ECOBAS Working Papers

2017 - 04

Title:

Capital accumulation when consumers are tempted by others consumption experience

Authors:

Jaime Alonso-Carrera
Universidade de Vigo

Stéphane Bouché
Universidade de Vigo

Economics and Business Administration for Society

www.ecobas.es

Capital accumulation when consumers are tempted by others' consumption experience*

Jaime Alonso-Carrera [a]
Departamento de Fundamentos del Análisis Económico
Universidade de Vigo

Stéphane Bouché [b]
ECOBAS
Universidade de Vigo

September 28, 2017

Abstract

This paper analyzes how temptations and costly self-control influence consumer's decisions on savings and the accumulation of wealth along the life-cycle. We consider an overlapping generations model where individuals are tempted to take the average consumption of agents living in the same period as their own aspiration or consumption reference. In addition, consumers also exhibit a preference for self-control. Therefore, they face a self-control problem and the degree of this problem is endogenously determined by the aggregate allocation of resources. We show that temptation and costly self-control may either increase or decrease the accumulation of capital. The crucial point would be whether or not consumers take the consumption of the individuals belonging to the other living generations as a determinant of their consumption reference. This point also affects the stability and welfare properties of the competitive equilibrium. In particular, we obtain that multiple equilibrium paths may exist and that consumption externalities may lead the capital stock to be either suboptimally small or large.

JEL classification codes: D91, E13, E21.

Keywords: temptation, self-control, consumption externalities, capital accumulation, overlapping generations.

---

*Financial support from the Government of Spain and FEDER through grant ECO2015-68367-R (MINECO/FEDER) is gratefully acknowledged. This paper has benefited from comments by X. Raurich, R. Dos Santos Ferrerira and by participants in the workshop "Economic Growth, Macroeconomic Dynamics and Agents' Heterogeneity" (European University at St. Petersburg) and the conference "Real and Financial Interdependencies: New Approaches with Dynamical General Equilibrium Model" (Paris School of Economics).

[a] Postal address: Universidade de Vigo, Facultad de Ciencias Económicas y Empresariales, Campus As Lagoas-Marcosende, 36310 Vigo (Spain). Phone: (34)986813516. E-mail: jalonso@uvigo.es

[b] Postal address: Universidade de Vigo, Facultad de Ciencias Económicas y Empresariales, Campus As Lagoas-Marcosende, 36310 Vigo (Spain). Phone: (34)986812524. E-mail: sbouche@uvigo.es

1. Introduction

In this paper we aim at analyzing how temptation and costly self-control influence consumer's decisions on savings and the accumulation of wealth along the life-cycle. We use an overlapping generations model where individuals are tempted to satisfy some aspirations in consumption. We specifically consider that consumers are tempted to use a reference level of consumption to compare the utility derived from their own consumption. We will assume that this reference consumption is a weighted average of the consumption of all agents living in the same period. However, consumers also exhibit a preference for self-control, so that their decisions result from accommodating two competing desires: the gratification from fully succumbing to their aspirations and the gratification derived from ex-ante ignoring their "social status" (i.e., their position in the social scale of consumption).

Several studies have investigated the empirical relevance of self-control problems by using different methodologies: (a) household-level data (see, Bucciol, 2012; and Hung et al., 2015); (b) experimental data (see, Frederick et al, 2002); and (c) survey data (see, Ameriks et al., 2007). They in general support the presence of temptation features and self-control problems. This empirical evidence has encouraged economic literature to investigate how the presence of self-control problems influences the economic decisions of individuals, as well as to study whether public authorities should intervene to alter the decentralized individuals' response to those problems.[1]

The standard macroeconomic model rests on geometric discounting of future utility. Thus, in light of the aforementioned empirical evidence, this model may be an inappropriate framework to study the intertemporal decisions on consumption. Strotz (1956) and Phelps and Pollak (1968) introduced an alternative framework with quasi-hyperbolic discounting that sets up a game between the different intertemporal selves cohabiting in an individual (i.e., a conflict between the opposite desires and aspirations that an individual may exhibit).[2] However, the preferences based on this quasi-hyperbolic discount are time-inconsistent and individuals do not have any commitment device. Gul and Pesendorfer (2001) introduced the *dynamic self-control* (DSC, henceforth) preferences: a class of preferences displaying self-control problems and preserving time consistency. The idea is that the objective function of consumers is given by the combination of two utility functions: the *commitment utility function*, which corresponds to the standard utility; and *the temptation utility*. Thus, the actual choice is a compromise between the commitment utility and the cost of self-control (i.e., how these actual decisions depart from what the temptation would dictate).

These DSC preferences then reveal to be a promising fundamental to explain some economic puzzles that cannot be easily reconcile with more traditional theories. For instance, those preferences were used for dealing with the question of whether or not individuals save enough for their retirement and, therefore, whether or not a social security program may be socially optimal (see, for instance, Kumru and Thanopoulos, 2008 and 2011; or Bucciol, 2011). A common assumption of this literature is that the commitment and the temptation utility functions only differ in the intensity of the

[1] See, for instance, Krusell et al. (2010) for a detailed discusion on this issue.

[2] Laibson (1997) uses this framework to prove that individuals with quasi-hyperbolic discount undersave.

trade-off between current and future satisfaction from consumption. These disparities in the marginal rate of substitution between consumption at different periods are driven by: (a) either different discount factors (see, e.g., Krusell and Smith, 2003; Krusell et al., 2007); or different intertemporal elasticities of substitution (see, e.g., Kumru and Thanopoulos, 2011). However, all aforementioned literature assume that these two fundamentals are dynamically constant. This has two important consequences for the predicted behavior of the corresponding economy. On the one hand, temptation and self-control problems have a monotonic effect on the accumulation of capital. Generally, this literature assumes that consumers are monotonously tempted for immediate gratification, so that it obtains that temptation and self-control problems reduce the capital stock. On the other hand, the degree of the self-control problem remains also constant along the process of economic growth. However, empirical evidence seems to contradict these two results. For instance, Ameriks et al. (2007) find that self-control problems are decreasing in scale with age and, furthermore, they lead some individuals to underconsume and others to overconsume.

In this paper, we aim to contribute to the literature on life-cycle saving and capital accumulation by endogenizing quite naturally the degree of the self-control problem.[3] We combine the literature on preference reversal with the one on consumption externalities. More specifically, we consider an overlapping generations model where individuals live for two periods. Preferences are represented by the combination of two momentary utility functions: a commitment utility function that only depends on own consumption; and a temptation utility function that depends on both own consumption and the weighted average consumption of all agents living in the same period. In evaluating the gratification from actual consumption, individuals take into account the disutility from self-control, i.e., from not fully succumbing to temptation. Consumers maximize the discounted sum of utilities from their two periods of life, where the factor used for discounting of commitment and temptation utility may be different.

As was mentioned before, the degree of the self-control problem faced by each individual in our model is then endogenous as temptation depends on the resource allocation of all agents. Apart from the exogenous difference in the discount factor between the commitment and the temptation utility functions, their intertemporal elasticities of substitution in consumption endogenously differ. While the intertemporal elasticity of substitution of the commitment utility function is constant, the temptation utility function exhibits a time-varying intertemporal elasticity because of the presence of consumption externalities.[4] We find in this framework that temptation and costly self-control may either increase or decrease the accumulation of capital as empirical evidence based on survey data suggests (see, Ameriks et al., 2007). The crucial assumption would be whether or not consumers take the consumption of the individuals belonging to the other living generation as a determinant of their consumption reference. The level of capital will be positively affected by the presence of endogenous self-control problems if at least one of the following conditions holds: (a) the discount

[3] Pavoni and Yazici (2015) assume that the individuals' ability to self-control exogenously varies with age, i.e, the degree of self-control problem is age-dependent.

[4] Fisher and Hof (2000) discuss how consumption externalities influence the intertemporal elasticity of substitution in consumption.

factor of temptation utility function is sufficiently large; or (b) individuals' aspirations depend on the consumption of the other living generation. The fact that preferences are intergenerationally dependent also affects the stability and welfare properties of the competitive equilibrium. In particular, we show that multiple equilibrium paths may exist in this case, so that the aggregate equilibrium allocation is indeterminate and displays coordination failure. In addition, we prove that consumption externalities may lead the competitive stock of capital to be either suboptimally large or small.

The paper is organized as follows. Section 2 presents the general model with the formulation of the self-control problem and interpersonally dependent preferences. We provide a detailed explanation of the novelty of this formulation with respect to the previous literature using DSC preferences. Section 3 explains how the proposed model of capital accumulation nests some well known models, which will be useful for the analysis in the rest of the paper. In Sections 4 and 5 we characterize the behavior of the capital stock at the steady-state equilibrium and along the transitional dynamics, respectively. In Section 6 we conduct the analysis of optimality by comparing the solution achieved by the social planner with the competitive solution. Section 7 contains some final comments and remarks. A final appendix contains all the proofs of the results.

2. The model

We consider an economy populated by overlapping generations of identical individuals uniformly distributed on the interval $[0, 1]$. Each of these individuals lives for two periods and has one descendant at the end of the first period of his life. Each agent supplies inelastically one unit of labor in the first period of his life and is retired in the second period. Each young individual distributes his labor income between consumption and saving. Therefore, an individual faces the following budget constraint during his first period of life:

$$c_t + s_t = w_t, \tag{2.1}$$

where w_t is the wage rate per unit of time, c_t is the amount of consumption of a young agent and s_t is the amount saved. In the second period of their life, individuals receive the returns on the amount of their saving, which is spent on consumption. Therefore, the budget constraint of an old individual is

$$d_{t+1} = R_{t+1}s_t, \tag{2.2}$$

where d_{t+1} is the amount of consumption of an old individual and R_{t+1} is the gross rate of return on saving.

Agents derive utility from the amount consumed in both periods of their life. We assume that individuals exhibit a kind of DSC preferences due to Gul and Pesendorfer (2001, 2004). More precisely, individuals feature a temptation to take some external aspiration as a reference with respect their own consumption will be compared to. However, they are still able to exercise some level of self-control at the psychological cost of deviating from the private welfare associated with fully falling into temptation. In particular, individual's preferences are then represented by two utility functions:

$u(c_t, d_{t+1})$ and $v(c_t, a_t, d_{t+1}, h_{t+1})$, where $u(.)$ is the commitment utility, $v(.)$ is the temptation utility, while a_t and h_{t+1} are the consumption references which tempt the individual in his first and second period of life, respectively. Resisting this temptation gives rise to a self-control utility cost given by

$$\left[\max_{\{\tilde{c}_t, \tilde{d}_{t+1}\}} v(\tilde{c}_t, a_t, \tilde{d}_{t+1}, h_{t+1})\right] - v(c_t, a_t, d_{t+1}, h_{t+1}),$$

where c_t and d_{t+1} represent the commitment choice of consumption in both periods of life, whereas $\tilde{c}_t$ and $\tilde{d}_{t+1}$ are the temptation alternatives to those values of consumption (i.e., they represent the values of consumption that an individual would choose in the absence of self-control or commitment). Observe that the cost of self-control for a given choice of c_t and d_{t+1} is the amount of temptation utility that the agent forgoes as a result of self-control. By taking the consumption references a_t and h_{t+1} as given, the representative agent then solves the following problem:

$$\max_{\{c_t, d_{t+1}\}} [u(c_t, d_{t+1}) + v(c_t, a_t, d_{t+1}, h_{t+1})] - \max_{\{\tilde{c}_t, \tilde{d}_{t+1}\}} v(\tilde{c}_t, a_t, \tilde{d}_{t+1}, h_{t+1}), \tag{2.3}$$

subject to the budget constraints (2.1) and (2.2). The choice of the agent then maximizes the difference between the commitment utility and the self-control cost.

In most macroeconomic models using DSC preferences, the only difference between commitment and temptation utility is that the individual discount factor is lower in the latter. In this case, we would easily obtain that the usual time separability of the utility functions implies

$$v'_{c_t} = \varepsilon_1 u'_{c_t}, \tag{2.4}$$

and

$$v'_{d_{t+1}} = \varepsilon_2 u'_{d_{t+1}}, \tag{2.5}$$

where ε_1 and ε_2 are some particular constants, while v'_{c_t} and u'_{c_t} represent the marginal utilities for temptation and commitment, respectively, and with $\varepsilon_1 > \varepsilon_2$. Our model is thus more general since our utility function $v(.)$ also depends on consumption references a_t and h_{t+1} making the temptation effect endogenous, so that the two previous conditions (2.4) and (2.5) do not necessarily hold. In order to formalize this feature, we derive the first order conditions of the representative consumer's problem at period t. Thus, we obtain

$$\begin{aligned} u'_{c_t}(1 + v'_{c_t}/u'_{c_t}) &= \nu_t, \\ u'_{d_{t+1}}(1 + v'_{d_{t+1}}/u'_{d_{t+1}}) &= \frac{\nu_t}{R_{t+1}}, \end{aligned}$$

where ν_t is the multiplier of the intertemporal budget constraint

$$c_t + \frac{d_{t+1}}{R_{t+1}} = w_t.$$

Combining both conditions we obtain:

$$\frac{u'_{c_t}}{u'_{d_{t+1}}} = R_{t+1} \underbrace{\left(\frac{1 + v'_{d_{t+1}}/u'_{d_{t+1}}}{1 + v'_{c_t}/u'_{c_t}} \right)}_{\widehat{T}(c_t, a_t, d_{t+1}, h_{t+1})}. \tag{2.6}$$

The function $\widehat{T}(.)$, which we label as the *temptation function*, measures how the temptation feature of preferences determines the intertemporal allocation of consumption. Note that this temptation function is equal to one in the neoclassical case (i.e., in the absence of temptation) because $v'_{c_t} = v'_{d_{t+1}} = 0$ in this case. However, if the individuals are tempted to follow the external consumption references, then the function $\widehat{T}(.)$ can be larger than, smaller than or equal to unity and, moreover, this value can endogenously depend on the state of the economy represented by the stock of capital. When $\widehat{T}(.) > 1$, for a given rate of interest, the intertemporal marginal rate of substitution must increase in order to satisfy our equilibrium condition. The agent thus decides to increase future consumption at the expense of present consumption. When $\widehat{T}(.) < 1$, the opposite reasoning applies implying that the marginal rate of substitution must decrease while present consumption increases. In a standard temptation model where the only difference between both utilities is the subjective discount factor, $\widehat{T}(.)$ is a constant lower than one because Conditions (2.4) and (2.5) hold: $\widehat{T} = (1+\varepsilon_2)\,/\,(1+\varepsilon_1)$. In this case the agent is then induced to increase present consumption. One of the main contributions of the paper is to endogenize temptation through the introduction of consumption externalities. They introduce differences between the intertemporal elasticity of substitution of the commitment utility function $u\,(.)$ and the one of the temptation utility $v\,(.)$. Furthermore, this gap is time-varying because of the endogenous nature of the intertemporal elasticity of substitution of the temptation utility.

In order to proceed we will now consider specific functional forms for both utilities. We assume that the period utility functions $u(.)$ and $v(.)$ are of the CRRA type:

$$u(c_t, d_{t+1}) = \frac{c_t^{1-\sigma}}{1-\sigma} + \delta \frac{d_{t+1}^{1-\sigma}}{1-\sigma}, \tag{2.7}$$

and

$$v(c_t, a_t, d_{t+1}, h_{t+1}) = \psi \left[\frac{(c_t - \gamma_1 a_t)^{1-\sigma}}{1-\sigma} + \delta\beta \frac{(d_{t+1} - \gamma_2 h_{t+1})^{1-\sigma}}{1-\sigma} \right], \tag{2.8}$$

where $\delta > 0$ is the subjective discount factor in commitment utility; $\sigma > 0$ is the relative risk aversion coefficient; $\psi > 0$ is the weight of temptation utility (i.e., it measure the strength of temptation); $\beta < 1$ is a parameter reflecting the fact that the future is more heavily discounted in temptation utility; and $\gamma_1 \in [0, 1)$ and $\gamma_2 \in [0, 1)$ measure the intensity of consumption references in the first and second period of life, respectively. If $\psi = 0$, we obtain the neoclassical model, while if $\gamma_1 = \gamma_2 = 0$, we obtain the standard temptation model where the unique deviation between the commitment and the temptation utilities is the rate at which an individual discounts the utility of his second period of life: this rate is δ in the commitment utility and $\beta\delta$ in the temptation utility. In our framework, we can assume that $\beta > 1$ and we can still obtain $\widehat{T}(.) < 1$ because in our economy with consumption references the value of $\widehat{T}(.)$ also depends on equilibrium allocations. However, we assume $\beta < 1$ to allow for the comparison of the equilibrium allocation in the proposed economy with the allocation that would emerge in the standard temptation model (i.e., when $\gamma_1 = \gamma_2 = 0$).

The commitment and the temptation utility functions share the same coefficient of relative risk aversion σ. However, these utilities exhibit different intertemporal elasticities of substitution in consumption. While this elasticity is constant and equal to the inverse of the relative risk aversion coefficient in the case of the commitment utility function, the temptation utility function has an endogenous elasticity because of the presence of consumption externalities. Therefore, the dynamics of the capital stock in our economy crucially depend on the two following differences between the commitment and the temptation utility functions: (a) the constant gap between the discount factors; and (b) the endogenous gap between the intertemporal elasticities of substitution. As will be shown in our analysis, these two forces may operate in the opposite direction in determining the intertemporal allocation of consumption and, thus, capital accumulation.

We assume that the temptation utility does not depend on the absolute level of consumption as in the standard temptation model, but on the comparison between this level and the consumption references. In addition, we also assume that the consumption references depend on the current levels of consumption of the young and the old individuals. In particular, following Alonso-Carrera et al. (2008), these consumption references are assumed to be a weighted arithmetic average of the per capita consumption of the two living generations, which we denote by $\widehat{c}_t$ and $\widehat{d}_t$.[5] Preferences then display consumption externalities provided that they feature temptation (i.e., $\psi > 0$). On the one hand, we consider that

$$a_t = \frac{\widehat{c}_t + \theta_1 \widehat{d}_t}{1 + \theta_1}, \tag{2.9}$$

where $\theta_1 \in [0, 1]$ is the weight given to consumption of a representative old individual in the specification of the reference for the young individuals. On the other hand, we assume that

$$h_{t+1} = \frac{\theta_2 \widehat{c}_{t+1} + \widehat{d}_{t+1}}{1 + \theta_2}, \tag{2.10}$$

where $\theta_2 \in [0, 1]$ is the weight given to consumption of a representative young individual in the specification of the reference for the old individuals. Note that the restrictions imposed on the values of the parameters imply that we are giving a larger weight to the average consumption of the individuals belonging to the same generation.

With the specification of preferences given by (2.3), (2.7), (2.8), (2.9) and (2.10), we need to ensure that the indifference curves of both the commitment and the temptation utility functions, $u(.)$ and $v(.)$, be downward sloping.[6] This is always the case for commitment utility but not necessarily for temptation utility. Given a value of the consumption references a_t and h_{t+1}, the intertemporal marginal rate of substitution between consumptions at young and old ages for temptation utility is given by

$$\frac{\partial d_{t+1}}{\partial c_t} = -\frac{(c_t - \gamma_1 a_t)^{-\sigma}}{\delta\beta(d_{t+1} - \gamma_2 h_{t+1})^{-\sigma}}.$$

This expression is negative provided that effective consumption in both periods has the same sign. We rule out the possibility of negative effective consumption in order to

[5] Alternatively, Abel (2005) and Wendner (2010) propose a weighted geometric average of consumption of the two living generations. We conjecture that this is not relevant for the qualitative results derived in this paper.

[6] See Lahiri and Puhakka (1998) and Wendner (2002) for a detailed explantion of this requirement.

avoid an ill-defined temptation utility function. By using expressions (2.9) and (2.10), it is possible to derive the following necessary and sufficient conditions which ensure that effective consumption is positive in the first and the second period of life along a symmetric equilibrium with $c_t = \widehat{c}_t$ and $d_t = \widehat{d}_t$, respectively:

$$\frac{\gamma_1\theta_1}{1+\theta_1-\gamma_1} < \frac{c_t}{d_t} < \frac{1+\theta_2-\gamma_2}{\gamma_2\theta_2}. \tag{2.11}$$

From now on, we assume that these conditions are always satisfied. Note that they restrict the domain of the initial capital stock (which is the state variable in our economy) to have a well defined equilibrium path. The strong non-linearity of the policy function relating the equilibrium values of consumptions c_t and d_t with the capital stock k_t does not allow us to derive the explicit value of this threshold for the initial capital stock. In any case, the left-hand side of the first inequality is zero when $\theta_1 = 0$, so that this inequality always hold in this case. In addition, the second inequality always holds when $\theta_2 = 0$ because in this case the right-hand side converges to infinite.

By particularizing Condition (2.6) with our parametrization of preferences, we obtain that the optimal plan of consumers is characterized by the following condition:

$$\frac{c_t^{-\sigma}}{\delta d_{t+1}^{-\sigma}} = R_{t+1}\widehat{T}(c_t, a_t, d_{t+1}, h_{t+1}), \tag{2.12}$$

where

$$\widehat{T}(c_t, a_t, d_{t+1}, h_{t+1}) = \frac{1+\beta\psi(1-\gamma_2 h_{t+1}/d_{t+1})^{-\sigma}}{1+\psi(1-\gamma_1 a_t/c_t)^{-\sigma}}, \tag{2.13}$$

is our parametric form of the temptation function $\widehat{T}(.)$ in (2.6). Observe that the temptation function $\widehat{T}(.)$ is constant and equal to $(1+\beta\psi)/(1+\psi) < 1$ when $\gamma_1 = \gamma_2 = 0$ (i.e., in the case of standard temptation where commitment and temptation utility functions only differ in the subjective discount factor). Otherwise, this temptation function is endogenously determined by the equilibrium values of consumptions at the first and second periods of life. In fact, we can rewrite the temptation utility function (2.8) in equivalent terms as the one used by the standard temptation case. More precisely, we can express (2.8) as

$$v(c_t, a_t, d_{t+1}, h_{t+1}) = \tilde{\psi}_t\left[\frac{{c_t}^{1-\sigma}}{1-\sigma} + \delta\tilde{\beta}_{t+1}\frac{d_{t+1}^{1-\sigma}}{1-\sigma}\right],$$

with

$$\tilde{\psi}_t = \psi(1-\gamma_1 a_t/c_t)^{1-\sigma}, \tag{2.14}$$

and

$$\tilde{\beta}_{t+1} = \beta\left(\frac{1-\gamma_2 h_{t+1}/d_{t+1}}{1-\gamma_1 a_t/c_t}\right)^{1-\sigma}. \tag{2.15}$$

Observe that this expression of the temptation utility function only differs with the commitment one (2.7) because of the presence of the variables $\tilde{\psi}_t$ and $\tilde{\beta}_{t+1}$. We can then assert that the self-control problem in our formulation of preferences is also fully driven by the intensity of temptation $\tilde{\psi}_t$ and by the difference in the way of discounting the future in the commitment utility function (β) and in the temptation utility function ($\tilde{\beta}_{t+1}$). Our temptation formulation of preferences based on interpersonal comparisons of consumption endogenizes the intensity of temptation and the discount factor. Therefore, we naturally obtain a time varying (and endogenous) self-control feature without considering any ad-hoc structural variation in the parameters defining preferences as, for instance, in Pavoni and Yazici (2015). This endogeneity results from the balance between two forces determining the time-bias of the self-control problem: (a) the differences in the discount factors of the two utility functions; and (b) the differences in the intertemporal elasticities of substitution because of the presence of consumption externalities in the temptation utility function. Contrary to the standard model of temptation, even when $\beta = 1$, temptation and and self-control problems play a crucial role in the accumulation of capital because of consumption externalities.

We close the presentation of the model's fundamentals by showing the features of the production-side of the economy. There is a single commodity y_t that can be used for either consumption or investment. We assume that this good is produced by means of the Cobb-Douglas production function

$$y_t = Ak_t^{\alpha}, \tag{2.16}$$

where A is the constant total factor productivity, k_t is the stock of capital and $\alpha \in (0,1)$ is the share of capital income in output. We also suppose full depreciation of the capital stock after one period.[7] Perfect competition among firms leads the rental prices of the two inputs, capital and labor, to equal their marginal productivities, i.e.,

$$R_t = \alpha Ak_t^{\alpha-1}, \tag{2.17}$$

and

$$w_t = (1-\alpha)Ak_t^{\alpha}. \tag{2.18}$$

By imposing the consistency equilibrium conditions $c_t = \widehat{c}_t$ and $d_t = \widehat{d}_t$, we define the *competitive equilibrium* of this economy as a path $\{k_t\}_{t=0}^{\infty}$ that, for a given initial value of the capital stock k_0 satisfying Condition (2.11), solves the difference equation (2.12), together with (2.1), (2.2), (2.9), (2.10), (2.17), (2.18), the market clearing conditions for capital markets

$$k_{t+1} = s_t, \tag{2.19}$$

(i.e., the capital stock installed in period $t+1$ is equal to the aggregate saving in period t) and the aggregate resource constraint

$$y_t = c_t + d_t + k_{t+1}, \tag{2.20}$$

(i.e., aggregate output in period t is distributed between aggregate consumption of young individuals, consumption of old individuals and investment). By combining these equilibrium conditions, we obtain that, given a initial capital stock k_0, the equilibrium paths are fully defined by the following second-order difference equation in capital:

$$G(k_t, k_{t+1}) = \delta(\alpha A)^{1-\sigma}T(k_t, k_{t+1}, k_{t+2}), \tag{2.21}$$

[7] One can assert that one period of the model consists on about 30 years for the empirical evaluation. Hence, full depreciation of capital stock is not an unrealistic assumption.

where

$$G\left(k_t, k_{t+1}\right) = \frac{k_{t+1}^{1-\alpha(1-\sigma)}}{\left[\left(1-\alpha\right) A k_t^{\alpha} - k_{t+1}\right]^{\sigma}}, \tag{2.22}$$

and $T(.)$ is the temptation function (2.13) at the equilibrium, i.e.,

$$T(k_t, k_{t+1}, k_{t+2}) = \frac{1 + \beta\psi(1+\theta_2)^{\sigma}\left\{1 + \theta_2 - \gamma_2 - \frac{\gamma_2\theta_2\left[(1-\alpha)Ak_{t+1}^{\alpha} - k_{t+2}\right]}{\alpha A k_{t+1}^{\alpha}}\right\}^{-\sigma}}{1 + \psi(1+\theta_1)^{\sigma}\left\{1 + \theta_1 - \gamma_1 - \frac{\gamma_1\theta_1\alpha A k_t^{\alpha}}{\left[(1-\alpha)Ak_t^{\alpha} - k_{t+1}\right]}\right\}^{-\sigma}}. \tag{2.23}$$

Observe that Equation (2.21) reduces to a first-order difference equation if either $\gamma_2 = 0$ or $\theta_2 = 0$. Therefore, a second-order difference equation arises if the old individuals use the average consumption of the young individuals as a reference (i.e, $\theta_2 > 0$). Finally, we also directly conclude that the variables $\tilde{\psi}_t$ and $\tilde{\beta}_{t+1}$, which determine the degree of the self-control problem, and which were defined in (2.14) and (2.15), are endogenously determined by the dynamic evolution of the capital stock. Obviously, this only happens when preferences are intergenerationally dependent: individuals take the consumption experience of the individuals belonging to the other living generation as a consumption reference, i.e., either $\theta_1 > 0$ or $\theta_2 > 0$. Otherwise, the degree of the self-control problem is exogenous, although it still depend on the intensity at which individuals compare their consumption experience with those of the individuals belonging to their own generation (i.e., γ_1 and γ_2).

3. Some special cases

Our model with self-control and interpersonally dependent preferences nests some other models that have largely been studied by the literature on capital and wealth accumulation. This property is very useful to know how these proposed preferences affect the dynamic behavior of capital and wealth. Thus we next present how our model particularizes in these well known models.

Standard neoclassical model. We obtain this particular model if we eliminate the temptation feature from our model, i.e., we impose $\psi = 0$. In this way preferences are only characterized by the commitment utility function. In this case, our model then simply reduces to the standard overlapping generations model proposed by Diamond (1965). From now on, we will denote the capital stock of this model without self-control problems by k_t^n. Therefore, the equilibrium path of this particular economy is given by the path $\{k_t^n\}_{t=0}^{\infty}$ that solves the dynamic equation (2.21) with the temptation function $T(k_t, k_{t+1}, k_{t+2}) = 1$.

Standard temptation model. We eliminate the interpersonal dependence feature from our model, i.e., we impose $\gamma_1 = \gamma_2 = 0$. In this case, preferences are still characterized by the interaction between the commitment and the temptation utility functions. However, the self-control problem only arises because the subjective discount factors in both utility functions are different: this factor is δ for the commitment utility function, whereas it is $\delta\beta$ for temptation utility function. The intertemporal elasticities of substitution are the inverse of the relative risk aversion coefficient σ in both utility functions. In this case, our model reduces to the OLG version of the self-control model introduced by the seminal papers of Gul and Pesendorfer (2001, 2004) and Krussell et al. (2010). The degree of the self-control problem in this standard temptation model is exogenous (and constant). Effectively, we directly obtain from (2.14) and (2.15) that $\tilde{\psi}_t = \psi$ and $\tilde{\beta}_{t+1} = \beta$ for all generations born at period t.[8] From now on, we will denote the capital stock of this standard temptation model by k_t^s. Therefore, the equilibrium path of this particular economy is given by the path $\{k_t^s\}_{t=0}^{\infty}$ that solves the dynamic equation (2.21) with the temptation function $T(k_t, k_{t+1}, k_{t+2}) = (1 + \beta\psi) / (1 + \psi) < 1$.

Model with consumption externalities. Consider now that preferences are only characterized by the temptation utility function. This happens in our economy when ψ tends to infinite. This particular economy then corresponds to an economy where the individual fully succumbs to temptation. Hence, the intertemporal consumption plan chosen by each young consumer in this case is given by the solution to the following problem:

$$\max_{\{c_t, d_{t+1}\}} v(c_t, a_t, d_{t+1}, h_{t+1}),$$

subject to the budget constraints (2.1) and (2.2), with the consumption references (2.9) and (2.10) taken exogenously, and where the utility function $v(.)$ is given by (2.8).[9] Observe that this particular model is therefore fully equivalent to the standard model with consumption externalities largely studied by the literature. In fact, our model reduces in this case to the model without self-control problems but with consumption externalities analyzed by Abel (2005) and Alonso-Carrera et al. (2008).[10] From now on, we will denote the capital stock of this model with consumption externalities (i.e., with the absence of commitment or without any self-control device) by k_t^T. Therefore, the equilibrium path of this particular economy is given by the path $\left\{k_t^T\right\}_{t=0}^{\infty}$ that solves the dynamic equation

$$G\left(k_t, k_{t+1}\right) = \beta\delta(\alpha A)^{1-\sigma} T(k_t, k_{t+1}, k_{t+2}), \tag{3.1}$$

where

$$T(k_t, k_{t+1}, k_{t+2}) = \frac{(1+\theta_2)^{\sigma}\left\{1 + \theta_2 - \gamma_2 - \frac{\gamma_2\theta_2\left[(1-\alpha)Ak_{t+1}^{\alpha} - k_{t+2}\right]}{\alpha A k_{t+1}^{\alpha}}\right\}^{-\sigma}}{(1+\theta_1)^{\sigma}\left\{1 + \theta_1 - \gamma_1 - \frac{\gamma_1\theta_1\alpha A k_t^{\alpha}}{(1-\alpha)Ak_t^{\alpha} - k_{t+1}}\right\}^{-\sigma}},$$

which is derived from solving the representative consumers' problem and imposing the market clearing conditions (2.19) and (2.20), as well as the consistency equilibrium conditions $c_t = \widehat{c}_t$ and $d_t = \widehat{d}_t$. Obviously, this equilibrium allocation is equal to the hypothetical temptation allocation $\left\{\widetilde{k}_t, \widetilde{c}_t, \widetilde{d}_t\right\}_{t=0}^{\infty}$ of our model with the general

[8] Note that in this case the constants ε_1 and ε_2 in (2.4) and (2.5), respectively, reduce to $\varepsilon_1 = \psi$ and $\varepsilon_2 = \psi\beta$.

[9] Obviously, in this case there is no self-control cost because of the absence of commitment utility function.

[10] Alonso-Carrera et al. (2008) basically combines the model with consumption externalities by Abel (2005) and the model with dynastic altruism due to Barro (1974).

specification of self-control, interpersonally dependent preferences. Note finally that the dynamic equation (3.1) differs from the dynamic equation (2.21) of our general model in: (a) the discount factor, which is now $\delta\beta$ instead of δ; and (b) the function $T(.)$, although we should not properly label it as temptation function.

In order to analyze how endogenous temptation affects capital and wealth accumulation, we will next compare the equilibrium paths of our model with those corresponding to the previous special cases, mainly the standard neoclassical model.

4. Long-run effects of temptations

In this section we analyze how the self-control problems affect the stationary level of the capital stock. For that purpose, we first characterize the existence properties of the steady-state equilibrium. The steady state of this economy is characterized by the fixed points of the dynamic equation (2.21). Let us denote the stationary value of the capital stock by k. To proceed with the study of the existence of this fixed point, we decompose Equation (2.21) into the following two functions:

$$\widetilde{G}(k) = \frac{G\left(k,k\right)}{\delta(\alpha A)^{1-\sigma}} \equiv \frac{k^{1-\alpha(1-\sigma)}}{\delta(\alpha A)^{1-\sigma}\left(w-k\right)^{\sigma}}, \tag{4.1}$$

and

$$\widetilde{T}(k) = T\left(k,k,k\right) \equiv \frac{1+\beta\psi\left[1+\theta_2-\gamma_2-\frac{\gamma_2\theta_2(1-\alpha)(w-k)}{\alpha w}\right]^{-\sigma}(1+\theta_2)^{\sigma}}{1+\psi\left[1+\theta_1-\gamma_1-\frac{\gamma_1\theta_1\alpha w}{(1-\alpha)(w-k)}\right]^{-\sigma}(1+\theta_1)^{\sigma}}, \tag{4.2}$$

where w represents the stationary value of the wage rate, i.e., $w=(1-\alpha)\,Ak^{\alpha}$. Note that function (4.2) corresponds to our temptation function (2.23) evaluated at the state state, i.e., $\widetilde{T}\left(k\right)=T\left(k,k,k\right)$. We should also constrain the domain of the capital stock to ensure that the stationary value of the effective consumptions (i.e., the arguments of the temptarion function) are positive. We find that the stationary stock of capital has to satisfy three restrictions. Firstly, we have to impose that $w-k>0$ to insure that real consumption at young age is positive. Thus, we obtain the following condition:

$$k < \overline{k}_1 = \left[(1-\alpha)\,A\right]^{\frac{1}{1-\alpha}}. \tag{4.3}$$

Secondly, we also have to impose that $1-\gamma_1 a/c>0$, which happens when

$$k < \overline{k}_2 = \left[(1-\alpha)\,A - \frac{\alpha\gamma_1\theta_1 A}{1+\theta_1-\gamma_1}\right]^{\frac{1}{1-\alpha}}. \tag{4.4}$$

Observe that $\overline{k}_1 > \overline{k}_2$ unless $\gamma_1\theta_1=0$ in which case they are equal. Finally, we also have to impose that $1-\gamma_2 h/d>0$, which happens when

$$k > \overline{k}_3 = \left[(1-\alpha)\,A - \frac{\alpha\left(1+\theta_2-\gamma_2\right)A}{\gamma_2\theta_2}\right]^{\frac{1}{1-\alpha}}. \tag{4.5}$$

Note that $\overline{k}_3$ can be either positive or negative depending on the parameter values. If $\overline{k}_3<0$, then the stationary stock of capital has to satisfy $k>0$. In fact, $\overline{k}_3$ converges to minus infinite when $\gamma_1\theta_1$ tends to zero. Condition (2.11) guarantees that $\overline{k}_2>\overline{k}_3$.

Proposition 4.1. *In this economy, there is always a unique steady-state equilibrium with $k > 0$.*

Given that there is always a unique steady-state in our temptation economy, it will be easier to make comparative static analysis and a steady-state comparison with the standard neoclassical model. It should be noticed that since $\widetilde{T}'(k) < 0$, the following inequality always applies: $\min\left\{\widetilde{T}(0), \widetilde{T}(\overline{k}_3)\right\} > \max\left\{\widetilde{T}(\overline{k}_1), \widetilde{T}(\overline{k}_2)\right\}$.

Proposition 4.2. *The stationary value of the capital stock k increases with β, γ_2 and θ_2, wheras it decreases with γ_1 and θ_1. However, the response of k to changes in ψ is ambiguous.*

We must remark that the starionary value of the capital stock can either increase or decrease with the parameter ψ driving the intensity of the temptation and self-control problem. This response will be crucial for the influence of these phenomena on the equilibrium dynamics of the economy. We will illustrate this point in the next section.

Finally, we now compare the stationary levels of capital in our model of temptation with the standard neoclassical one. The first thing to notice is that since $\widetilde{G}'(k) > 0$, when $\widetilde{T}(k) > 1$, the steady-state capital stock of our economy k is higher than the one of the neoclassical economy k^n. In this case, the agent wishes to consume more in the future and tends to accumulate more capital. On the contrary, when $\widetilde{T}(k) < 1$, the neoclassical economy delivers a higher steady-state capital stock. It should be noticed that in the standard model of temptation where the only difference between commitment and temptation utility is the discount factor (i.e., $\gamma_1 = \gamma_2 = 0$) $\widetilde{T}(k) < 1$ and the neoclassical economy accumulates more capital. This discussion is summarized in the following proposition.

Proposition 4.3. *At the steady-state equilibriun, the capital stock k of our economy with temptation and self-control problems is larger than the capital stock k^n of the standard neoclassical economy if and only if $\widetilde{T}(k^n) > 1$.*

Since k^n is implicitly given by the equation $\widetilde{G}\left(k^n\right) = 1$, we cannot derive a general condition on parameters driving the comparison between the steady-state of our model with temptation and self-control problems and the standard neoclassical model. The next result provides this condition for some particular cases.

Proposition 4.4. *The following statements hold:*

(a) When $\theta_1 = \theta_2 = 0$, then $k > k^n$ if and only if

$$\beta^{1/\sigma} > \frac{1-\gamma_2}{1-\gamma_1}.$$

(b) When $\theta_1 > 0$ and $\theta_2 = 0$, a sufficient condition for $k < k^n$ is

$$\beta^{1/\sigma} < \frac{\left(1+\theta_1\right)\left(1-\gamma_2\right)}{1+\theta_1-\gamma_1-\frac{\gamma_1\theta_1\alpha}{(1-\alpha)}}.$$

(c) When $\theta_1 = 0$ and $\theta_2 > 0$, a sufficient condition for $k > k^n$ is:

$$\beta^{1/\sigma} > \frac{1 + \theta_2 - \gamma_2}{(1 - \gamma_1)(1 - \theta_2)}.$$

We can also derive from Proposition 4.3 the stationary property of the capital stock k^s of the standard temptation model, where the self-control problem arises from the fact that consumers are only tempted toward immediate gratification (i.e., $\beta < 1$). The next result states this property.

Corollary 4.5. *If $\gamma_1 = \gamma_2 = 0$, then $k = k^s < k^n$.*

The previous three results illustrate the importance of temptation and self-control problems based on interpersonal comparison of consumption concerning the steady-state equilibrium. The main conclusion is that the introduction of consumption externalities in the temptation utility function $v(.)$ may reverse the result of the standard temptation model where the presence of $\beta < 1$ implies that the stationary capital stock is always smaller than the one of the neoclassical model. Effectively, Corollary 4.5 remarks the crucial role of γ_1 and γ_2 for the consumption-saving decision. However, the two kind of consumption externalities driving the temptation have an opposite effect. On the one hand, the presence of consumption externalities in the temptation of young individuals (i.e., $\gamma_1 > 0$) encourages their consumption. On the contrary, the presence of consumption externalities in the temptation of old individuals (i.e., $\gamma_2 > 0$) encourages young individuals to save in order to finance the competition for social status when old.

The results in this section also suggest that whether or not individuals consider the consumption of the other living generation as a determinant of temptation has a second order effect on the value of the stationary capital stock. Effectively, both θ_1 and θ_2 can either reinforce or partially compensate the aforementioned effects of γ_1 and γ_2. When $\theta_1 > 0$, the young agent compares his current consumption level with the one of other young agents as well as with the one of older agents. This fact encourages capital accumulation if the consumption externality in the first period of life decreases with θ_1. Similarly, we can explain the effect of θ_2 on the stationary value of the capital stock. When $\theta_2 > 0$, old individuals compare their current level of consumption with the one of other old agents as well as with the one of younger agents. This fact encourages capital accumulation if the second period consumption externality in the first period of life increases with θ_2.

In any case, the presence of temptation based on the intergenerational comparison of consumption is very important for the behavior of the capital stock along the transitional dynamics. Observe that in this case the temptation function (2.23) is endogenous. In particular, we observe from the proof of Proposition 4.1 that the previous function is decreasing in capital, so that the degree of temptation also decreases when individuals accumulate capital. As will be shown in the next section, this will be crucial to the dynamic properties of the model.

5. Comparative dynamic analysis of temptations

In this section we study how temptation and self-control problems affect the equilibrium dynamics of the proposed model when the initial capital stock differs from its stationary value. To this end, we should first characterize the stability properties of this equilibrium. The fact that individuals are tempted by the average consumption of the economy might have a significant influence on the stability properties of the steady-state equilibrium. In fact, we cannot exclude the steady-state equilibrium to be locally indeterminate. We next discuss the logical behind this stability property. This is a clear consequence of the second-order nature of the dynamic equation (2.21) defining the equilibrium path in this case. To this end, we define $\lambda = \{A, \psi, \sigma, \alpha, \delta, \beta, \gamma_1, \gamma_2, \theta_1, \theta_1\}$ and $\lambda \in \Lambda = R_{++} \times R_+^2 \times (0,1) \times [0,1]^6$. We also divide Λ in several subsets by using the following conditions:

$$G'_{k_{t+1}} - \delta (\alpha A)^{1-\sigma} T'_{k_t+1} > 0, \tag{5.1}$$

$$T'_{k_{t+2}} < \frac{G'_{k_t} - G'_{k_{t+1}} + \delta (\alpha A)^{1-\sigma} \left(T'_{k_{t+1}} - T'_{k_t} \right)}{\delta (\alpha A)^{1-\sigma}}, \tag{5.2}$$

and

$$T'_{k_{t+2}} < \frac{G'_{k_{t+1}} + G'_{k_t} - \delta (\alpha A)^{1-\sigma} \left(T'_{k_{t+1}} + T'_{k_t} \right)}{\delta (\alpha A)^{1-\sigma}}, \tag{5.3}$$

where $T'_{k_{t+2}}$, T'_{k_t+1} and T'_{k_t} are the derivatives of the temptation function (2.23) with respect to k_{t+2}, k_{t+1} and k_t at the steady state, repectively; and G'_{k_t} and $G'_{k_{t+1}}$ are the derivatives of the function (2.22) with respect to k_t and k_{t+1} at the steady state, respectively. Hence, we consider the following subsets of Λ :

- $\Lambda_1 = \{\lambda \in \Lambda \,|\text{Conditions (5.1) and (5.2) hold}\}$,
- $\Lambda_2 = \{\lambda \in \Lambda \,|\text{Conditions (5.1) and (5.3) hold}\}$,
- $\Lambda_3 = \{\lambda \in \Lambda \,|\text{Condition (5.1) does not hold and Condition (5.3) hold}\}$,
- $\Lambda_4 = \{\lambda \in \Lambda \,|\text{Condition (5.1) does not hold and Condition (5.2) hold}\}$,
- $\Lambda_5 = \Lambda \diagdown \bigcup_{i=1}^4 \Lambda_i$.

Proposition 5.1. *The stability properties of the steady-state equilibrium are the following:*

(a) If $\lambda \in \Lambda_5$, then the equilibrium is locally saddle-path stable.

(b) If $\lambda \in \Lambda_1 \cup \Lambda_2$, then the equilibrium is locally indeterminate.

(c) If $\lambda \in \Lambda_3 \cup \Lambda_4$, then the equilibrium is locally unstable.

Given a value for k_0 multiple equilibrium paths converging to the stationary value k might then exist when old individuals are tempted by the consumption of living young individuals, i.e., when $\theta_2 > 0$. Conditions (5.2) and (5.3) do not hold when $\theta_2 = 0$. The right-hand side of these conditions are negative as can be checked by using the proof of Proposition 5.1. Hence, since the derivative $T'_{k_{t+2}}$ is equal to zero when $\theta_2 = 0$, indeterminacy does not arise in this case. The existence of multiple equilibrium paths then requires θ_2 to be sufficiently large.

Proposition 5.2. *If $\theta_2 = 0$, the steady-state equilibrium is always locally determinate, i.e., there is a unique equilibrium path converging to the steady state.*

Unfortunately, we cannot find a threshold value of θ_2 determining the existence of local indeterminacy. However, we can still numerically illustrate that this result is feasible for reasonable values of the parameters, and, moreover, we can also provide some economic intuition concerning the mechanism behind this result. To this end, we set the values of parameter as follows. We consider that each model period corresponds to 30 years. We first calibrate the corresponding neoclassical model (i.e., when $\gamma_1 = \gamma_2 = 0$) as in de la Croix and Michel (2002). We set the scale parameter $A = 6.75$ to obtain a stationary capital stock around unity in the neoclassical version of the model. Following the RBC literature, we consider $\alpha = 1/3$ and $\delta = 0.3$ to replicate the share of labor income in aggregate output and a quarterly subjective discount factor of 0.99, respectively. We consider $\sigma = 1.1$ to replicate a saving rate of 15.36%, which is in line with the evidence reported by Maddison (1992). After setting the standard parameter, we finally fix the rest of the parameters, $\{\psi, \beta, \gamma_1, \gamma_2, \theta_1, \theta_2\}$, which characterize equilibrium dynamics of our model with self-control problems. To this end, we should observe that the dynamic behavior of the capital stock crucially depends on the values of these parameters. For instance, consider the following values: $\psi = 1$, $\beta = 0.85$, $\gamma_1 = 0.5$, $\gamma_2 = 0.8$ and $\theta_1 = 0.5$. We obtain local indeterminacy in our numerical example for values of θ_2 sufficiently large. For the proposed numerical example, indeterminacy emerges for values of θ_2 larger than 0.84. This numerical result is independent of the value of θ_1, but it requires a sufficiently large value of γ_1.[11]

The mechanism underlying the existence of multiple equilibrium paths is based on the existence of strong complementarities in the saving decisions. In our model with temptation, the returns on saving have two components: (a) the standard market return given by the marginal productivity of capital; and (b) the one given by the fact that savings would allow individuals to satisfy temptation when old. Consider that the economy is in an equilibrium path and young individuals coordinate into an increase in savings. Obviously, the increase in savings can only be an equilibrium decision if their return increases. We proceed to show that consumption externalities in temptation can cause a complementarity between current savings and next period's response to temptation of old individuals. Current young individuals will increase savings if they expect that the next generation will consume a lot when young. In this case, the consumption reference when old will be large and the net returns on augmenting savings will increase even when the market return decreases. This expectation will

[11] For this numerical example, local indeterminacy emerges even with $\theta_1 = 0$ provided that γ_1 is not smaller than 0.5.

be self-fulfilling if θ_2 is sufficiently large. The current increase in savings leads to a high production in the next period, which will result in an increase in the consumption of young individuals. This translates into an increase in the consumption reference of old individuals, which will be larger the higher the value of θ_2. Observe that indeterminacy will require the consumption of future young individuals to increase a lot with the current increase in savings. Our numerical simulations suggest that this is only possible if γ_1 is sufficiently large. In this case, the weight of the consumption reference of young individuals is large and, therefore, the marginal utility of their consumption is also large.

We now proceed with the comparative dynamic analysis to determine the impact of temptation and self-control problems. To this end, we will assume from now on that θ_2 is sufficiently small and there is a unique equilibrium path converging to the steady state. However, the second order nature of the dynamic equation (2.21) governing the equilbrium dynamics still makes the proposed analysis hard. To overcome this difficulty, we will study separately different parametric cases. We first focus on the *exgonenous temptation case* where $\theta_1 = \theta_2 = 0$ which implies that the temptation function is independent of the capital stock. We then study the *endogenous first-order temptation case* where $\theta_1 > 0$ and $\theta_2 = 0$ making the temptation function dependent of k_t and k_{t+1} but independent of k_{t+2}. In this case, an analytical analysis is still possible. Finally, we study numerically the *endogenous second-order temptation case* where $\theta_2 > 0$, so that the temptation function depends on the capital stock in three different periods.

5.1. Exogenous temptation case: $\theta_1 = \theta_2 = 0$

In this case, we are able to rank the capital stocks of three economies which differ in terms of the temptation weight (ψ): the neoclassical economy (k_t^n) in which $\psi = 0$, our temptation economy (k_t) where $0 < \psi < \infty$ and finally the economy where the individual succumbs fully to temptation (k_t^T) in which $\psi \to \infty$. The only difference between these economies is the form of the temptation function $T(.)$ as was shown in Section 3. In the three economies, the temptation function does not depend on the capital stock which greatly simplifies the analysis. The next result states the dynamic response of the capital stock to marginal variations in the parameters driving temptation and self-control problems: ψ, β, γ_1 and γ_2.

Proposition 5.3. *If $\theta_1 = \theta_2 = 0$, then the capital stock k_t at any period t:*

(a) increases with β and γ_2;

(b) decreases with γ_1; and

(c) increases with ψ if and only if $\beta^{1/\sigma} > \frac{1-\gamma_2}{1-\gamma_1}$.

Finally, we next compare the equilibrium path of the capital stock in the three aforementioned economies: our economy with self-control problems arising from interpersonally dependent preferences, the neoclassical economy and the economy where consumers fully succumb to temptation.

Proposition 5.4. *The following statements hold when $\theta_1 = \theta_2 = 0$:*

(a) If $\beta^{1/\sigma} > \frac{1-\gamma_2}{1-\gamma_1}$, $k_t^n < k_t < k_t^T$.

(b) If $\beta^{1/\sigma} < \frac{1-\gamma_2}{1-\gamma_1}$, $k_t^T < k_t < k_t^n$.

When β and γ_2 are sufficiently large compared to γ_1, the economies with temptation will generate a capital stock larger than the neoclassical economy. In this case, the temptation degree is given by $T > 1$ and, therefore, the young agent wishes to increase future consumption at a given rate of interest. Since temptation induces the agent to accumulate more capital, an economy in which the latter fully succumbs to temptation will generate the highest capital stock.

When γ_1 is sufficiently large, it is the neoclassical economy that will generate the largest capital stock. In this case, the economies with temptation generate a function $T < 1$ implying that the agent wishes to increase present consumption. Since temptation induces now the agent to accumulate less capital, it is the economy where the agent does not fully succumbs to temptation that delivers the largest capital stock among the two temptation economies. It is interesting to notice that when $\gamma_1 \geq \gamma_2$, case (a) always applies and a temptation economy cannot generate more capital than a neoclassical one. A sufficiently high value of γ_2 (i.e., when temptation is more intense in the second period of life) is thus necessary in order to observe a temptation economy with a larger capital.

The comparison between the neoclassical case and our economy including temptation and externalities is valid in a case without distortionary taxes. If a government imposes a set of taxes on the temptation economy, our equilibrium condition becomes:

$$G(k_{t+1}, k_t) = \delta(\alpha A)^{1-\sigma} \frac{(1+\tau_t)(1+\tau_{t+1}^k)[1+\beta\psi(1-\gamma_2)]}{(1+\tau_{t+1})[1+\psi(1-\gamma_1)]},$$

where τ_t is a tax on consumption and τ_{t+1}^k is a subsidy on the capital stock. When the value of T without taxes is larger than one, a combination of taxes such that $(1+\tau_t)(1+\tau_{t+1}^k) < 1+\tau_{t+1}$ generates a temptation economy closer to the neoclassical solution (compared to a temptation economy without taxation). Clearly such a policy imposes larger taxes on future consumption to reduce the impact of temptation on the marginal rate of substitution between current and future consumption. Obviously, when the value of T without taxes is smaller than one„ a similar reasoning can be made with a combination of taxes such that $(1+\tau_t)(1+\tau_{t+1}^k) > 1+\tau_{t+1}$, i.e., by imposing higher taxes on current consumption. It is even possible that the economy with temptation and the neoclassical economy exhibit the same capital stock level (i.e., $k_t = k_t^n$) if

$$\frac{(1+\tau_t)(1+\tau_{t+1}^k)}{1+\tau_{t+1}} = \frac{1+\psi(1-\gamma_1)}{1+\beta\psi(1-\gamma_2)}.$$

5.2. Endogenous first-order temptation case: $\theta_1 > 0$ and $\theta_2 = 0$

We will now focus on the case where T is a function of the capital stock. However, when $\theta_2 > 0$, the function depends on k_{t+2}, so that the dynamics are charecterized by a second order difference equation, which does not allow us to obtain general analytical results. Setting $\theta_2 = 0$ greatly simplifies the analysis and we should follow this path in the following. In this case, we also proceed by analyzing the response of the equilibrium value of k_{t+1} with repect to the weight of the temptation utility ψ for a given value of k_t. We next state the results from the comparative dynamic analysis to determine the impact of the self-control problem.

Proposition 5.5. *If $\theta_1 > 0$ and $\theta_2 = 0$, then the capital stock k_t at any period t:*

(a) increases with β and γ_2;

(b) decreases with γ_1 and θ_1; and

(c) decreases with ψ when

$$\beta^{1/\sigma} < \frac{(1+\theta_1)(1-\gamma_2)}{1+\theta_1-\gamma_1-\frac{\gamma_1\theta_1\alpha}{1-\alpha}}. \tag{5.4}$$

Proposition 5.5 allow us to compare the dynamics of the capital stock in our economy with self-control problems arising from interpersonally dependent preferences and the one in the neoclassical economy.

Proposition 5.6. *When $\theta_1 > 0$ and $\theta_2 = 0$, then $k_t < k_t^n$ if Condition 5.4 holds.*

Proposition 5.6 does not exclude the capital stock of our temptation economy to be larger than the one of the neoclassical model. In fact, we know from Proposition 5.5 that $T'_\psi > 0$ for sufficiently small values of the capital stock when Condition 5.4 does not hold. Hence, we conclude that $k_t > k_t^n$ for this range of the capital stock. Furthermore, in that case, we cannot discard that the sign of T'_ψ becomes negative as the capital stock increases. If this happens, then the paths of the capital stocks corresponding to the two aforementioned economies will cross at some level of the capital stock. At this point we assert that the later property of the equilibrium dynamics is very unlikely or it occurs for very large values of the capital stock. Observe from the proof of Proposition 5.5 that $T'_\psi < 0$ requires either a large value of the capital stock or a small value of $\beta(1-\gamma_2)^{-\sigma}$. However, small values of the later expression leads Condition 5.4 to hold and, therefore the derivative T'_ψ is negative for the entire domain of the capital stock. In fact, we did not find a numerical example where this derivative becomes negative when Condition 5.4 does not hold.

The main conclusion from these first two subsections is that the sign of the comparison between the capital stock in our model with self-control problems and the one in the neoclassical model is likely to be monotone along the entire transition path provided that $\theta_2 = 0$. Along the entire transition path, the former capital is always either larger or smaller than the neoclassical one. However, this is not necessarily the case when $\theta_2 > 0$, i.e., when old individuals include the living young individuals in their reference set. In this case, the paths of these two aforementioned capital stocks might cross, so that the sign of the comparison between them can change. We will numerically illustrate this point in the next subsection.

5.3. Endogenous second-order temptation case: $\theta_2 > 0$

We finally study the case with $\theta_2 > 0$, which implies that our function T now depends on k_t, k_{t+1} as well as k_{t+2}. This property does not allow us to give an analytical characterization of the comparative dynamic analysis. In this subsection, we numerically compare the equilibrium behavior of capital in the model with self-control problems and in the standard neoclassical model. To this end, we consider the numeric example provided at the beginning of this section: $A = 6.75$, $\alpha = 1/3$, $\delta = 0.3$, $\sigma = 1.1$, $\psi = 1$, $\beta = 0.85$, $\gamma_1 = 0.5$, $\gamma_2 = 0.8$ and $\theta_1 = 0.5$. We will next illustrate how the comparison of the equilibrium dynamics of the capital stock crucially depends on the values of θ_2, i.e., the weigth that the consumption of young individuals has on the consumption reference of old individuals. In fact, starting from the same initial value of the capital stock k_0, the capital stock of our model with self-control problems may be larger or smaller than the one corresponding to the neoclassical model along the entire equilibrium path. More interesting, for some parameter values, the two paths of the capital stock corresponding to the two aforementioned economies cross once. For instance, by considering $\theta_2 = 0.5$, Figure 1 illustrates the dynamics of the capital stock corresponding to the two models. Panel (a) gives the policy functions relating the present and the future capital stock, whereas Panel (b) provides the equilibrium paths when k_0 is equal to 25% of the stationary capital stock of the neoclassical model (i.e., when $\psi = 0$).[12] By starting at the same initial value, the neoclassical capital stock is larger (smaller) than the one in our model with self-control problems in the first (last) part of the transition dynamics.

[Insert Figure 1]

The economic intuition for the previous conclusion is as follows. Since $\gamma_2 > \gamma_1$ in our benchmark example, temptation is future-biased, i.e., consumption benchmark guiding the temptation of old individuals is more intense than the one for young individuals. Furthermore, the consumption references for the two periods of life are both increasing functions of the capital stock. Therefore, temptation faced by individuals tends to be more future-biased as the economy accumulates capital since $\theta_2 \geq \theta_1$. This effect on the time-bias of the temptation stimulates consumers to shift the intertemporal composition of their consumption towards the second period of life. The corresponding response of savings to this endogenous change in the intensity of temptation leads the capital stock to overtake the one that would be achieved in the absence of temptation.

With slight changes in the benchmark values of the parameters, we can illustrate how important is the assumption that temptation is based on intergenerational consumption comparisons. We numerically obtain that $k_t > k_t^n$ for all t if we reset $\theta_2 = 0$, whereas if we instead fix $\theta_1 = 0$ we get a comparison between k_t and k_t^n qualitatively identical to the one in Figure 1. In addition, we get $k_t < k_t^n$ by modifying the benchmark numerical example and setting $\theta_2 = 0$ and $\gamma_2 = 0.5$. However, the comparison in Figure 1 qualitatively maintains in the case where $\theta_2 = 0.5$ and $\gamma_2 = 0.5$. Therefore, our numerical simulations suggest that a non-monotonic

[12] To facilitate the illustration, we have plotted the path of the capital stock by years. To this end, we have assumed that the value corresponding to one model period is generated in each of the 30 years at the same proportion. We have then plotted the accumulated stock at each of these years.

relationship between the capital stock of our model with self-control problems and the one of the neoclassical model requires the old individuals to be tempted by the consumption of the living young individuals (i.e., $\theta_2 > 0$).

Panel (b) of Figure 1 also shows the difference between our model with DSC preferences and the model with externalities and without self-control problem. For this parametrization, the capital stock of the later model is larger than the stock in the model with DSC preferences along the entire equilibrium path. Although the model with externalities and without temptation also exhibits an endogenous intertemporal elasticity of substitution, the time bias of preferences is given by the exogenous factor $\delta\beta$. However, as was pointed out in Section 2, the time bias of preferences are endogenously determined by the endogenous gas in the intertemporal elasticity of substitution between the commitment and the temptation utility functions.

The previous numerical results confirm that the existence of endogenous self-control problems has large consequences for the intertemporal decisions on consumption and saving. This is specially true when old individuals take the consumption of living young individuals as a important determinant of the consumption reference driving their temptation.

6. Optimal allocation

Since we are in an OLG framework, the neoclassical solution is not necessarily the optimal one. Hence, it is necessary to analyze how self-control problems affect the welfare properties of the equilibrium path. We should start by establishing what is a socially optimal allocation in the proposed model. Following Drugeon and Wigniolle (2017), we consider that the social planner omits the cost arising from the self-control problem. Therefore, the social planner solve the following problem:

$$\max_{\{c_t,d_t,k_{t+1}\}} \sum_{t=0}^{\infty} \zeta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \delta \frac{d_{t+1}^{1-\sigma}}{1-\sigma} \right),$$

subject to the aggregate resource constraint

$$Ak_t^{\alpha} = c_t + d_t + k_{t+1},$$

where ζ is the weight that the social planner assigns to each generation. This approach will allow us to compare the optimal allocation with the one of the temptation economy. By following the standard procedure, we obtain the first order conditions and, after a simple manipulation, we obtain the following conditions characterizing the socially planned solution:

$$\frac{\zeta}{\delta} \left(\frac{c_t^*}{d_t^*} \right)^{-\sigma} = 1, \tag{6.1}$$

$$\frac{1}{\zeta} \left(\frac{c_t^*}{c_{t+1}^*} \right)^{-\sigma} = \alpha A \left(k_{t+1}^* \right)^{\alpha-1}, \tag{6.2}$$

where starred variables denote optimal outcomes. The first condition determines the optimal allocation of aggregate consumption between the two living generation in any period t. In addition, Condition (6.2) characterizes the optimal intertemporal allocation of consumption for a generation born in period t.

Combining (6.1) and (6.2) we obtain

$$\frac{1}{\delta}\left(\frac{c_t^*}{d_{t+1}^*}\right)^{-\sigma} = \alpha A\left(k_{t+1}^*\right)^{\alpha-1}, \tag{6.3}$$

which can be compared to a similar condition from the competitive equilibrium:

$$\frac{1}{\delta}\left(\frac{c_t}{d_{t+1}}\right)^{-\sigma} = \alpha A k_{t+1}^{\alpha-1} T(k_{t+2}, k_{t+1}, k_t). \tag{6.4}$$

Comparing both expressions, we should be able to determine which economy will generate the largest capital stock. There are two sources of inefficiency. On the one hand, the presence of consumption externalities in temptation utility distorts the marginal rate of substitution between present and future consumption. This inefficiency is given by the value of the temptation function $T(k_{t+2}, k_{t+1}, k_t)$. Obviously, this distortion does not emerge in the neoclassical economy where $T(k_{t+2}, k_{t+1}, k_t) = 1$. On the other hand, in our temptation economy and in the neoclassical one, there is the standard distortion in the marginal rate of transformation that determines the intergenerational allocation of consumption. To show the consequences of these two sources of inefficiency, we focus on the steady-state allocations. By combining (6.1) and (6.3), we obtain the modified golden rule in our economy as follows

$$\alpha A\left(k^*\right)^{\alpha-1} = \frac{1}{\zeta}.$$

Since $\widetilde{T}(k) = T(k, k, k)$ is decreasing (see Proposition 4.1), the capital stock at the steady-state equilibrium is suboptimal if $\zeta \alpha A k^{\alpha-1}\widetilde{T}(k) \neq 1$. By using this fact, the next proposition establishes the efficiency properties of our model with temptation.

Proposition 6.1. *If* $\zeta \alpha A k^{\alpha-1}\widetilde{T}(k) > 1$ *the capital stock at the competitive equilibrium is suboptimally small, whereas this capital stock is suboptimally large when* $\zeta \alpha A k^{\alpha-1}\widetilde{T}(k) < 1$.

Observe that efficiency of the competitive equilibrium crucially depends on the value of the temptation function $\widetilde{T}(k)$. In particular, we can obtain that the suboptimal nature of the capital stock in our temptation economy can be the opposite of the one of the neoclassical economy. The reason for this conclusion is that the temptation function $\widetilde{T}(k)$ is equal to one in the neoclassical economy but can be different than one in our economy.

Regarding the policies to decentralize the optimal outcome, we should note that "the sole replication of the optimal consumption sequence does not suffice when there are temptation motives" (Drugeon and Wigniolle, 2017). In defining the optimal allocation the social planner does not take into account that individuals suffer a disutility from not fully succumbing to temptation. Therefore, the optimal policies consist in restricting the menu of actions that individuals have access to. As Gul and Pesendorfer (2004) point out, one should not allow individuals to face those activities or goods generating self-control problems. In our framework, this policy reduces to restrict in each period the level of consumption that young individuals can choose as in Drugeon and Wigniolle (2017). If individuals face present-biased temptation, then one should impose the level of consumption at young age to be smaller than the optimal one. On the contrary, the consumption of young individuals should be larger than the optimal one when temptation is future-biased.

7. Concluding remarks

We have analyzed how temptation and costly self-control influence consumer's decisions on savings and the accumulation of wealth along the life-cycle. We have used an overlapping generations model where individuals were tempted to take the average consumption of agents living in the same period as a consumption reference. Their decisions result from the compromise between two competing desires: the gratification from fully succumbing to temptation and the gratification derived from ignoring how their choices will determine their position in the social scale of consumption. The presence of consumption externalities generates a time-varying and endogenous gap between the intertemporal elasticity of substitution for commitment consumption and the one for temptation consumption. Therefore, the degree of the self-control problem is endogenously determined by the aggregate allocation of resources. We have showed that the influence of temptation and costly self-control on consumption-saving decisions crucially depend on whether or not consumers take the consumption of the individuals belonging to the other living generation as a determinant of their consumption reference. In particular, we have obtained that consumption externalities may generate a suboptimal accumulation of capital.

The main conclusion from the results of the paper is that our preferences displaying endogenous temptation and self-control problems may have important implications for the dynamics of income and wealth inequality. Since the degree of the self-control problem is driven by consumption externalities, the aggregate performance of the economy would significantly depend on income inequality and wealth distribution. In this case, the intensity of the self-control problem faced up by each individual is crucially determined by his position in the social scale of consumption. Hence, our framework contains an interesting propagation mechanism for income inequality and structural shocks.

Finally, future research may also focus on extending our analysis to consider other forms of consumption references for the temptation utility function. More precisely, we may include phenomena as, for instance, habit formation. In contrast with what happens in our framework, individuals internalize in that alternative case that their current decisions affect the degree of their future self-control problem. This makes the problem more complex and, therefore, deserves some effort to investigate its consequences on economic decisions.

References

[1] Abel, A., (2005). Optimal taxation when consumers have endogenous benchmark levels of consumption. *Review of Economic Studies* 75, 21-42.

[2] Alonso-Carrera, J., J. Caballé and X. Raurich, (2008). Estate taxes, consumption externalities, and altruism. *Journal of Public Economics* 92, 1751-1764.

[3] Ameriks, J., A. Caplin, J. Leahy and T. Tyler, (2007). Measuring self-control problems. *American Economic Review* 97, 966-972.

[4] Barro, R.J., (1974). Are bonds net wealth? *Journal of Political Economy* 82, 1095-1117.

[5] Bucciol, A., (2011). A note on social security welfare with self-control problems. *Macroeconomic Dynamics* 15, 579-594.

[6] Bucciol, A., (2012). Measuring self-control problems: a structural estimation. *Journal of the European Economic Association* 10, 1084-1115.

[7] de la Croix, D. and P. Michel, (2002). *A theory of economic Growth*. Cambridge: Cambridge University Press.

[8] Diamond, P.A., (1965). National debt in a neoclassical groth model. *American Economic Review* 55, 1126-1150.

[9] Drugeon, J.P. and B. Wigniolle, (2017). On impatience, temptation and Ramsey's conjecture. *Economic Theory* 63, 73-98.

[10] Fisher, W.H. and F.X. Hof, (2000). Relative consumption, economic growth, and taxation. *Journal of Economics* 3, 241-262.

[11] Frederick, S., G. Loewenstein, and T. O'Donoughe, (2002). Time discounting and time preference: a critical review. *Journal of Economic Literature* 40, 351-401.

[12] Gul, F. and W. Pesendorfer, (2001). Temptation and self-control. *Econometrica* 69, 1403–1436.

[13] Gul, F. and W. Pesendorfer, (2004). Self control and the theory of consumption. *Econometrica* 72, 119–158.

[14] Hung, K.X.D., Z. Liu and J.Q. Zhu, (2015). Temptation and self-control: some evidence and applications. *Journal of Money, Credi, and Banking* 47, 581–615.

[15] Krusell, P., B. Kuruscu and A.A. Smith, (2010). Temptation and taxation. *Econometrica* 78, 2063-2084.

[16] Krusell, P. and A.A. Smith, (3003). Consumption-savings decisions with quasi-geometric discounting. *Econometrica* 71, 365-375.

[17] Kumru, C.G. and A.C. Thanopoulos, (2008). Social security and self control preferences. *Journal of Economic Dynamics and Control* 32, 757-778.

[18] Kumru, C.G. and A.C. Thanopoulos, (2011). Social security reform with self-control preferences. *Journal of Public Economics* 95, 886-899.

[19] Laibson, D. , (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics* 112, 443-478.

[20] Lahiri, A. and M. Puhakka, (1998). Habit persistence in overlapping generations economies under pure exchange. *Journal of Economic Theory* 78, 176-188.

[21] Maddison, A., (1992). A long-run perspective of saving. *Scandinavian Journal of Economics* 94, 181-196.

[22] Pavoni, N. and H. Yazici, (2015). Life-cycle taxes and self-control problems. *The Economic Journal*, 1-29.

[23] Phelps, E.S. and R. A. Pollak, (1968). On second-best national savings and game-equilibrium growth. *Review of Economic Studies* 35, 185-199.

[24] Strotz, R.H. (1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies* 23, 165-180.

[25] Wendner, R., (2002). Capital accumulation and habit formation. *Economics Bulletin* 4, 1-10.

[26] Wendner, R., (2010). Conspicuous consumption and generation replacement in a model of perpetual youth. *Journal of Public Economics* 94, pages 1093-1107.

Appendix

A. Proofs of the results

Proof of Proposition 4.1. We proceed with the following three steps:

(a) We compute the derivates of functions $\widetilde{G}(k)$ and $\widetilde{T}(k)$. We obtain

$$\widetilde{G}'(k) = \frac{(1-\alpha)\left[w-(1-\sigma)k\right]k^{\alpha(\sigma-1)}}{\delta(\alpha A)^{1-\sigma}(w-k)^{1+\sigma}} > 0,$$

and

$$\widetilde{T}'(k) = -\left\{\frac{\sigma\psi}{1+\psi\left[\zeta_1-\eta_1\phi(k)\right]^{-\sigma}}\right\}\left\{\begin{array}{l} \beta\eta_2\left[\zeta_2-\eta_2/\phi(k)\right]^{-\sigma-1}\left[\frac{\phi'(k)}{\phi(k)}\right] \\ \\ +\frac{\eta_1\phi'(k)[\zeta_1-\eta_1\phi(k)]^{-\sigma-1}\left\{1+\beta\psi[\zeta_2-\eta_2/\phi(k)]^{-\sigma}\right\}}{[1+\psi([\zeta_1-\eta_1\phi(k)])^{-\sigma}]^2}\end{array}\right\},$$

with

$$\zeta_1 = \frac{1+\theta_1-\gamma_1}{1+\theta_1},$$
$$\eta_1 = \frac{\alpha\gamma_1\theta_1}{(1-\alpha)(1+\theta_1)},$$
$$\zeta_2 = \frac{1+\theta_2-\gamma_2}{1+\theta_2},$$
$$\eta_2 = \frac{(1-\alpha)\gamma_2\theta_2}{\alpha(1+\theta_2)},$$

and

$$\phi(k) = \frac{w}{w-k}.$$

Since effective consumption is positive for the young and the old and given that $\phi'(k) = \frac{(1-\alpha)^2Ak^\alpha}{(w-k)^2} > 0$, $T'(k) < 0$. Given the behavior of our functions $\widetilde{G}(.)$ and $\widetilde{T}(.)$, they will cross at most once.

(b) We characterize the limits of functions $\widetilde{G}(.)$ and $\widetilde{T}(.)$ at the upper bound of the domain for k. We obtain that

$$\lim_{k\to\min\left\{\overline{k}_1,\overline{k}_2\right\}}\left[\widetilde{G}\left(k\right)-\widetilde{T}\left(k\right)\right] > 0,$$

because

$$\lim_{k\to\min\left\{\overline{k}_1,\overline{k}_2\right\}}\widetilde{G}\left(k\right) = \left\{\begin{array}{cc} \infty & \text{if } \theta_1=0 \text{ (i.e., } \overline{k}_1=\overline{k}_2) \\ \\ M\in(0,\infty) & \text{otherwise}\end{array}\right\},$$

and

$$\lim_{k\to\min\left\{\overline{k}_1,\overline{k}_2\right\}}\widetilde{T}\left(k\right) = \left\{\begin{array}{cc} \frac{1+\beta\psi(1+\theta_2-\gamma_2)^{-\sigma}(1+\theta_2)^\sigma}{1+\psi(1-\gamma_1)^{-\sigma}} & \text{if } \theta_1=0 \text{ (i.e., } \overline{k}_1=\overline{k}_2) \\ \\ 0 & \text{otherwise}\end{array}\right\}.$$

The previous limits derive from the fact that $w-k=0$ at $k=\overline{k}_1$.

(c) We characterize the limits of functions $\widetilde{G}(.)$ and $\widetilde{T}(.)$ at the lower bound of the domain for k. We obtain that

$$\lim_{k \to \max\left\{0, \overline{k}_3\right\}} \left[\widetilde{G}(k) - \widetilde{T}(k)\right] < 0,$$

because

$$\lim_{k \to \max\left\{0, \overline{k}_3\right\}} \widetilde{G}(k) = \left\{ \begin{array}{cc} 0 & \text{if } \theta_2 = 0 \text{ (i.e., } \overline{k}_3 < 0) \\ & \\ M \in (0, \infty) & \text{otherwise} \end{array} \right\},$$

and

$$\lim_{k \to \max\left\{0, \overline{k}_3\right\}} \widetilde{T}(k) = \left\{ \begin{array}{cc} \frac{1+\beta\psi(1-\gamma_2)^{-\sigma}}{1+\psi\left[1+\theta_1-\gamma_1-\frac{\gamma_1\theta_1\alpha}{(1-\alpha)}\right]^{-\sigma}(1+\theta_1)^{\sigma}} & \text{if } \theta_2 = 0 \text{ (i.e., } \overline{k}_3 < 0) \\ & \\ \infty & \text{otherwise} \end{array} \right\}.$$

The propostion directly follows from the previous three statements. ■

Proof of Proposition 4.2. Define $\pi = \{\psi, \beta, \gamma_1, \gamma_2, \theta_1, \theta_2\}$. Applying the implicit function theorem to (2.21) evaluated at the steady-state equilibrium, we obtain that

$$\frac{\partial k}{\partial \pi} = \frac{\widetilde{T}'_{\pi}}{\widetilde{G}'_k - \widetilde{T}'_k},$$

where $\widetilde{G}'_k$ is the derivative of Function (4.1) with respect to k_t; and $\widetilde{T}'_k$ and $\widetilde{T}'_{\pi}$ denote the derivatives of Function (4.2) with respect to k and π, respectively. We know from the proof of Propostion 4.1 that $\widetilde{G}'_k > 0$ and $\widetilde{T}'_k < 0$. Hence, the sign of $\partial k/\partial \pi$ coincides with the sign of $\widetilde{T}'_{\pi}$. We directly obtain from Function (4.2) that $\widetilde{T}'_{\beta} > 0$, $\widetilde{T}'_{\gamma_1} < 0$, $\widetilde{T}'_{\gamma_2} > 0$, $\widetilde{T}'_{\theta_1} < 0$, and $\widetilde{T}'_{\theta_2} > 0$. In addition, we get

$$\widetilde{T}'_{\psi} = \frac{\beta\left[1 - \gamma_2 (h/d)\right]^{-\sigma} - \left[1 - \gamma_1 (a/c)\right]^{-\sigma}}{\left\{1 + \psi\left[1 - \gamma_1 (a/c)\right]^{-\sigma}\right\}^2},$$

whose sign cannot be established. The proposition then follows. ■

Proof of Proposition 4.4. We separately prove the three statements.

(a) When $\theta_1 = \theta_2 = 0$, then $\widetilde{T}(k)$ is constant and equal to

$$\widetilde{T}(k) = \frac{1 + \beta\psi(1 - \gamma_2)^{-\sigma}}{1 + \psi(1 - \gamma_1)^{-\sigma}}.$$

Therefore, using Propostion 4.3, we directly obtain the first statement in the corollary.

(b) When $\theta_1 > 0$ and $\theta_2 = 0$, the domain for the capital stock is $\left(0, \overline{k}_2\right)$ and, moreover, we know from the proof of Proposition 4.1 that

$$\lim_{k \longrightarrow 0} \widetilde{T}(k) = \frac{1 + \beta\psi (1 - \gamma_2)^{-\sigma}}{1 + \psi \left[1 + \theta_1 - \gamma_1 - \frac{\gamma_1 \theta_1 \alpha}{(1-\alpha)}\right]^{-\sigma} (1 + \theta_1)^{\sigma}},$$

and

$$\lim_{k \longrightarrow \overline{k}_2} \widetilde{T}(k) = 0.$$

Therefore, if $\lim_{k \longrightarrow 0} \widetilde{T}(k) < 1$, then $\widetilde{T}(k) < 1$ for all $k_t \in \left(0, \overline{k}_2\right)$. The second statement of the corollary then directly follows from using Propositon 4.1.

(c) When $\theta_1 = 0$ and $\theta_2 > 0$, the domain for the capital stock is $\left(\overline{k}_3, \overline{k}_2\right)$ and, moreover, we know from the proof of Proposition 4.1 that

$$\lim_{k \longrightarrow \overline{k}_3} \widetilde{T}(k) = \infty,$$

and

$$\lim_{k \longrightarrow \overline{k}_2} \widetilde{T}(k) = \frac{1 + \beta\psi (1 + \theta_2 - \gamma_2)^{-\sigma} (1 + \theta_2)^{\sigma}}{1 + \psi (1 - \gamma_1)^{-\sigma}}.$$

Therefore, if $\lim_{k \longrightarrow \overline{k}_2} \widetilde{T}(k) > 1$, then $\widetilde{T}(k) > 1$ for all $k_t \in \left(\overline{k}_3, \overline{k}_2\right)$. The third statement of the proposition then directly follows from using Propositon 4.1. ■

Proof of Corollary 4.5. When $\gamma_1 = \gamma_2 = 0$, we derive from (2.23)

$$\widetilde{T}(k) = \frac{1 + \beta\psi}{1 + \psi} < 1.$$

Therefore, using Propostion 4.3, we directly obtain the corollary. ■

Proof of Proposition 5.1. Given the second-order nature of the dynamic equation (2.21) characterizing the equilibrium, we make use of the following variable transformation to study the stability:

$$k_{t+1} = x_t. \tag{A.1}$$

Hence, the equilibrium dynamics are given by (A.1) and the implicit function derived from (2.21):

$$x_{t+1} = \Phi(x_t, k_t). \tag{A.2}$$

The determinant and the trace of the Jacobian of the dynamic system (A.1)-(A.2) are respectively given by

$$Det = -\frac{\partial x_{t+1}}{\partial k_t} = -\frac{G'_{k_t} - \delta (\alpha A)^{1-\sigma} T'_{k_t}}{\delta (\alpha A)^{1-\sigma} T'_{k_{t+2}}},$$

and

$$Tr = \frac{\partial x_{t+1}}{\partial x_t} = \frac{G'_{k_{t+1}} - \delta\left(\alpha A\right)^{1-\sigma} T'_{k_t+1}}{\delta\left(\alpha A\right)^{1-\sigma} T'_{k_{t+2}}},$$

where $T'_{k_{t+2}}$, T'_{k_t+1} and T'_{k_t} are the derivatives of the temptation function (2.23) with respect to k_{t+2}, k_{t+1} and k_t at the steady state, respectively; and G'_{k_t} and $G'_{k_{t+1}}$ are the derivatives of function (2.22) with respect to k_t and k_{t+1} at the steady state, respectively. For the purpose of this proof, we have to determine how many eigenvalues of the Jacobian are in the interval $(-1, 1)$. To this end, we write our function $T(k_t, k_{t+1})$ as

$$T(k_t, k_{t+1}) = \frac{1 + \beta\psi[\zeta_2 - \eta_2\varphi(k_{t+1}, k_{t+2})]^{-\sigma}}{1 + \psi[\zeta_1 - \eta_1\phi(k_t, k_{t+1})]^{-\sigma}},$$

where $\zeta_1 = 1 + \theta_1 - \gamma_1$, $\eta_1 = \frac{\gamma_1\theta_1\alpha}{1-\alpha}$, $\zeta_2 = 1 + \theta_2 - \gamma_2$, $\eta_2 = \frac{\gamma_1\theta_1(1-\alpha)}{\alpha}$

$$\phi(k_t, k_{t+1}) = \frac{w_t}{w_t - k_{t+1}},$$

and

$$\varphi(k_{t+1}, k_{t+2}) = \frac{w_{t+1} - k_{t+2}}{w_{t+1}},$$

with

$$\phi'_{k_{t+1}} = \frac{w_t}{(w_t - k_{t+1})^2},$$

$$\phi'_{k_t} = -\frac{w_t k_{t+1}}{k_t(w_t - k_{t+1})^2},$$

$$\varphi'_{k_{t+1}} = \frac{\alpha k_{t+2}}{w_{t+1}k_{t+1}},$$

and

$$\varphi'_{k_{t+2}} = -\frac{1}{w_{t+1}}.$$

We then obtain:

$$G'_{k_{t+1}} = \frac{[1 - \alpha(1-\sigma)]k_{t+1}^{-\alpha(1-\sigma)} + \sigma k_{t+1}^{1-\alpha(1-\sigma)}(w_t - k_{t+1})^{-1}}{(w_t - k_{t+1})^{\sigma}} > 0,$$

$$G'_{k_t} = -\frac{\sigma w'_{k_t} k_{t+1}^{1-\alpha(1-\sigma)}}{(w_t - k_{t+1})^{1+\sigma}} < 0,$$

$$T'_{k_t} = -\frac{\sigma\psi\eta_1\phi'_{k_t}[\zeta_1 - \eta_1\phi(k_t, k_{t+1})]^{-\sigma-1}\left\{1 + \beta\psi[\zeta_2 - \eta_2\varphi(k_{t+1}, k_{t+2})]^{-\sigma}\right\}}{\{1 + \psi[\zeta_1 - \eta_1\phi(k_t, k_{t+1})]^{-\sigma}\}^2} > 0,$$

and

$$T'_{k_{t+2}} = \frac{\sigma\beta\psi\eta_1\varphi'_{k_{t+2}}[\zeta_2 - \eta_2\varphi(k_{t+1}, k_{t+2})]^{-\sigma-1}\left\{1 + \psi[\zeta_1 - \eta_1\phi(k_t, k_{t+1})]^{-\sigma}\right\}}{\{1 + \psi[\zeta_1 - \eta_1\phi(k_t, k_{t+1})]^{-\sigma}} < 0.$$

When $\theta_2 = 0$ we obtain that

$$T'_{k_{t+1}} = -\frac{(1 + \beta\psi(1-\gamma_2)^{-\sigma})\sigma\psi[\zeta_1 - \eta_1\phi(k_t, k_{t+1})]^{-\sigma-1}\eta_1\phi'_{k_{t+1}}}{\{1 + \psi[\zeta_1 - \eta_1\phi(k_t, k_{t+1})]^{-\sigma}\}^2} < 0.$$

However, we cannot determine the sign of $T'_{k_{t+1}}$ when $\theta_2 > 0$.

Since $G'_{k_t} < 0$, $T'_{k_t} > 0$ and $T'_{k_{t+2}} < 0$, then we conclude that $Det < 0$. However, the sign of the trace is ambiguous since we cannot determine the sign of the derivative T'_{k_t+1}. Hence, we must consider two cases.

(a) If $G'_{k_{t+1}} - \delta\left(\alpha A\right)^{1-\sigma} T'_{k_t+1} > 0$, then the trace is negative since $T'_{k_{t+2}} < 0$. First, the Jacobian of the dynamic system (A.1)-(A.2) has two eigenvalues in the interval $(-1, 1)$ if and only if $1 + Tr + Det > 0$. Hence, we conclude that the steady state is locally indeterminate when Condition (5.2) holds. Second, the Jacobian has two eigenvalues out of the unit circle if and only if $1 - Tr + Det < 0$. Hence, the steady state is locally unstable when Condition (5.3) holds. Finally, the steady-state equilibrium is locally saddle-path stable otherwise.

(b) If $G'_{k_{t+1}} - \delta\left(\alpha A\right)^{1-\sigma} T'_{k_t+1} < 0$, then the trace is positive since $T'_{k_{t+2}} < 0$. First, the Jacobian of the dynamic system (A.1)-(A.2) has two eigenvalues in the interval $(-1, 1)$ if and only if $1 - Tr + Det > 0$. Hence, we conclude that the steady state is locally indeterminate when Condition (5.3) holds. Second, the Jacobian has two eigenvalues out the unit circle if and only if $1 + Tr + Det < 0$. Hence, the steady state is locally unstable when Condition (5.2) holds. Finally, the steady-state equilibrium is locally saddle-path stable otherwise.

■

Proof of Proposition 5.2. When $\theta_2 = 0$, the equilibrium condition (2.21) reduces to a first-order dynamic equation. By using the implicit function theorem, we obtain the slope of the policy function for k_{t+1} at the steady state as

$$\frac{\partial k_{t+1}}{\partial k_t} = -\frac{G'_{k_t} - \delta\left(\alpha A\right)^{1-\sigma} T'_{k_t}}{G'_{k_{t+1}} - \delta\left(\alpha A\right)^{1-\sigma} T'_{k_t+1}}. \tag{A.3}$$

This derivative is positive since $G'_{k_t} < 0$, $G'_{k_{t+1}} > 0$, $T'_{k_t} > 0$ and $T'_{k_{t+1}} < 0$ as follows from the proof of Proposition 5.1. For our dynamical system to be locally stable, we need the derivative (A.3) evaluated at the steady state to be lower than one. The following condition should then be satisfied:

$$G'_{k_t} + G'_{k_{t+1}} > \delta\left(\alpha A\right)^{1-\sigma}\left(T'_{k_{t+1}} + T'_{k_t}\right). \tag{A.4}$$

Comparing the expressions of G'_{k_t} and $G'_{k_{t+1}}$, and using the fact that $w'_k = \alpha(w/k)$, we can conclude that $G'_{k_{t+1}} + G'_{k_t} > 0$ at the steady state provided that $w > k$, which is always the case in order to obtain positive consumption in the first period. With this result in hand, we know from inequality (A.4) that a sufficient condition for local stability is that the right hand side evaluated at the steady-state be positive. By using the fact that $w'_k = \alpha(w/k)$, we obtain that $T'_{k_{t+1}} + T'_{k_t} < 0$ at the steady state. This result implies that condition (A.4) always holds and our dynamic system is locally stable. ■

Proof of Proposition 5.3. Define $\pi = \{\psi, \beta, \gamma_1, \gamma_2\}$. Applying the implicit function theorem to (2.21), and using the fact that

$$T(k_t, k_{t+1}, k_{t+2}) = \frac{1 + \beta\psi(1-\gamma_2)^{-\sigma}}{1 + \psi(1-\gamma_1)^{-\sigma}},$$

in this case, we obtain

$$\frac{\partial k_{t+1}}{\partial \pi} = \frac{\delta(\alpha A)^{1-\sigma} T'_\pi}{G'_{k_{t+1}}},$$

where T'_π is the derivative of T with respect to the corresponding parameter in π. Since $G'_{k_{t+1}} > 0$ as was proved in Proposition 5.1, the sign of $\partial k_{t+1}/\partial \pi$ coincides with the sign of T'_π. We obtain that

$$T'_\beta \equiv \frac{\partial T}{\partial \beta} = \frac{\psi(1-\gamma_2)^{-\sigma}}{1 + \psi(1-\gamma_1)^{-\sigma 2}} > 0,$$

$$T'_{\gamma_1} \equiv \frac{\partial T}{\partial \gamma_1} = -\frac{\sigma\psi\left[1 + \beta\psi(1-\gamma_2)^{-\sigma}\right]}{\left[(1-\gamma_1)^{1+\sigma}\right]\left[1 + \psi(1-\gamma_1)^{-\sigma}\right]^2} < 0,$$

$$T'_{\gamma_2} \equiv \frac{\partial T}{\partial \gamma_2} = \frac{\sigma\beta\psi}{\left[(1-\gamma_2)^{1+\sigma}\right]\left[1 + \psi(1-\gamma_1)^{-\sigma}\right]} > 0,$$

and

$$T'_\psi \equiv \frac{\partial T}{\partial \psi} = \frac{\beta(1-\gamma_2)^{-\sigma} - (1-\gamma_1)^{-\sigma}}{\left[1 + \psi(1-\gamma_1)^{-\sigma}\right]^2}.$$

The proposition then directly follows. ■

Proof of Proposition 5.4. The result directly follows from Proposition 5.3 by noting that the neoclassical model arises when $\psi = 0$, the model where consumers fully succumb to temptation emerges when ψ tends to infinite, and the model with self-control problems from interpersonally dependent preferences is given by $\psi \in (0, \infty)$. ■

Proof of Proposition 5.5. Define $\pi = \{\psi, \beta, \gamma_1, \gamma_2, \theta_1\}$. Applying the implicit function theorem to (2.21), and using the fact that

$$T(k_t, k_{t+1}) = \frac{1 + \beta\psi\,(1-\gamma_2)^{-\sigma}}{1 + \psi(1+\theta_1)^{\sigma}\left\{1 + \theta_1 - \gamma_1 - \frac{\gamma_1\theta_1\alpha A k_t^\alpha}{[(1-\alpha)A k_t^\alpha - k_{t+1}]}\right\}^{-\sigma}},$$

in this case, we obtain

$$\frac{\partial k_{t+1}}{\partial \pi} = \frac{\delta(\alpha A)^{1-\sigma} T'_\pi}{G'_{k_{t+1}} - \delta(\alpha A)^{1-\sigma} T'_{k_{t+1}}}, \tag{A.5}$$

where $G'_{k_{t+1}}$ is the derivative of function $G(.)$ with respect to k_{t+1}; and $T'_{k_{t+1}}$ and T'_π denote the derivatives of function $T(.)$ with respect to k_{t+1} and π, respectively. We already know from the proof of Proposition 5.1 that $G'_{k_{t+1}} > 0$, and it is straightforward to check that $T'_{k_{t+1}} < 0$ when $\theta_2 = 0$. Thus, the sign of $\partial k_{t+1}/\partial \pi$ coincides with the sign of T'_π. We easily obtain that $T'_\beta > 0$, $T'_{\gamma_1} < 0$, $T'_{\gamma_2} > 0$ and $T'_{\theta_1} < 0$. In addition, we get

$$T'_\psi = \frac{\beta(1-\gamma_2)^{-\sigma} - \left[1+\theta_1-\gamma_1-\frac{\gamma_1\theta_1\alpha w_t}{(1-\alpha)(w_t-k_{t+1})}\right]^{-\sigma}(1+\theta_1)^\sigma}{\left\{1+\psi\left[1+\theta_1-\gamma_1-\frac{\gamma_1\theta_1\alpha w_t}{(1-\alpha)(w_t-k_{t+1})}\right]^{-\sigma}(1+\theta_1)^\sigma\right\}^2}. \tag{A.6}$$

We proceed in several steps to characterize the sign of T'_ψ.

(a) Since consumptions c_t and d_{t+1} are normal goods, we can assert that $\partial s_t/\partial w_t \in (0,1)$ and, therefore, we conclude that

$$\frac{w_t}{(w_t-k_{t+1})}, \tag{A.7}$$

is an increasing function of k_t. Obviously, this implies that the term

$$\left[1+\theta_1-\gamma_1-\frac{\gamma_1\theta_1\alpha w_t}{(1-\alpha)(w_t-k_{t+1})}\right]^{-\sigma},$$

also increases with k_t.

(b) Notice that the ratio k_{t+1}/k_t converges to one as k_t tends to zero. Therefore, we directly obtain

$$\lim_{k_t\to 0} T'_\psi = \frac{\beta(1-\gamma_2)^{-\sigma} - [1+\theta_1-\gamma_1(1+\theta_1\alpha/1-\alpha)]^{-\sigma}(1+\theta_1)^\sigma}{\{1+\psi[1+\theta_1-\gamma_1(1+\theta_1\alpha/1-\alpha)]^{-\sigma}(1+\theta_1)^\sigma\}^2}.$$

This limit is positive if and only if Condition (5.4) holds.

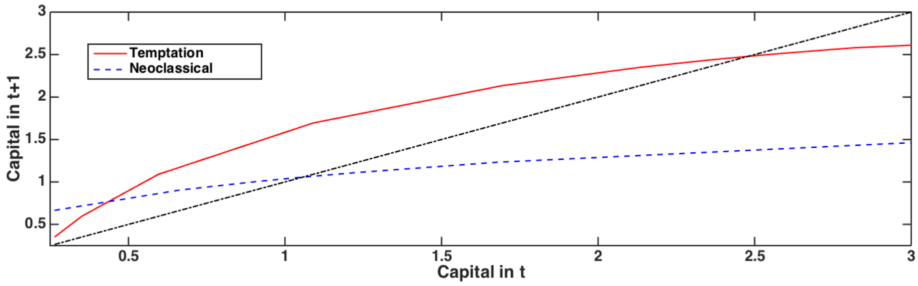
(c) Note that the effective consumption in the first period of life decreases with k_t because Expression (A.7) is an increasing function of k_t. Therefore, there exists a threshold $\overline{k}$ such that this effective consumption would be negative for a $k_t > \overline{k}$. In fact, the value of $\overline{k}$ is implicitly given by the first condition in (2.11). This threshold $\overline{k}$ is then the upper bound of the domain of the capital stock for which equilibrium is defined. Furthermore, we also obtain

$$\lim_{k_t\to\overline{k}} T'_\psi = 0.$$

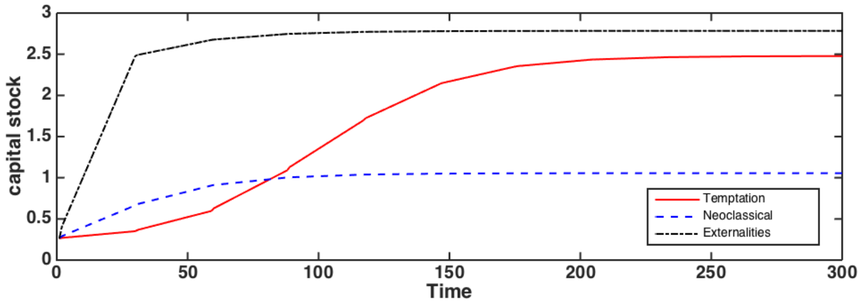
Combining these different results we can conclude that $T'_\psi < 0$ on the interval $k_t \in (0,\overline{k})$ when Condition (5.4) holds. Therefore, the proposition directly follows. ■

Proof of Proposition 5.6. The result directly follows from Proposition 5.5 by noting that the neoclassical model arises when $\psi = 0$ and the model with self-control problems from interpersonally dependent preferences is given by $\psi > 0$. ■

Figure 1. Comparative dynamics ($\theta_1 > 0$ and $\theta_2 > 0$)



(a) Policy function of k_{t+1} on k_t



(b) Equilibrium path as $k_0 < k$