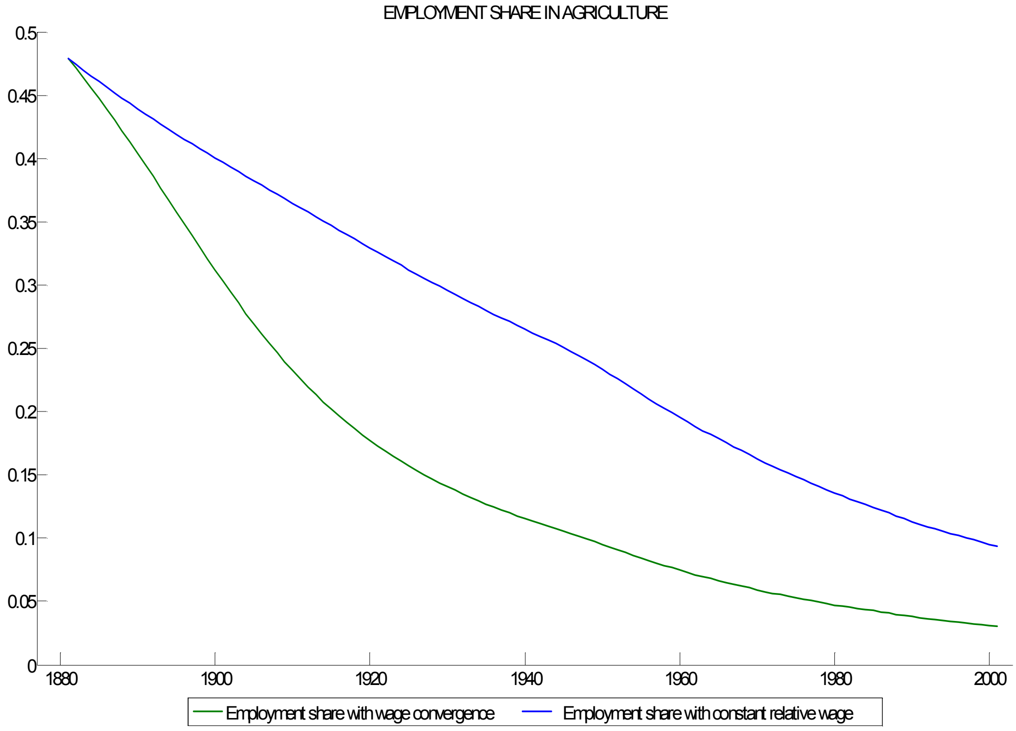

Figure 3. Demand and supply factors governing structural change

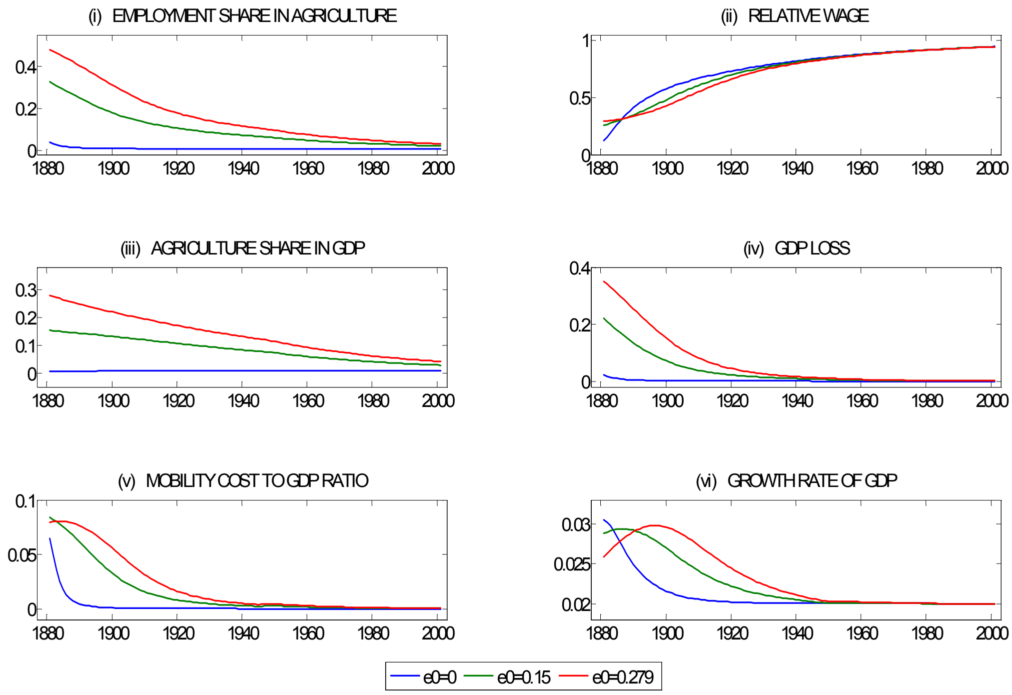

Figure 4. Economies with different initial minimum consumption intensity

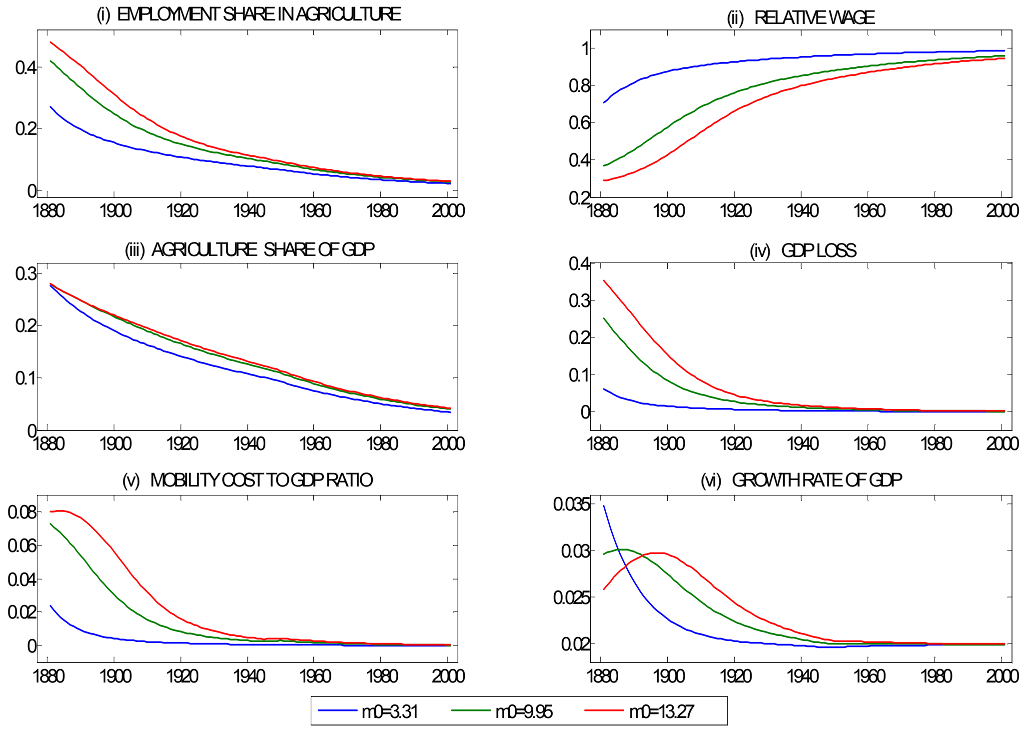

Figure 5. Economies with different labor mobility cost

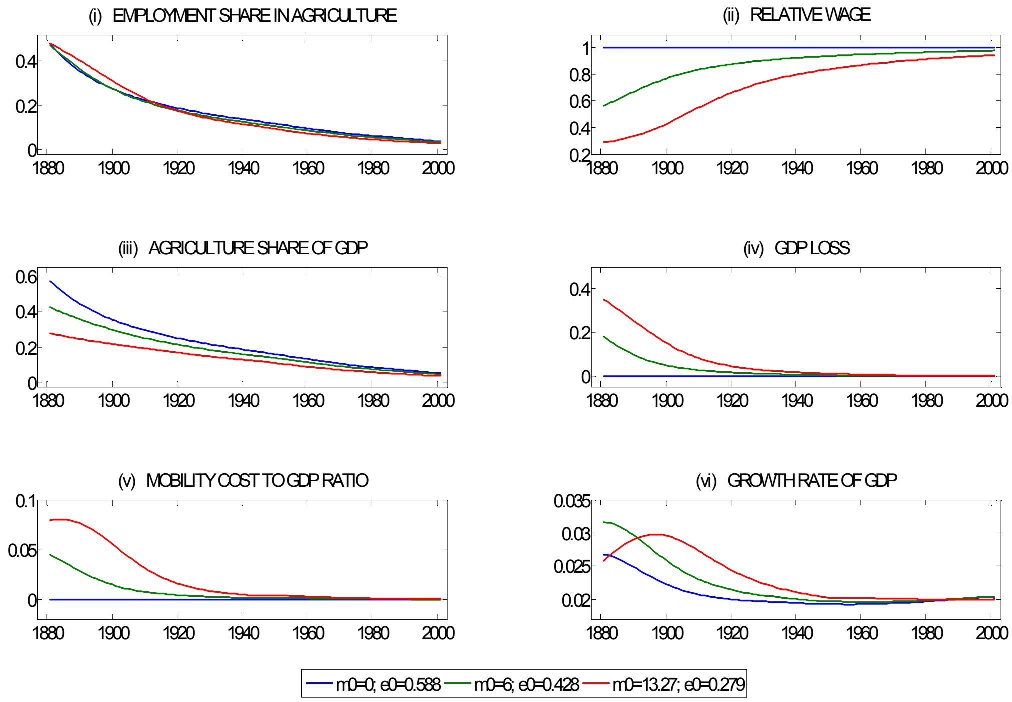

Figure 6. Economies with different labor mobility cost and minimum consumption requirements.

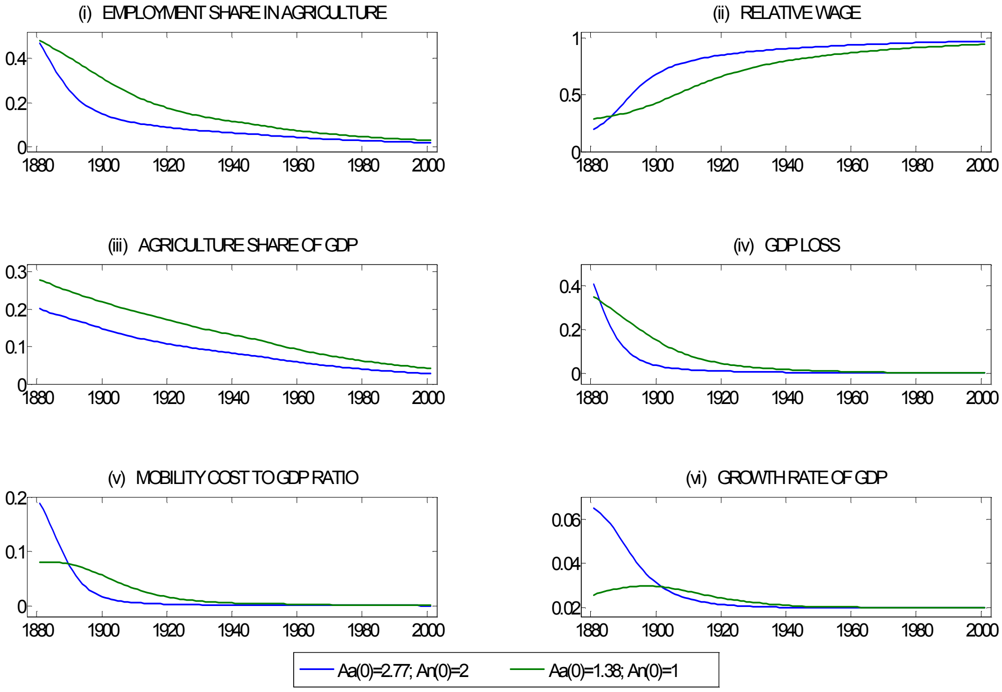

Figure 7 Economies with different initial technological levels.

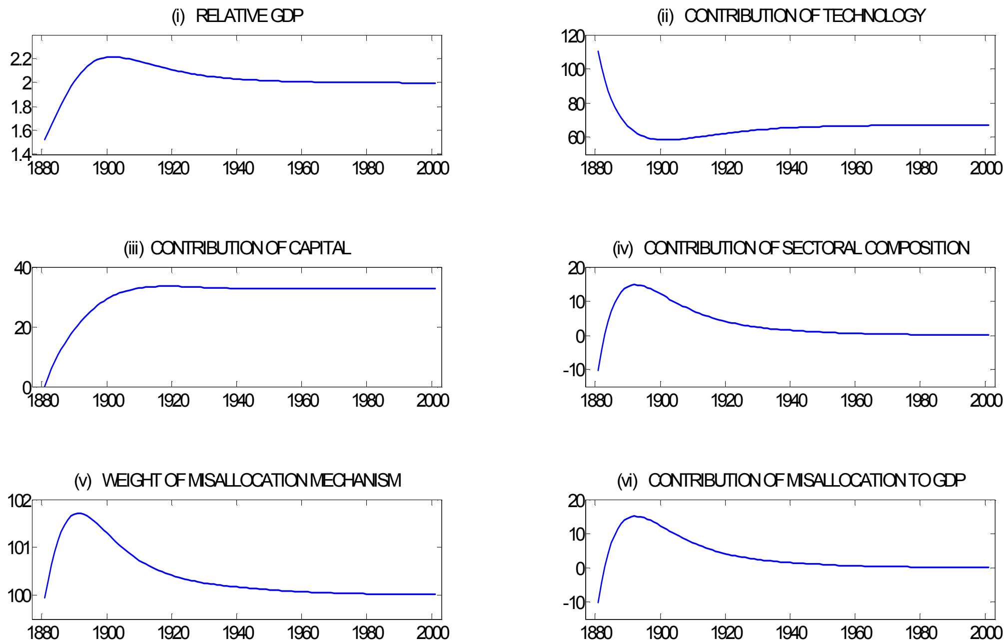

Figure 8. Development accounting between two economies with different technology.