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Aspirations, environmental quality and optimal tax policy

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Abstract

In this paper, we combine a standard overlapping generations model with aspirations and environmental quality in the utility function. The combination of both externalities (environmental degradation and aspirations) generates a steady-state capital stock that can be higher or lower than in the standard Diamond Economy. The study focuses next on the analysis of the optimal allocation and its decentralization by means of an appropriate tax policy. A sufficiently high social discount factor is necessary in order to avoid possible local oscillations. Moreover, investment should either be subsidized or taxed depending on the magnitude of both externalities while maintenance investment should always be subsidized.

Keywords: Overlapping generations, aspirations, environmental quality, optimal taxation.

JEL classification: D62, H21, O41, Q29

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1 Introduction

In a dynamic setting, the main problem associated to environmental conservation is that short-lived individuals fail to internalize the long term effects of their decisions on environmental degradation. Present choices only affect future environmental quality implying that current generations impose costs on future ones. The intergenerational aspects of the problem justifies the use of the overlapping generations model (OLG) developed by Diamond (1965) in order to study the problem of environmental conservation in a dynamic economy (see, for example, John and Pecchenino, 1994; Ono, 1996; Bovenberg and Heijdra, 1998). In the present paper, we follow this strand of the literature but we introduce an additional intergenerational externality under the form of aspirations. The latter are inherited from the previous generation and are a frame of reference against which adulthood consumption is evaluated. This formulation introduces an additional state-variable to our analysis beyond capital and environmental quality. While aspirations have already been introduced in the standard Diamond framework (see, for example, De la Croix, 1996; De la Croix and Michel, 1999; Artige et al., 2004; Alonso-Carrera et al., 2007), their implications in terms of environmental policy have been largely overlooked up to now. A notable exception is the work of Aronsson and Johansson-Stenman (2014) who study the optimal provision of state variable public goods (using global climate as the main example) in a model where agents care about relative consumption (including aspirations).

The empirical validity of our assumption is firstly related to the importance of relative well-being highlighted in works like the ones of Clark and Oswald (1996) or Ferrer-i Carbonell (2005) among others. These authors show that utility depends obviously on present consumption but also on some reference point. Additionally, as noted by Becker (1992), individual behavior is affected by inherited tastes that are transmitted from parents to children. For example, Waldkirch et al. (2004) estimate that parental preferences explain between 5% and 10% of their children's preferences after controlling for income. Moreover, Senik (2009) presents evidence showing that an individual's well-being increases if he has done better in life than his parents. This evidence combined with the role that endogenous preferences can play concerning the problem of environmental degradation (see, for example, Brekke and Howarth, 2003; Welsch, 2009) leads us to introduce consumption aspirations in our model.

Our paper can be related to models that have introduced habit formation and status effects in dynamic models dealing with environmental concerns. We can cite the works of Wendner (2003, 2005), Brekke and Howarth (2003), Howarth (2006) as well as Schumacher and Zou (2008). While the first three study the implications of environmental externalities when relative consumption matters, the last ones introduce habits in pollution and study the complex dynamic implications of this assumption. In the present paper, we choose to study the implications of environmental externalities when consumption aspirations are introduced in the utility function.

An account of the results is as follows. The combination of aspirations and environmental externalities generates a competitive steady-state capital stock level that can be higher or lower than in the standard Diamond economy. Concerning the stability of the dynamical system, it is only guaranteed if the impact of aspirations is not too large. We also study the optimal allocation by solving the social planner's problem and derive conclusions concerning its decentralization. The steady-state capital stock is not necessarily higher in the optimal case and we derive a condition under which this is the case. In order to decentralize the optimal allocation, investment should be subsidized (or taxed) if the impact of consumption on future aspirations is higher (lower, respectively) than the impact of consumption on future environmental quality in terms of utility. Maintenance investment on the other hand should always be subsidized. The optimal solution can be subject to local oscillations if the social discount factor is too low implying that an appropriate policy should focus on solving the problems linked to environmental externalities as well as on choosing an adequate social discount factor.

The remainder of this paper is organized as follows. Section 2 introduces the model, derives the first order conditions and argues that these are also sufficient to ensure the existence of a maximum. The intertemporal competitive equilibrium is studied in section 3 while section 4 focuses on the optimal allocation and its decentralization trough an adequate tax policy. Finally, section 5 is devoted to the conclusion.

2 The model

We consider an overlapping generations model where a given generation lives for three periods and has perfect foresight. Population is constant and normalized to one. When young, the agent does not take any decision and inherits aspirations from the previous adult generation, h_t. In adulthood, the agent supplies inelastically one unit of labor and earns in exchange the real wage w_t. This wage is split between present consumption c_t, savings s_t and maintenance investment m_t:

$$w_t = c_t + s_t + m_t. \tag{1}$$

In old age, the agent retires and earns the gross return R_{t+1} on his savings from which he consumes d_{t+1}:

$$d_{t+1} = R_{t+1} s_t. \tag{2}$$

Environmental quality evolves according to:

$$E_{t+1} = E_t - \chi(c_t + d_t) + \xi m_t, \tag{3}$$

where $\chi > 0$ measures the impact of total consumption on environmental quality and $\xi > 0$ the effectiveness of maintenance investment. The initial level of environmental quality is given by $E_0 > 0$. It should be noticed that we don't include a natural regeneration rate concerning environmental quality such that without human activities (or in the case where consumption and maintenance activities exactly offset each other), environmental quality is constant.

The fact that present decisions only affect future levels of environmental quality creates an intergenerational externality, which is justified by the fact that the evolution of environmental quality is a long term process. The negative externalities can in general arise several years after the consumption decisions.

The life-cycle utility function of the representative generation is defined over present and future consumption, future environmental quality as well as consumption aspirations and takes the following form:

$$U(c_t, h_t, d_{t+1}, E_{t+1}) = \theta \ln(c_t - \gamma h_t) + \delta \ln(d_{t+1}) + \eta \ln(E_{t+1}). \tag{4}$$

Aspirations can be seen as a frame of reference against which present consumption is evaluated and γ represents the intensity of the aspiration effect. The assumption that only future environmental quality is included in the utility function is standard and follows the work of John and Pecchenino (1994) as well as that of Ono (1996). The parameters θ, δ and η are the weights associated respectively to present consumption, future consumption and environmental quality. Furthermore, we should assume that $\theta + \delta + \eta = 1$ and that $\theta > \delta + \eta$. The last assumption implies that the representative generation exhibits a preference toward the present.

As in De la Croix (1996) and De la Croix and Michel (1999), we will consider that aspirations are equivalent to the consumption of the previous adult generation such that children get used to particular consumption standards when living with their parents:

$$h_t = c_{t-1}. \tag{5}$$

We also assume that preferences do not exhibit habits so that the old generation does not compare its consumption level to the one enjoyed during adulthood. This assumption can be justified by empirical evidence showing that aspirations are less important for older persons. Clark and Oswald (1996) show for example that reported satisfaction levels increase with age. Older persons putting less weight on comparisons in their welfare evaluation. In the present framework, the economy faces two kind of intergenerational externalities. The first one is due to aspirations as a frame of reference originating in the consumption of the previous adult generation. The second is due to the impact of current consumption decisions on the level of environmental quality enjoyed by future generations.

Concerning production, there is a representative firm which produces an homogeneous good with a Cobb-Douglas production function, $y_t = Ak_t^{\alpha}$ where α is the share of capital in the production process, A is a time-invariant productivity parameter and we assume complete depreciation after one period. The representative firm maximizes profits in a competitive market that clears:

$$R_t = \alpha A k_t^{\alpha-1}, \tag{6}$$

$$w_t = (1-\alpha) A k_t^{\alpha}, \tag{7}$$

$$s_t = k_{t+1}. \tag{8}$$

A representative generation faces the following problem:

$$\max_{c_t, d_{t+1}, s_t, m_t} \theta \ln(c_t - \gamma h_t) + \delta \ln(d_{t+1}) + \eta \ln(E_{t+1}) \tag{9}$$

subject to

$$\begin{cases} w_t = c_t + s_t + m_t, \\ d_{t+1} = R_{t+1} s_t, \\ E_{t+1} = E_t - \chi(c_t + d_t) + \xi m_t, \\ c_t, d_{t+1}, s_t, m_t \geq 0, \end{cases}$$

given w_t, R_{t+1}, E_t and d_t.

Substituting for c_t, d_{t+1} and E_{t+1} in expression (9) and taking the derivative with respect to s_t and m_t we obtain the following first order conditions:

$$\frac{\partial U}{\partial s_t} = \frac{-\theta}{c_t - \gamma h_t} + \frac{\delta R_{t+1}}{d_{t+1}} + \frac{\eta \chi}{E_{t+1}} = 0, \tag{10}$$

$$\frac{\partial U}{\partial m_t} = \frac{-\theta}{c_t - \gamma h_t} + \frac{\eta(\chi + \xi)}{E_{t+1}} = 0. \tag{11}$$

Concerning second order conditions, we can see that our constraints are linear. We thus only need to prove that our utility function is concave. The computation of the Hessian matrix concerning the utility function gives:

$$H = \begin{bmatrix} -\frac{\theta}{(c_t-\gamma h_t)^2} & 0 & 0 \\ 0 & -\frac{\delta}{(d_{t+1})^2} & 0 \\ 0 & 0 & -\frac{\eta}{(E_{t+1})^2} \end{bmatrix}. \quad (12)$$

As can be seen immediately, the leading principal minors alternate in sign such that the Hessian matrix is negative definite and our objective function is indeed concave. We can proceed with the first order conditions which at equilibrium can be written as

$$\delta E_{t+1} = \eta \xi k_{t+1}, \quad (13)$$

$$\theta E_{t+1} = \eta(\chi + \xi)(c_t - \gamma c_{t-1}). \quad (14)$$

Expression (13) represents the equality between the marginal benefit of increasing savings and the marginal cost of decreasing maintenance investment. In order to identify the impact of the different parameters, we use the implicit function theorem and obtain

$$s_\delta = \frac{s}{\delta} > 0, s_\eta = -\frac{\xi s^2}{\delta E_{t+1}} < 0, s_\xi = -\frac{\eta s^2}{\delta E_{t+1}} < 0.$$

From these results, we can see that a higher relative preference for future consumption δ will imply higher savings. A higher relative preference toward environmental quality η will push the representative generation to save less and invest more in maintenance investment in order to increase future environmental quality. Finally, if maintenance effectiveness ξ is relatively low, the generation will prefer to save more in order to consume more in the future and to avoid environmental degradation through immediate consumption. Expression (14) represents the equality between the marginal benefit of increasing maintenance investment and the marginal cost of decreasing present consumption. As before, we use the implicit function theorem and obtain

$$c_\theta = \frac{(c_t - \gamma h_t)}{\theta} > 0, c_\gamma = h_t > 0, c_\eta = -\frac{(\chi + \xi)(c_t - \gamma h_t)^2}{\theta E_{t+1}} < 0,$$

$$c_\chi = -\frac{\eta(c_t - \gamma h_t)^2}{\theta E_{t+1}} < 0, c_\xi = -\frac{\eta(c_t - \gamma h_t)^2}{\theta E_{t+1}} < 0.$$

A higher relative preference for present consumption θ as well as a higher effect of habits γ imply higher present consumption and thus a decrease in future environmental quality. A higher relative preference toward environmental quality η will imply lower present consumption in order to avoid environmental degradation. Similarly, a higher impact of total consumption on environmental quality χ and of maintenance effectiveness ξ will imply a substitution toward savings or maintenance investment. We can now proceed with the study of the intertemporal equilibrium which is the subject of the next section.

3 Intertemporal competitive equilibrium

3.1 Steady-state analysis

By using the first order conditions and the budget constraints, we are able to define the intertemporal equilibrium of this economy.

Definition 1:
An intertemporal equilibrium of this economy is a sequence $\{k_t, h_t\}_0^\infty$ with initial conditions $\{k_0, h_0\}$ that satisfies the following difference equations:

$$k_{t+1} = \eta k_t + \frac{\delta}{\xi}[\xi - (\chi+\xi)\alpha]Ak_t^\alpha - \frac{\delta}{\xi}(\chi+\xi)\gamma h_t, \tag{15}$$

$$h_{t+1} = \frac{\theta\xi\eta}{\delta(\chi+\xi)}k_t + \frac{\theta[\xi-(\chi+\xi)\alpha]}{\chi+\xi}Ak_t^\alpha + (1-\theta)\gamma h_t. \tag{16}$$

By setting $k_{t+1} = k_t = \overline{k}$ and $h_{t+1} = h_t = \overline{h} = \overline{c}$, we derive the steady-states of this economy. There exist two steady-states, the first one is trivial with $\{\overline{k}, \overline{c}\} = (0, 0)$. The other steady-state is given by

$$\overline{k} = \left\{\frac{A\delta[\xi-(\chi+\xi)\alpha](1-\gamma)}{\xi(1-\gamma)(1-\eta)+\theta\xi\gamma}\right\}^{\frac{1}{1-\alpha}}, \tag{17}$$

$$\overline{c} = \frac{\theta\xi}{\delta(\chi+\xi)(1-\gamma)}\overline{k}. \tag{18}$$

The condition for the existence of a positive steady-state is that the numerator and the denominator of expression (17) have the same sign. The denominator is always positive given our assumptions concerning the parameters of the model. The numerator is positive only if the following condition is satisfied:

$$\frac{\chi}{\xi} < \frac{1-\alpha}{\alpha}. \tag{19}$$

This condition implies that the impact of pollution can be higher than the one of maintenance investment provided that the difference is not too large

and bounded above by a ratio involving the elasticity of capital in the production function. Moreover, if this condition is met, output has a positive impact on future capital and present consumption as can be seen from expressions (15) and (16).
In order to assess if the steady-state capital stock is higher or lower compared to the standard Diamond framework we will study the influence of key parameters on the steady-state capital stock. We need to focus on parameters which are not present in the standard model. In order to do so, we take the derivative of expression (17) with respect to γ, ξ, χ and η:

$$\frac{\partial \bar{k}}{\partial \gamma} = -\frac{A\theta\delta[\xi - (\chi + \xi)\alpha]}{\xi(1-\alpha)\{(1-\gamma)[1-\eta] + \theta\gamma\}^2}\bar{k}^{\alpha} < 0. \tag{20}$$

The negative impact of γ on the steady-state capital stock can be explained by the deterring effect that aspirations play on savings and thus on capital accumulation.

$$\frac{\partial \bar{k}}{\partial \xi} = \frac{\{(1-\gamma)[1-\eta] + \theta\gamma\}A\delta(1-\gamma)\chi\alpha}{\xi^2(1-\alpha)\{(1-\gamma)[1-\eta] + \theta\gamma\}^2}\bar{k}^{\alpha} > 0. \tag{21}$$

In this case, the impact is positive since a high maintenance effectiveness allows the adult generation to devote more resources to savings.

$$\frac{\partial \bar{k}}{\partial \chi} = -\frac{A\delta\alpha(1-\gamma)}{(1-\alpha)\xi\{(1-\gamma)[1-\eta] + \theta\gamma\}}\bar{k}^{\alpha} < 0. \tag{22}$$

Intuitively, a large impact of consumption on environmental degradation pushes the adult generation to invest in maintenance at the expenses of the capital stock.

$$\frac{\partial \bar{k}}{\partial \eta} = \frac{A\delta\xi[\xi - (\chi + \xi)\alpha](1-\gamma)^2}{(1-\alpha)\{\xi(1-\gamma)[1-\eta] + \theta\xi\gamma\}^2}\bar{k}^{\alpha} > 0. \tag{23}$$

Finally, a larger relative preference for future environmental quality induces the adult generation to save more in order to avoid environmental degradation through immediate consumption.
The opposite effects that we have identified imply that the steady-state capital stock might be higher or lower than in the standard Diamond model. This is a clear difference with the model without environmental quality studied by De la Croix (1996) where the presence of aspirations guarantees that the steady-state capital stock is always lower compared to the standard Diamond framework. We can now proceed with the dynamics of the model in the following section.

3.2 Dynamics

In this section, we linearize the model around the non-trivial steady-state in order to assess the local stability of our dynamical system (see, for example, Azariadis, 1993). We start by computing the Jacobian matrix around the non-trivial steady-state $(\overline{k}, \overline{c})$:

$$J = \begin{bmatrix} \frac{\eta(1-\alpha)(1-\gamma)+\alpha[1-\gamma(1-\theta)]}{1-\gamma} & -\frac{\delta(\chi+\xi)\gamma}{\xi} \\ \frac{\theta\xi}{\delta(\chi+\xi)}\left\{\frac{\eta(1-\alpha)(1-\gamma)+\alpha[1-\gamma(1-\theta)]}{1-\gamma}\right\} & (1-\theta)\gamma \end{bmatrix}. \tag{24}$$

The characteristic function $P(\lambda)$ is given by:

$$P(\lambda) = \lambda^2 - Tr(J)\lambda + Det(J) = 0,$$

where

$$Det(J) = \frac{\eta\gamma(1-\alpha)(1-\gamma)+\alpha\gamma[1-\gamma(1-\theta)]}{1-\gamma}, \tag{25}$$

$$Tr(J) = \frac{Det(J)}{\gamma} + (1-\theta)\gamma. \tag{26}$$

It is useful to notice that given our assumptions concerning the parameters of the model, both the determinant and the trace of the Jacobian matrix can only take positive values. The following proposition assesses the possible dynamic behavior of our planar system.

Proposition 1:
Consider an interior competitive equilibrium of this economy and define:

$$\hat{\gamma} = \frac{1+\alpha+\eta(1-\alpha) - \sqrt{[1+\alpha+\eta(1-\alpha)]^2 - 4[\alpha(1-\theta)+\eta(1-\alpha)]}}{2[\alpha(1-\theta)+\eta(1-\alpha)]}. \tag{27}$$

(i) The fixed point $(\overline{k}(\gamma), \overline{h}(\gamma))$ is asymptotically stable if $\gamma < \hat{\gamma}$ and unstable if $\gamma > \hat{\gamma}$.

(ii) Let $(\hat{k}, \hat{h})$ be the fixed point associated to $\gamma = \hat{\gamma}$ and $\hat{\gamma} > (1-\sqrt{\theta})/(1-\theta)$. There is a neighborhood U of $\hat{\gamma}$ for which there is, either for $\gamma < \hat{\gamma}$ or for $\gamma > \hat{\gamma}$, a closed invariant curve Γ which encircles $(\hat{k}, \hat{h})$ implying the existence of a Neimark-Sacker bifurcation.

Proof. (i) In planar maps, the steady-state is asymptotically stable if $1 + Tr(J) + Det(J) > 0$, $1 - Tr(J) + Det(J) > 0$ and $Det(J) < 0$. In our case, the first condition is always satisfied since $Tr(J) > 0$ and $Det(J) > 0$. The second condition is equivalent to $1 > Det(J)\left(\frac{1-\gamma}{\gamma}\right) + (1-\theta)\gamma$ which can be written as a condition on the parameter γ by using expression (25). The second condition is satisfied if:

$$\gamma < \frac{1-\eta}{1-\theta-\eta}, \tag{28}$$

which is always the case since the right hand side of the previous expression is higher than one.
We are left with the condition $Det(J) < 1$ which can be written as a quadratic inequality whose roots are given by

$$\hat{\gamma} = \frac{1+\alpha+\eta(1-\alpha) \pm \sqrt{[1+\alpha+\eta(1-\alpha)]^2 - 4[\alpha(1-\theta)+\eta(1-\alpha)]}}{2[\alpha(1-\theta)+\eta(1-\alpha)]}. \tag{29}$$

However, we can prove that the larger root should be discarded since it can only take values larger than one. In order to see this it is enough to notice that $1+\alpha+\eta(1-\alpha) > 2[\alpha(1-\theta)+\eta(1-\alpha)]$. We are thus left with the smaller root which gives us a value for $\hat{\gamma}$ under which the steady-state is asymptotically stable.
(ii) In the present case, it is possible that the system generates a Neimark-Sacker bifurcation provided that $Det(J) = 1$ and $Tr(J) \in [-2, 2]$. We know that $Det(J) = 1$ implies $\gamma = \hat{\gamma}$ and given that the Trace can only take positive values we can focus on the case where $Tr(J) < 2$. The latter condition is satisfied if $\hat{\gamma} \in [\frac{1-\sqrt{\theta}}{1-\theta}, \frac{1+\sqrt{\theta}}{1-\theta}]$. However, the higher bound of the interval is higher than one and can thus be discarded by assumption. Under these two conditions, the two eigenvalues are complex conjugates, they cross the unit circle at non-zero speed when γ changes around $\hat{\gamma}$ and none of them may be of the first four roots of unity. The fulfilment of these conditions implies the existence of a Neimark-Sacker bifurcation. □

As can be seen from the proposition, the critical value determining the stability outcome $\hat{\gamma}$ only depends on a limited number of technology and preference parameters (α, θ, η). The parameters affecting environmental quality (χ, ξ) do not play any role in this result despite the fact that they influence the two difference equations defining our intertemporal equilibrium. The main message of Proposition 1 is that the aspiration parameter γ should not be too high in order to obtain the relevant case of asymptotic stability.
The introduction of environmental quality in this framework do not modify substantially the results concerning transitional dynamics compared to the model without environmental quality studied by De la Croix (1996). In his model, there is also a critical value γ under which the model is asymptotically

stable and at which the system can generate a Neimark-Sacker bifurcation. Of course, the critical value is not the same and in our case it also depends on the relative preference toward environmental quality η. The Neimark-Sacker bifurcation implies the possible existence of a limit-cycle (the invariant curve Γ) inducing constant fluctuations of endogenous variables around the steady-state. It is important to explain the mechanism by which cyclical behavior is possible in the present framework. At the intertemporal equilibrium, savings finance the capital stock that is used to produce and to pay the wages of the adult generation. This process exhibits decreasing returns given our assumption on the production function. On the other hand, aspirations generate an increase in present consumption which in turn decreases environmental quality. At some point, the negative impact on environmental quality increases the demand for maintenance investment. The combination of aspirations and maintenance investment has a depressing effect on savings inducing a recession. As a consequence, the capital stock decreases followed by consumption. Maintenance investment and the decrease in consumption having a positive impact on environmental quality. Once the decrease is sufficiently strong, a rise in savings occurs together with the start of an expansion period[1].
The main difference between the model with and without environmental quality is then that in the present case the competitive steady-state capital stock can be higher compared to the standard Diamond framework while this is not possible in the model of De la Croix (1996). This result will play an important role concerning policy issues. We should now turn our attention to the study of the optimal allocation and the question of its decentralization.

4 Optimality

4.1 Optimal allocation

In this section, we consider the case of a central planner who chooses the allocation of resources in order to maximize the discounted welfare of current and future generations. Contrary to the representative generation in the competitive equilibrium case, the planner takes into account the impact of aspirations as well as the one of old age consumption on environmental quality. The social discount factor is given by β and the planner will maximise the discounted sum of utilities subject to the feasibility constraint, the evolution of aspirations and the law of motion for environmental quality. The

[1]De la Croix also shows that his model can generate local oscillations on a interval determined by specific values of γ. It is possible to obtain a similar result in our framework.

optimization problem is the following:

$$\max_{\{c_t,d_{t+1},m_t,h_{t+1},k_{t+1},E_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t[\theta \ln(c_t - \gamma h_t) + \delta \ln(d_{t+1}) + \eta \ln(E_{t+1})] \tag{30}$$

subject to:

$$\begin{cases} Ak_t^\alpha = c_t + d_t + k_{t+1} + m_t, \\ h_t = c_{t-1}, \\ E_{t+1} = E_t - \chi(c_t + d_t) + \xi m_t, \end{cases}$$

given initial conditions $\{k_0, E_0, h_0\}$.
The Lagrangian function is the following:

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^{\infty} \beta^t[\theta \ln(c_t - \gamma h_t) + \delta \ln(d_{t+1}) + \eta \ln(E_{t+1})] \qquad (31) \\ &+ \sum_{t=0}^{\infty} \beta^t \lambda_t (Ak_t^\alpha - c_t - d_t - m_t - k_{t+1}) \\ &+ \sum_{t=0}^{\infty} \beta^t \mu_t (E_{t+1} - E_t + \chi(c_t + d_t) - \xi m_t) \\ &+ \sum_{t=0}^{\infty} \beta^t \nu_t (h_t - c_{t-1}). \end{aligned}$$

The first order conditions of the maximization problem are

$$\frac{\partial \mathcal{L}}{\partial c_t} = \frac{\beta^t \theta}{c_t - \gamma h_t} - \beta^t \lambda_t + \beta^t \mu_t \chi - \beta^{t+1} \nu_{t+1} = 0, \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = \frac{\beta^t \delta}{d_{t+1}} - \beta^{t+1} \lambda_{t+1} + \beta^{t+1} \mu_{t+1} \chi = 0, \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = -\beta^t \lambda_t - \beta^t \mu_t \xi = 0, \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} A \alpha k_{t+1}^{\alpha-1} = 0, \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial h_{t+1}} = -\frac{\beta^{t+1} \theta \gamma}{c_{t+1} - \gamma c_t} + \beta^{t+1} \nu_{t+1} = 0, \tag{36}$$

$$\frac{\partial \mathcal{L}}{\partial E_{t+1}} = \frac{\beta^t \eta}{E_{t+1}} + \beta^t \mu_t - \beta^{t+1} \mu_{t+1} = 0, \tag{37}$$

$$\lim_{t\to\infty} \beta^t \lambda_t k_t = 0, \tag{38}$$

$$\lim_{t\to\infty} \beta^t \mu_t E_t = 0, \tag{39}$$

$$\lim_{t\to\infty} \beta^t \nu_t h_t = 0. \tag{40}$$

Concerning second order conditions, given our constraints, we once again need to prove that our utility function is concave. In the planner's case, the Hessian matrix is given by

$$H = \begin{bmatrix} -\frac{\theta}{(c_t-\gamma h_t)^2} & \frac{\theta\gamma}{(c_t-\gamma h_t)^2} & 0 & 0 \\ \frac{\theta\gamma}{(c_t-\gamma h_t)^2} & -\frac{\theta\gamma^2}{(c_t-\gamma h_t)^2} & 0 & 0 \\ 0 & 0 & -\frac{\delta}{(d_{t+1})^2} & 0 \\ 0 & 0 & 0 & -\frac{\eta}{(E_{t+1})^2} \end{bmatrix}. \tag{41}$$

The leading principal minors (Δ) are $\Delta_1 = -\theta/(c_t - \gamma h_t)^2 < 0$ and $\Delta_2 = \Delta_3 = \Delta_4 = 0$ such that the Hessian matrix is semi-negative definite and our objective function is indeed concave. The first order conditions are also sufficient for optimality and we obtain:

$$\frac{\delta}{\beta d_t^*} = \frac{\theta}{c_t^* - \gamma h_t^*} - \frac{\beta\gamma\theta}{c_{t+1}^* - \gamma c_t^*}, \tag{42}$$

$$d_{t+1}^* = \beta d_t^* A\alpha k_{t+1}^{*\alpha-1}, \tag{43}$$

$$\frac{\eta(\chi+\xi)}{E_{t+1}^*} = \frac{\delta}{\beta d_t^*} - \frac{\delta}{d_{t+1}^*}, \tag{44}$$

where starred variables denote the optimal outcome. Expression (42) describes the allocation of consumption between generations alive at the same time. The marginal utility of consumption of the adult is corrected to internalize the future impact of aspirations and is equalized to the marginal utility of consumption of the old. Expression (43) is standard and describes the inter-temporal allocation of consumption. Finally, expression (44) describes the allocation between consumption and maintenance investment at a point in time. The marginal benefit of maintenance investment is equalized to the marginal utility of consumption in adulthood corrected to internalize two effects: the impact of future aspirations and the one of future old-age consumption.

Definition 2:
An intertemporal optimal allocation of this economy is a sequence $\{c_t^*, d_t^*, m_t^*, k_t^*, h_t^*, E_t^*\}_0^\infty$ with initial conditions $\{k_0, h_0, E_0\}$ that satisfies the following difference equations:

$$c_{t+1}^* = \frac{\beta^2\theta\gamma(c_t^* - \gamma h_t^*)d_t^*}{\beta\theta d_t^* - \delta(c_t^* - \gamma h_t^*)} + \gamma c_t^*, \tag{45}$$

$$d_{t+1}^* = \frac{\beta^2\delta d_t^* E_{t+1}^*}{\delta E_{t+1}^* - \eta\beta(\chi+\xi)d_t^*}, \tag{46}$$

$$
\begin{aligned}
d^*_{t+1} &= \beta d^*_t A\alpha k^{*\alpha-1}_{t+1}, & (47)\\
k^*_{t+1} &= Ak^{*\alpha}_t - c^*_t - d^*_t - m^*_t, & (48)\\
E^*_{t+1} &= E^*_t - \chi(c^*_t + d^*_t) + \xi m^*_t, & (49)\\
h^*_{t+1} &= c^*_t. & (50)
\end{aligned}
$$

The steady-state of this optimal allocation is given by

$$
\begin{aligned}
\overline{k}^* &= (A\alpha\beta)^{\frac{1}{1-\alpha}}, & (51)\\
\overline{E}^* &= \frac{\xi\eta\beta(1-\alpha\beta)(1-\gamma)}{\alpha\beta(1-\beta)[\theta\beta(1-\beta\gamma)+\delta(1-\gamma)]}\overline{k}^*, & (52)\\
\overline{c}^* &= \frac{\xi\theta\beta(1-\alpha\beta)(1-\beta\gamma)}{\alpha\beta(\chi+\xi)[\theta\beta(1-\beta\gamma)+\delta(1-\gamma)]}\overline{k}^*, & (53)\\
\overline{d}^* &= \frac{\xi\delta(1-\alpha\beta)(1-\gamma)}{\alpha\beta(\chi+\xi)[\theta\beta(1-\beta\gamma)+\delta(1-\gamma)]}\overline{k}^*, & (54)\\
\overline{m}^* &= \frac{\chi(1-\alpha\beta)}{\alpha\beta(\chi+\xi)}\overline{k}^*. & (55)
\end{aligned}
$$

Expression (51) is the modified golden rule such that the optimal steady-state capital stock level is the same as in the standard overlapping generations model. The allocation of consumption between adults and old as well as the level of environmental quality at the steady-state differ clearly from the competitive equilibrium. It can also be seen that the steady-state capital stock level is not necessarily higher in the optimal case.

Proposition 2:
The steady-state capital stock is higher in the optimal case if and only if the following condition is satisfied:

$$
\gamma > \frac{\delta[\xi-(\chi+\xi)]-\beta\alpha\xi(1-\eta)}{\beta\theta+\delta[\xi-(\chi+\xi)]-\beta\alpha\xi(1-\eta)}. \qquad (56)
$$

Proof. It is straightforward to compare the steady-state capital stock level of the competitive economy given by expression (17) with the one from the modified golden rule giving rise to inequality (56). □

The condition imposes a lower bound on the aspiration parameter so that when the impact of aspirations is sufficiently low the economy displays over-accumulation of capital as in the model of Ono (1996). In this case, the appropriate policy goes against the one proposed in the case without environmental degradation studied by De la Croix and Michel (1999) where investment always needs to be subsidized. This is due to the fact that in the competitive case, the adult generation has an incentive to increase savings in order to avoid environmental degradation when old. However, by not taking into account future externalities, the representative generation might generate a steady-state capital stock level higher than the one of the modified golden rule. It can also be noticed that the right hand side of expression (56) is increasing in β implying that the choice of the social discount factor is fundamental concerning the policy that should be implemented. A sufficiently high social discount factor might ensure a steady-state capital stock that is higher in the optimal case.

As we will see in the following, the social discount factor can also play an important role concerning the dynamic behavior of the optimal solution. In general, optimal solutions are characterized by monotonic convergence in the one-sector overlapping generations model. This is however only true if the utility function is separable across generations and periods of life. When the utility function is non-separable across periods of life, Michel and Venditti (1997) have shown that optimal paths can be characterized by endogenous fluctuations under the form of optimal two-cycles. Concerning non-separability across generations, which is also the case in the present framework, De la Croix and Michel (1999) have shown that local converging oscillations are a possible outcome in the optimal case. Our objective here is to show that when the social discount factor is too low, our optimal solution can also exhibit local converging oscillations. In order to do so, we will proceed numerically since the number of endogenous variables does not allow us to solve analytically for the eigenvalues of the Jacobian matrix. We will proceed with the numerical simulations in two different cases that only differ in terms of their social discount factors.

Table 1: Value for the parameters

Parameter	Value
A	10
α	0.33
β	0.8 or 0.99
θ	0.6
δ	0.25
η	0.15
ξ	0.2
χ	0.4
γ	0.65

The value for the parameters are given in Table 1. The elasticity of capital in the production function α is set at 0.33 which is in accordance with empirical evidence. The relative preference for present consumption θ is higher then the ones for future consumption δ and environmental quality η while their sum is equal to one in accordance with our assumptions. In order to respect the necessary condition for a positive steady-state in the competitive equilibrium case, the ratio χ/ξ should be lower than 2.33 given that $\alpha = 0.33$ so that we choose a ratio equal to 2 with $\xi = 0.2$ and $\chi = 0.4$. We also choose to focus on a quite high level of aspirations with $\gamma = 0.65$. The difference in terms of social discount factor distinguishes between a case with $\beta = 0.8$ and the quasi golden rule case where $\beta = 0.99$. The results concerning the eigenvalues for the case where $\beta = 0.8$ are given in Table 2.

Table 2: Eigenvalues $\beta = 0.8$

Modulus	Real	Imaginary
1.038e-16	-1.038e-16	0
0.6036	0.5763	0.1795
0.6036	0.5763	-0.1795
2.071	1.977	0.6159
2.071	1.977	-0.6159

In this case the system exhibits damped oscillations and thus convergence to the steady-state implying short-run fluctuations. We will now simulate the model when $\beta = 0.99$ in order to see if the quasi golden rule case can generate monotonic convergence toward the steady-state. The results are given in Table 3.

Table 3: Eigenvalues $\beta = 0.99$

Modulus	Real	Imaginary
1.202e-15	-1.202e-15	0
0.6503	0.6503	0
0.9806	0.9806	0
1.03	1.03	0
1.553	1.553	0

In this case, the system exhibits monotonic convergence to the steady-state and endogenous fluctuations are not present any more. This suggest that choosing a sufficiently high discount factor might avoid possible endogenous fluctuations in the optimal case.

4.2 Decentralizing the first best solution

Beyond choosing an appropriate social discount factor, a policy intervention should focus on three objectives: adjusting the consumption of the adult correcting for the future impact of aspirations, adjusting savings in order to reach the modified golden rule and adjusting old age consumption correcting for future environmental degradation. As we will see shortly, these objectives can be reached by taxing (or subsidizing) savings, subsidizing maintenance investment and using appropriate lump sum transfers. A different policy based for example on consumption taxes is also possible, however, in order to be able to compare our policy to the case without environmental quality (De la Croix and Michel, 1999) as well as the case without aspirations (Ono, 1996), we choose to rely on a policy that has been studied in both cases. Let's denote the investment subsidy by i_t and the maintenance investment subsidy by l_t. We will also use lump-sum transfers to the adult, g_t, and to the old, n_t.
The maximization problem of the individual becomes

$$\max_{c_t, d_{t+1}, s_t, m_t} \theta \ln(c_t - \gamma h_t) + \delta \ln(d_{t+1}) + \eta \ln(E_{t+1}) \tag{57}$$

subject to

$$\begin{cases} w_t + g_t = c_t + s_t + m_t, \\ d_{t+1} = R_{t+1}(1 + i_{t+1})s_t + n_{t+1}, \\ E_{t+1} = E_t - \chi(c_t + d_t) + \xi(1 + l_t)m_t, \\ c_t, d_{t+1}, s_t, m_t \geq 0, \end{cases}$$

given w_t, R_{t+1}, E_t and d_t.
We then obtain the following first order conditions:

$$\frac{\delta R_{t+1}}{d_{t+1}}(1 + i_{t+1}) = \frac{\theta}{c_t - \gamma h_t} - \frac{\eta\chi}{E_{t+1}}, \tag{58}$$

$$\frac{\eta[\chi + \xi(1 + l_t)]}{E_{t+1}} = \frac{\theta}{c_t - \gamma h_t}. \tag{59}$$

Proposition 3:
In order to decentralize the optimal allocation:

(i) The investment subsidy is given by

$$i_{t+1} = \frac{\beta d_t^*}{\delta}\left(\frac{\beta\theta\gamma}{c_{t+1}^* - \gamma c_t^*} - \frac{\eta\chi}{E_{t+1}^*}\right) \gtreqless 0. \tag{60}$$

(ii) The maintenance investment subsidy is given by

$$l_t = \frac{E_{t+1}^*}{\eta\xi}\left(\frac{\beta\theta\gamma}{c_{t+1}^* - \gamma c_t^*} + \frac{\delta}{d_{t+1}^*}\right) > 0. \tag{61}$$

Proof. Combining the first order conditions of the planner's problem with expressions (58) and (59) it is straightforward to compute the values of both tax rates. Contrary to what could be thought the policy is not forward looking since c^*_{t+1}, d^*_{t+1} and E^*_{t+1} can be defined from the first order conditions and the law of motion for environmental quality. □

The decentralization of the first best solution is completed by the following lump-sum transfers:

$$g_t = k^*_{t+1} + c^*_t + m^*_t - w^*_t, \quad (62)$$
$$n_t = -g_t - i_t R^*_t k^*_t - l_t m^*_t. \quad (63)$$

Expression (62) ensures that the capital stock is set at the level of the modified golden rule while expression (63) is the planner's budget constraint.
As can be seen from expression (60), the planner might decide to either subsidize or tax capital. The choice is based on the comparison between the impact of consumption on future aspirations and on future environmental quality in terms of utility. If the impact of aspirations is higher, then the planner decides to subsidize investment. On the contrary, if the impact of consumption on the environment is higher, then the planner decides to tax capital.
The appropriate policy also requires a subsidy to maintenance investment (expression (61)) in order to internalize the impact of present consumption on future aspirations as well as the impact of old age consumption on future environmental quality.
In a model without aspirations like the one of Ono (1996), the planner always taxes capital and subsidizes maintenance investment while in a model without environmental externalities De la Croix and Michel (1999) show that investment should always be subsidized. Our result concerning capital taxation is thus an intermediary one between the two cases. Maintenance investment should always be subsidized once environmental externalities are present since the planner needs to induce the adult generation to consume less of the polluting good.
Concerning the social discount factor, a relatively high value for β implies that it is more probable to obtain a capital subsidy since the term between brackets in expression (60) is increasing in β. Clearly, the possibility of taxing capital in the present framework is reduced if the planner puts a high weight on future generations. Even if he decides to accumulate more capital than in the competitive equilibrium, the planner will also increase maintenance investment in order to avoid too much environmental degradation in the future (notice that the maintenance subsidy is also increasing in β).

The fact that both instruments are implemented together provides an economic intuition concerning the possible existence of oscillations in the case where the social discount factor is sufficiently low. The necessary increase in maintenance investment in the optimal case coupled with the existence of aspirations exerts a negative impact on capital accumulation which can generate cyclical behavior. This is even more the case if the optimal policy suggests that investment should be taxed.
The possibility of taxing capital in the present model is also closely related to the fact that capital is indirectly responsible for environmental degradation. Here we do not take into account the possibility of having a clean sector from which capital could be accumulated or a second non-polluting consumption good. Concerning the behavior of actual economies, the policy would imply taxing only investment devoted to polluting activities or reducing the consumption of specific polluting goods.

5 Conclusion

In this paper, we have extended the overlapping generations literature by introducing aspirations in a model with environmental quality and maintenance investment. In this economy, the competitive steady-state capital stock can be higher or lower compared to the standard Diamond economy depending on the value of the parameters.
Concerning the dynamics of the model, asymptotic stability is only guaranteed if the parameter governing the impact of aspirations on the utility function is not too large. At the boundary value of this parameter, the system looses stability and can generate a potential limit-cycle.
We then focused on the optimal allocation which converges to the modified golden rule for the capital stock and is clearly different from the competitive equilibrium in terms of consumption and environmental quality allocations. We found evidence showing that a sufficiently high discount factor is necessary in order to avoid possible local oscillations. We then derived an optimal tax policy allowing the decentralization of the first best solution. This one is characterized by a tax or a subsidy on investment depending on the magnitude of both externalities (environmental degradation and aspirations) and by a subsidy to maintenance investment.
Further research could focus on the possible combination of aspirations and habits in environmental quality as well as on the inclusion of a non-polluting sector.

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