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Abstract

By using robustness methods we design HCRs that explicitly include scientific uncertainty. Under scientific uncertainty –when the perceived model can be generated by a nearby operating model– robust HCRs are designed assuming that the (inferred) operating model is more persistent than the perceived model. As a result, a robust HCR has a steeper ratio between fishing mortality and biomass than a non robust one. We prove that constant effort HCRs are not robust. Moreover, rather than decreasing fishing mortality reference points for exploitation, the optimal robust response to scientific uncertainty is to increases biomass precautionary limit points when knowledge about the stock status decreases. Finally, we show that robustness can be implemented if fishing mortality is increased faster than linearly –by a factor of 2-fold– when a stock is assessed as above $0.5B_{\mathrm{MSY}}$. We illustrate our findings by designing HCRs for 17 ICES stocks using this rule of thumb.

Keywords: HCR, Robustness.
JEL codes: O1, AMS 91B76, 92D25.

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1 Introduction

Scientific advice on fishery stocks is provided by evaluating the performance of the fishery system relative to harvesting control rules (HCR) based on reference points. For example, the International Council for the Exploration of the Sea (ICES) advises 250 stocks by using a precautionary approach (PA) within a maximum sustainable yield (MSY) framework.[1] To attain the long-term maximum yield, maintaining the stock above a limit point, B_{lim}, ICES uses the *MSY advice rule*: fishing mortality is set as F_{MSY} (F at which the maximum sustainable yield is achieved) if the spawning–stock biomass, B, is at or above the MSY Biomass trigger point, B_{trigger}, (the lower bound of spawning–stock biomass fluctuation around B_{MSY}); fishing mortality is reduced linearly if a stock is assessed to be below B_{trigger}, and finally, fishing mortality is set to zero if a stock is depleted below B_{lim} (see Figure 1).

This HCR requires a relatively high level of data and knowledge on the dynamics of the stocks concerned. As Punt and Donovan (2007) point out *"uncertainty attributable to lack of understanding of the true underlying system and to ineffective implementation may dominate the sources of error that must be accounted for if management is to be successful"*.

Unfortunately, the ICES *MSY advice rule* does not explicitly include any way of dealing with those uncertainties. Therefore, under scientific uncertainty –i.e, when the data and knowledge requirements are not fulfilled –ICES *increases margins when knowledge about the stock status decreases*.[2] Based on experience obtained by using a simulation modeling approach[3] lower reference points for exploitation – such as $F_{0.1}$, for example– have been

[1] Recurring advice is provided to the European Commission, the North Atlantic Salmon Conservation Organization (NASCO), and the North East Atlantic Fisheries Commission (NEAFC)). In addition to this recurring advice ICES also provides advice in response to special requests from the Commissions mentioned above and from the Helsinki Commission (HELCOM), the OSPAR Commission (OSPAR) and ICES Member Countries.

[2] ICES Advice 2016, Book 1, http://www.ices.dk/sites/pub/Publication%20Reports/Advice/2016/2016/Introduction_to_advice_2016.pdf.

[3] Several stocks use FLR methodology – Kell et al. (2007)– to determine the effectiveness of this management procedure.

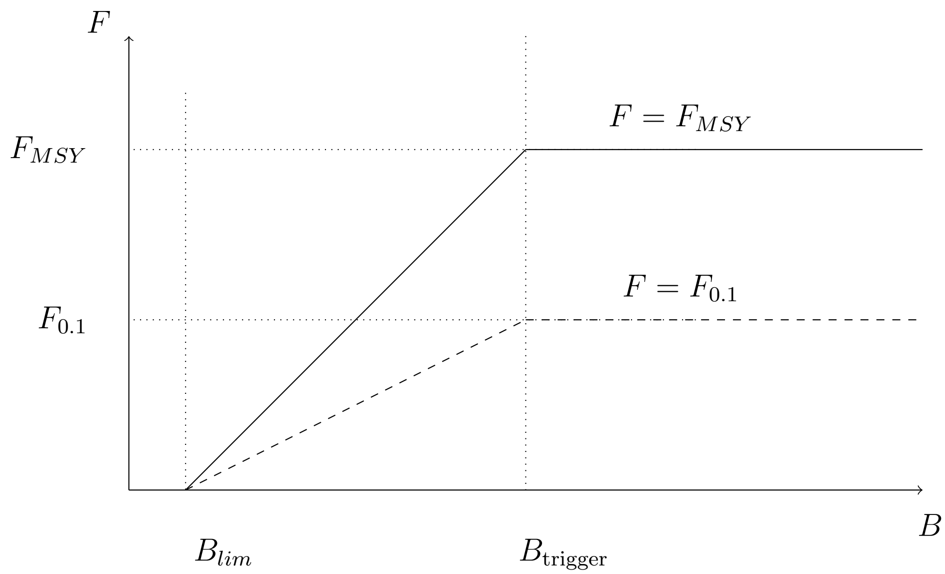

Figure 1: ICES MSY advice rule: the solid line represents the advice rule without scientific uncertainty; the dashed line represents the advice rule with scientific uncertainty.

applied.[4]

In this paper we design HCRs that explicitly include scientific uncertainty. We assume that managers understand that perceived dynamics -a version of Hannesson (1975) with stochastic recruitment- is an approximation of the real (operating) model. Following the idea of Fellner (1965) and Hansen and Sargent (2008), we characterize robust HCR by distorting the perceived dynamics with the worst case estimate of the operating model. In this context, a robust precautionary HCR is characterized by solving an extremization problem: managers maximize the performance of the fishery assuming that a hypothetical malevolent nature chooses the level of scientific uncertainty –the distortion of the operating model– in order to minimize fishery performance.

What does this reveal? When managers are concerned about scientific uncertainty, they know that a fraction of the volatility observed in the data is generated by the ignorance of the observer. The size of that uncertainty, –when a hypothetical malevolent nature so

[4] $F_{0.1}$ is the value of F where the yield per recruit slope is 10 percent of the maximum yield per recruit slope.

chooses – is proportional to the perceived recruitment shock. Therefore, they infer that the operating model –which is generating the perceived recruitment shock– is more persistent than the perceived model. As a result HCRs are designed by assuming a more persistent process.

We prove that constant effort HCRs –which provide precautionary advice when recruitments are uncorrelated- are not robust under scientific uncertainty. A robust HCR has a steeper ratio between fishing mortality and biomass than a non-robust one. Rather than decreases fishing mortality reference points for exploitation, the optimal robust response to scientific uncertainty is *to increase biomass limits when knowledge about the stock status decreases*.

We illustrate our findings with a numerical example. We show that robustness can be implemented if fishing mortality is increased faster than linearly –by a factor of 2-fold– when a stock is assessed as above *an endogenous robust limit point*, $0.5B_{\mathrm{MSY}}$. We illustrate our findings by designing HCRs for 17 ICES stocks using this rule of thumb.

Our paper studies robust control in natural resources and is thus closely linked to Roseta-Palma and Xepapadeas (2004), Athanassogloua and Xepapadeas (2012) and Xepapadeas and Roseta-Palma (2013). However two key distinctions should be emphasized in our work: First, we characterize robust HCRs in closed form. Second, we show that for log linear dynamics a simple rule of thumb for robust HCRs can be obtained.

The rest of the paper is organized as follows: Section 2 describes the model. In Section 3 we characterize robust HCRs. In Section 4 we derive a rule of thumb to reclassify the status of 40 ICES stocks. Section 5 concludes.

2 Model

We build on Da-Rocha and Mato-Amboage (2016). We consider a stochastic version of the fishery of Hannesson (1975) with two age classes: juveniles and adults. Let $N_{t,1}$, and $N_{t,2}$ be the population of juveniles and adults in period t, respectively. Each year, t, a stochastic exogenous number of juvenile fish are born, $N_{t,1} = exp(z_t)$, where z_t is a random variable that determines the recruitment of the fishery. The managers of the fishery perceive that the stochastic recruitment process follows an AR(1) process,

$$z_{t+1} = \rho z_t + \tilde{\varepsilon}_{t+1}, \tag{1}$$

where $\tilde{\varepsilon}_{t+1}$ is a Gaussian i.i.d. process with zero mean and variance σ_ε^2, and $|\rho| \in [0, 1)$ is the correlation coefficient. Managers understand this perceived model as an approximation to the actual operating model. Following Hansen and Sargent (2008), this scientific uncertainty is represented with a set of alternative models of the form

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} + \omega_{t+1}, \tag{2}$$

where ε_{t+1} is another Gaussian i.i.d. process with zero mean and identity variance and ω_{t+1} is a vector of perturbations in the mean of $\tilde{\varepsilon}_{t+1}$, that can feed back on the history of the state, z.

The dynamics of the second age group is then given by $N_{t+1,2} = N_{t,1}e^{-F_t - m}$, where F_t represents the fishing mortality applied in period t and m is the natural mortality rate, which for the sake of simplicity is assumed to be constant over time. Finally, the spawning stock biomass of the fishery is defined as $B_t = \log N_{t,2}$.[5]

[5]This equation implies that the spawning stock biomass is an increasing function of the number of adults in the population and that only a non constant fraction of adults are spawners.

3 Robust HCRs

We assume that fishery managers want to design a precautionary robust HCR, λ, to reach an exogenous target, (B_{tar}, F_{tar}) and avoid the risk of the stock dropping below a limit point, B_{lim}, subject to the stock dynamics. Given that they know that the perceived model (1) is an approximation of the operating model, (2), they are aware of the dynamic misspecification of the model (the scientific uncertainty). Therefore, fishery managers are looking for robust HCRs that minimize the expected net present gap between the fishing mortality, F_t, and biomass, B_t, relative to the targets

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left[(F_t - F_{tar})^2 + \lambda (B_t - B_{tar})^2 \right],$$

avoiding the risk of the stock dropping below a limit point[6], B_{lim},

$$Pr(B \leq B_{lim}) = v, \tag{3}$$

admitting a certain scientific uncertainty level η, i.e.

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} \omega_{t+1}^2 = \eta. \tag{4}$$

The left hand side of equation (4), $E_0 \sum_{t=0}^{\infty} \beta^{t+1} \omega_{t+1}^2$, is an intertemporal measure of the size of model misspecification –the conditional relative entropy. This constraint is used to measure the statistical discrepancy between the perceived model (1) and the operating model (2).[7] Therefore, equation (4) expresses the idea that managers know that the operating model can be any nearby model around the perceived model. Greater scientific uncertainty implies a larger set of alternative models. Therefore, η measures scientific uncertainty. Formally,

[6]ICES set $v = 0.05$, i.e. HCR is precautionary if the stock is above the limit point, B_{lim} with at least a 95% probability.

[7]See Hansen and Sargent (2008).

the conditional relative entropy is constrained to be lower than or equal to an exogenous scientific uncertainty level η.

We characterize the precautionary robust HCR in two steps. In the first step we solve an *extremization problem*

$$\max_{\{F_t,B_{t+1}\}_{t=0}^{\infty}} \min_{\{\omega_{t+1}\}_{t=0}^{\infty}} E_0 \qquad \sum_{t=0}^{\infty} \beta^t \left\{-(F_t - F_{tar})^2 - \lambda(B_t - B_{tar})^2 + \beta\theta\omega_{t+1}^2\right\}, \tag{5}$$

$$s.t. \qquad \begin{cases} B_{t+1} = z_t - F_t - m, \\ z_{t+1} = \rho z_t + \varepsilon_{t+1} + \omega_{t+1}. \end{cases}$$

Problem (5) highlights that a robust HCR is a decision where managers seek to maximize their intertemporal target while a hypothetical malevolent nature minimizes that same target by selecting the worst perturbation process. In the second step we find the HCR, λ, and the multiplier θ (associated with the scientific uncertainty level, η) that satisfies $Pr(B \leq B_{lim}) = v$, and $E_0 \sum_{t=0}^{\infty} \beta^{t+1}\omega_{t+1}^2 = \eta$.

3.1 The extremization problem

The extremization problem (5) can be simplified with a change of variables, $\Delta F_t = F_t - F_{tar}$ and $\Delta B_t = B_t - B_{tar}$, and expressed as the following nonstochastic problem,

$$\max_{\{\Delta F_t,\Delta B_{t+1}\}_{t=0}^{\infty}} \min_{\{\omega_{t+1}\}_{t=0}^{\infty}} \qquad \sum_{t=0}^{\infty} \beta^t \left\{-\Delta F_t^2 - \lambda \Delta B_t^2 + \beta\theta\omega_{t+1}^2\right\},$$

$$s.t. \qquad \begin{cases} \Delta B_{t+1} = z_t - \Delta F_t, \\ z_{t+1} = \rho z_t + \omega_{t+1}, \end{cases}$$

where ε_{t+1} is set to zero in the second constraint by the modified certainty equivalent principle that applies to robust control problems.[8]

Substituting the restrictions into the objective function, the problem can easily be converted into an unconstrained optimization problem. Any solution for ΔF_t and ω_{t+1} must be the solution to the following two-period problem

$$\max_{\Delta F_t} \min_{\omega_{t+1}} -\Delta F_t^2 + \beta\theta\omega_{t+1}^2 - \beta\lambda(z_t - \Delta F_t)^2 - \beta^2\lambda(\rho z_t + \omega_{t+1} - \Delta F_{t+1})^2.$$

The first order conditions (f.o.c.) for this optimization problem are

$$\begin{aligned} -\Delta F_t + \beta\lambda(z_t - \Delta F_t) &= 0, \\ \theta\omega_{t+1} - \beta\lambda(\rho z_t + \omega_{t+1} - \Delta F_{t+1}) &= 0. \end{aligned}$$

Solving thess f.o.c.'s for ΔF_t and ω_{t+1}, and considering the first constraint of the nonstochastic problem, we have

$$\Delta F_t = \frac{\beta\lambda}{1+\beta\lambda} z_t, \tag{6}$$

$$\Delta B_{t+1} = \frac{1}{1+\beta\lambda} z_t, \tag{7}$$

$$\omega_{t+1} = \frac{\rho\beta\lambda}{\theta(1+\beta\lambda) - \beta\lambda} z_t. \tag{8}$$

It is worth highlighting that the multiplier $\lambda = \frac{1}{\rho\beta}\frac{\Delta F_t}{\Delta B_t}$, represents the HCR. Note that for the uncorrelated process, $\rho = 0$, constant effort is an optimal rule, i.e. $\Delta F_t = \rho\beta\Delta B_t = 0$.

[8] Hansen and Sargent (2008) show that the robust version of a stochastic optimal linear regulator can be computed by its corresponding nonstochastic version.

3.2 Robust precautionary HCR

Given the solution of the extremization problem we solve for θ and λ using equations (3) and (4). We start by substituting the first order condition (8) into (7), that is[9]

$$\Delta B_{t+1} = \hat{\rho}\Delta B_t + \frac{\varepsilon_t}{1+\beta\lambda},$$

where

$$\hat{\rho}(\theta,\lambda) = \rho\left[\frac{\theta(1+\beta\lambda)}{\theta(1+\beta\lambda)-\beta\lambda}\right]. \tag{9}$$

Since ε_t is a Gaussian process, B_{t+1} also follows a Gaussian distribution whose mean and variance are given respectively by $\mu_B = B_{tar}$, and $\sigma_B^2 = \dfrac{\sigma_\varepsilon^2}{(1+\beta\lambda)^2(1-\hat{\rho}^2)}$. Therefore

$$Pr(B \leq B_{lim}) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{B_{lim}-\mu_B}{\sqrt{2}\sigma_B(\theta,\lambda)}\right)\right] = v,$$

where erf is the Gaussian error function. Taking into account the mean and the variance of the biomass equation (3) can be rewritten as:

$$(1+\beta\lambda)\left(1-\hat{\rho}^2\right)^{1/2} = \frac{\sigma_\varepsilon \mathrm{erf}^{-1}(2v-1)\sqrt{2}}{(B_{lim}-B_{tar})}. \tag{10}$$

For a given, $\hat{\rho}$, managers can prevent that biomass from dropping below B_{lim} with a probability v, if the harvest control rule λ satisfies equation (10). In order to compute $\hat{\rho}$, we generate the vector of perturbations in the mean of $\tilde{\varepsilon}_{t+1}$

$$\omega_{t+1} = \frac{\rho\beta\lambda}{\theta(1+\beta\lambda)-\beta\lambda} z_t = \hat{\rho}\omega_t + \frac{\rho\beta\lambda}{\theta(1+\beta\lambda)-\beta\lambda}\varepsilon_t \tag{11}$$

by using the worst-case nature response, equation (8). Note that conditional means are not zero and can feed back on the history of the state, z. Therefore equation (4) can be written

[9] Note that we make use of $z_t = \rho z_{t-1} + \omega_t + \varepsilon_t = \hat{\rho} z_{t-1} + \varepsilon_t$.

as[10]

$$\eta = E_0 \sum_{t=0}^{\infty} \beta^{t+1} \omega_{t+1}^2 = \frac{\beta}{1-\beta} \frac{(\rho\beta\lambda)^{2\beta}\sigma_\varepsilon^2}{(1-\hat{\rho}^2)\left[\theta(1+\beta\lambda)-\beta\lambda\right]^2} = \frac{\beta}{1-\beta} \frac{(\hat{\rho}-\rho)^2\sigma_\varepsilon^2}{(1-\hat{\rho}^2)}. \tag{12}$$

Notice that when managers are not concerned about scientific uncertainty (and $\eta \to 0$), $\hat{\rho} = \rho$. However when managers are concerned about scientific uncertainty (and η is finite), $\hat{\rho} > \rho$ meaning that the (inferred) operating model is more persistent than the perceived one.

By solving equations (9), (10) and (12) it is possible to find the robust precautionary HCR. Formally, for the scientific uncertainty level η and a precautionary level v, the robust precautionary HCR is given by

$$\lambda \;=\; \frac{1}{\beta}\left[\frac{\sigma_\varepsilon \mathrm{erf}^{-1}(2v-1)\sqrt{2}}{(B_{lim}-B_{tar})\sqrt{1-\hat{\rho}^2}} - 1\right], \tag{13}$$

where equation (12) determines $\hat{\rho}$ as a function of the scientific uncertainty level η.

Note that under scientific uncertainty –when data can be generated by a nearby operating model, at a distance η –, even for an uncorrelated perceived process, $\rho = 0$, robust HCR is designed assuming the existence of persistence, i.e. $\hat{\rho}^2 = \dfrac{\eta}{\left(\frac{\beta}{1-\beta}\right)\sigma_\varepsilon^2 + \eta} > 0$. Greater scientific uncertainty, a higher η, is associated with a higher $\hat{\rho}^2$ and a higher λ.

Two conclusions can be highlighted from the characterization of robust precautionary HCRs. First, an HCR consisting of keeping fishing mortality constant at F_{tar} cannot be a robust precautionary rule.

Proposition 1. *A constant effort rule, $F_t = F_{tar}$ is not a robust precautionary HCR.*

Proof See Appendix A.1

[10]Notice that from (9) we make use of $\frac{\beta\lambda}{\theta(1+\beta\lambda)} = \frac{\hat{\rho}-\rho}{\hat{\rho}}$.

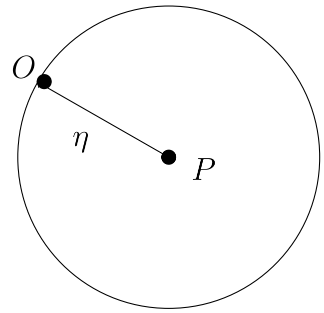

Figure 2: Set of nearby operating models for which the decision rule will work well using the perceived model. O and P stand for operating and perceived, respectively.

Constant effort HCRs, which can be good precautionary HCRs for uncorralated processes, i.e $\rho = 0$, are not robust. Under scientific uncertainty it can be inferred that the operating process is correlated, even when the stochastic process obtained form the perceived model is not. Robustness implies the use of biomass based HCRs, $\lambda > 0$. This result is consistent with the numerical findings of Da-Rocha and Mato-Amboage (2016).

The sense in which a constant effort HCR generates a stock performance that is not robust under scientific uncertainty can be ilustrated. A naive manager considers that perceived model (1) generates the data. Under this assumption, a constant effort HCR can achieve a precautionary level v by using a constant effort HCR, $\lambda^{NR} = 0$, when a process with correlation ρ and variance $\sigma_z = \dfrac{(B_{lim} - B_{tar})\sqrt{(1-\rho^2)}}{\text{erf}^{-1}(2v-1)\sqrt{2}}$ is expected.[11] When the perceived process is uncorrelated, $\rho = 0$, the expected Biomass volatility–based on naive expectations– is $\sigma_B^2 = \sigma_z^2$.

However, data are generated by the operating model (2) by chosing w_{t+1} using equation (8), and the mean of the perceived model (1) $\tilde{\varepsilon}_{t+1}$ is perturbed by the process given by equation (11). Biomass volatility –generated by the operating model – is $\sigma_B^2 = \dfrac{\sigma_z^2}{1-\hat{\rho}^2}$.[12]

Managers concerned with robustness seek an HCR that is reliable for all close operating

[11]Note that variance is given by solving equation (13) for ρ and $\lambda = 0$.

[12]When $\tilde{\varepsilon}_{t+1}$ is perturbed, biomass evolves following $\Delta B_{t+1} = \Delta z_t = \hat{\rho}\Delta z_{t-1} + \omega_t \varepsilon_t = \hat{\rho}\Delta B_t + \varepsilon_{t-1}$.

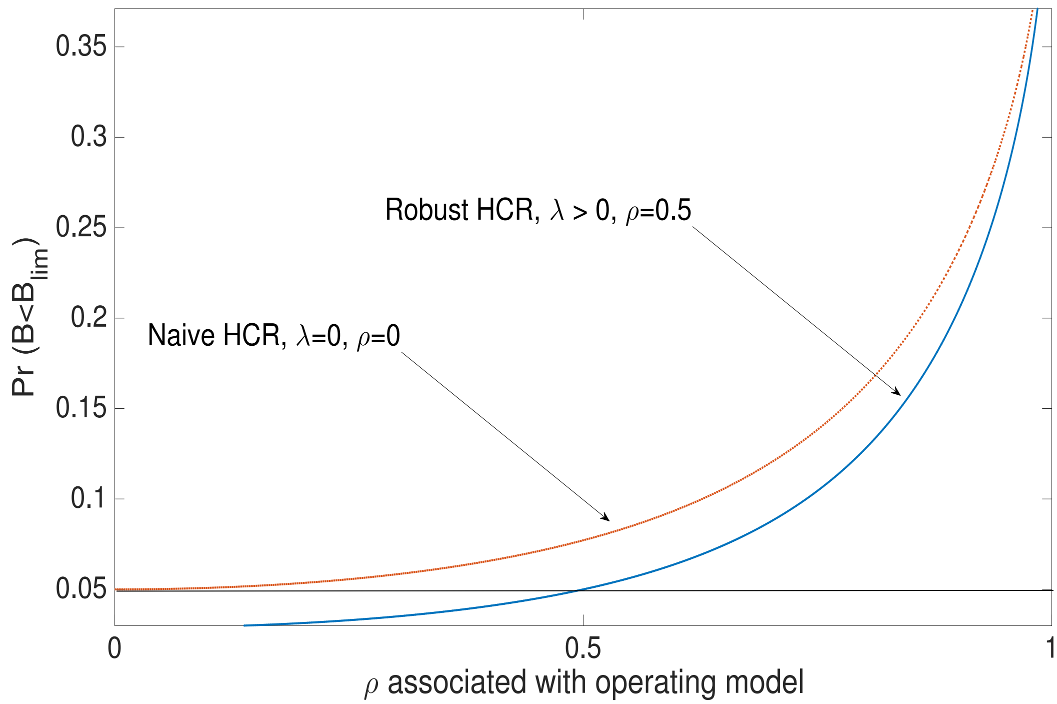

Figure 3: Risk of biomass dropping below B_{lim} for a naïve HCR, $\lambda = 0$ (equation 15) and a robust HCR, $\lambda > 0$ (equation 14). The robust HCR was designed by assuming $\hat{\rho}$ to be 0.5. Notice that when the correlation generated by the operating model is 0.5 the probability of the biomass dropping below B_{lim} is exactly $v = 0.05$.

models (2) in the set displayed in Figure 2. This is equivalent to designing an HCR by assuming that $\hat{\rho} > \rho = 0$. The robust HCR, given by equation (13), is equal to $\lambda^R = \frac{1}{\beta}\left[\frac{1}{\sqrt{(1-\hat{\rho}^2)}} - 1\right] > 0$, and generates a risk measure equal to

$$Pr(B \leq B_{lim}) = \frac{1}{2}\left[1 + \text{erf}\left((1+\lambda^R\beta)\sqrt{(1-\hat{\rho}^2)}\text{erf}^{-1}(2v-1)\right)\right]. \tag{14}$$

Figure 3 show how naive HCR performance deteriorates more quickly than robust HCR rules as scientific uncertainty (the correlation generated by the perturbation process, $\hat{\rho}$) increases.

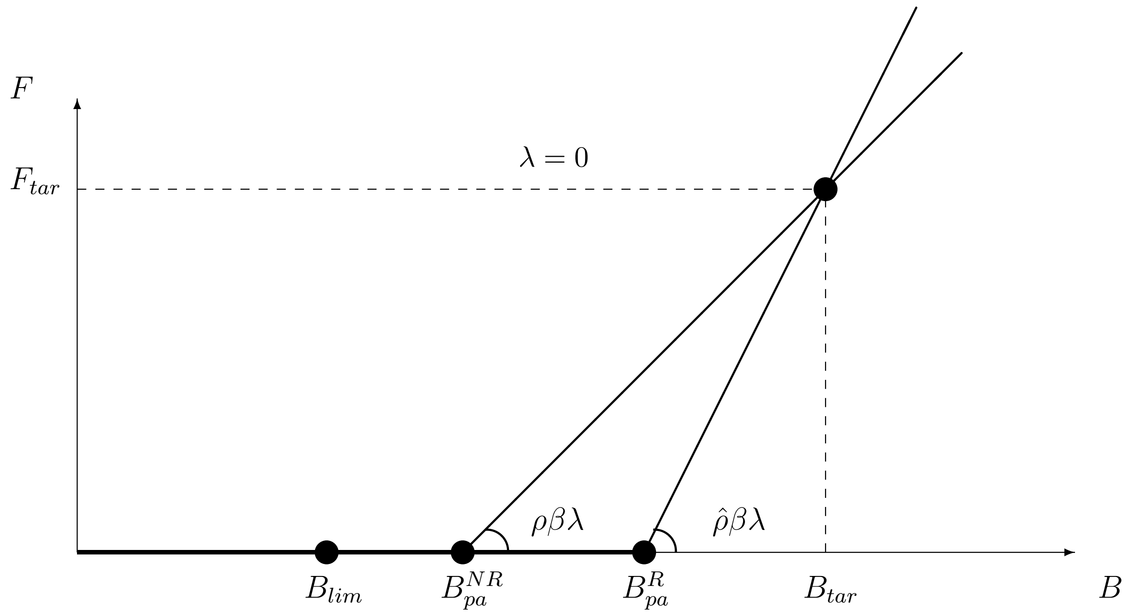

Figure 4: Robust HCR versus non-robust HCR. Robust design of HCR leads to higher precautionary biomass. B_{pa}^{NR} and B_{pa}^{R} stand for non-robust and robust precautionary biomass, respectively.

When the naive constant effort rule is applied the precautionary constraint is violated, i.e.

$$Pr(B \leq B_{lim}) = \frac{1}{2}\left[1 + \text{erf}\left(\sqrt{(1-\hat{\rho}^2)}\text{erf}^{-1}(2v-1)\right)\right] > v. \tag{15}$$

HCR reduces precautionary levels (low v's), when the perceived model is correct. However, the performance becomes more precautionar as scientific uncertainty increases.

Second, how much faster fishing mortality is reduced if a stock is assessed to be below the target biomass depends on the level of scientific uncertainty. Proposition 2 establishes a precautionary management procedure: the higher concern about scientific uncertainty is the higher the proportionality is between biomass and fishing mortality in the HCR.

Proposition 2. *Greater scientific uncertainty levels imply: i) a steeper relationship between biomass and robust fishing mortality in the harvesting control rule, and ii) higher precautionary biomass, i.e* $B_{pa} = B_{tar} - F_{tar}/\hat{\rho}\beta\lambda$.

Proof See Appendix A.2.

Figure 4 compares robust and non robust precautionary HCRs. For the same precautionary criteria (given by B_{lim} and v) the robust HCR selects higher λ and B_{pa} than the non-robust HCR.

4 A robust rule of thumb for ICES

To compute robust precautionary HCR time series data are needed –for each stock– to compute $\sigma_{\varepsilon}, \rho, B_{lim}$. However, if the idea is only to explore the impact of scientific uncertainty –for the given levels of v– there is no need to compute these statistics.

Assume that a stock has been assessed as above B_{trigger}, and the ICES *MSY (constant effort) advice rule* has been applied. In that case, (see Appendix A.3), the robust precautionary HCR satisfies $\lambda = \dfrac{\widehat{\sigma} - \sigma}{\sigma}$. Robutness is proportional to the difference between the standard deviation of the perceived model and the variance associated with the vector of perturbations, $\widehat{\sigma}$.

Assume that λ is set to 2. In that case, $\widehat{\sigma} = 3\sigma$, which is the three-sigma rule which guarantees that 99.7% of random events lie around the mean of its normal distribution, (see Pukelsheim (1994)). According to this rule $\mathrm{B_{pa}} = 0.5\mathrm{B_{MSY}}$. By setting $\lambda = 2$ a rule of thumb can be defined for robustness: fishing mortality is increased faster than linearly –by a factor of 2-fold– when a stock is assessed as above *an endogenous robust limit point* –$0.5B_{\text{MSY}}$.

We compare robust HCRs with the MSY ICES advice rule for 17 ICES stocks. For those stocks ICES provides $\mathrm{F_{lim}}$, $\mathrm{F_{pa}}$, $\mathrm{B_{lim}}$, $\mathrm{B_{pa}}$, $\mathrm{F_{MSY}}$ and $\mathrm{MSYB_{trigger}}$ (see Table 3). Therefore, the ICES MSY advice rule can be computed, i.e.

$$\mathrm{F}_{\mathrm{advice}} = \begin{cases} 1 & \text{if} \quad \frac{\mathrm{B}}{\mathrm{MSYB}_{\mathrm{trigger}}} \geq 1, \\ \\ \frac{\mathrm{B}-\mathrm{B}_{\mathrm{lim}}}{\mathrm{MSYB}_{\mathrm{trigger}}-B_{lim}} & \text{if} \quad \frac{\mathrm{B}}{\mathrm{B}_{\mathrm{MSY}}} \in (\frac{B_{lim}}{\mathrm{B}_{\mathrm{MSY}}}, 1), \\ \\ 0 & \text{if} \quad \frac{\mathrm{B}}{\mathrm{B}_{\mathrm{MSY}}} < \frac{\mathrm{B}_{\mathrm{lim}}}{\mathrm{B}_{\mathrm{MSY}}}. \end{cases} \tag{16}$$

We compute the robust HCR by applying the rule of thumb associated with $\lambda = 2$, i.e

$$\mathrm{F}_{\mathrm{robust}} = \begin{cases} 1 + 2 \times \left(\frac{\mathrm{B}}{\mathrm{B}_{\mathrm{MSY}}} - 1\right) & \text{if} \quad \frac{\mathrm{B}}{\mathrm{B}_{\mathrm{MSY}}} \geq 0.5, \\ \\ 0 & \text{if} \quad \frac{\mathrm{B}}{\mathrm{B}_{\mathrm{MSY}}} < 0.5. \end{cases} \tag{17}$$

ICES does not provide $\mathrm{B}_{\mathrm{MSY}}$. We approximate this value by exploiting the fact that for all those stocks the ICES sets $\mathrm{MSYB}_{\mathrm{trigger}}$ equal to B_{pa}. Therefore, we compute a $\mathrm{B}_{\mathrm{MSY}}$ proxy using $\mathrm{F}_{\mathrm{MSY}}$ and F_{pa}. We assume that $\mathrm{B}_{\mathrm{MSY}} = \left[1 + \frac{(\mathrm{F}_{\mathrm{pa}}-\mathrm{F}_{\mathrm{MSY}})}{\mathrm{Fpa}}\right] \times \mathrm{MSYB}_{\mathrm{trigger}}$.

Figure 5 compares the two HCRs and Table 1 shows the resulting advice –the fishing mortality rate recommended for exploitation by each HCR– associated with the 2014 biomass level. The robust HCR provides more precautionary advice for stocks assessed as below $\mathrm{B}_{\mathrm{MSY}}$.

For stocks assessed as above $\mathrm{B}_{\mathrm{MSY}}$ the ICES MSY advice rule recommends lower fishing mortality rates than advice based on the robust rule of thumb. It is worth highlighting that –in general– systems are not in equilibrium and fishing mortality rates higher than $\mathrm{F}_{\mathrm{MSY}}$ generate higher total allowed catches (T.A.C.s).

Table 1: ICES MSY advice vs Robust HCR

FishStock(*)	Robust HCR		MSY ICES advice rule		
	B/B_{MSY}	F_{robust}(1)	Blim/$MSYB_{trigger}$	B/$MSYB_{trigger}$	F_{advice} (2)
cod-arct	3.91	6.82	0.48	3.91	1.00
whb-comb	2.29	3.58	0.67	2.43	1.00
ple-nsea	1.84	2.68	0.70	2.91	1.00
spr-2232	1.45	1.89	0.72	1.58	1.00
sai-faro	1.27	1.54	0.00	1.27	1.00
her-2532-gor	1.05	1.10	0.72	1.43	1.00
sol-eche	0.99	0.98	0.00	1.26	1.00
sol-celt	0.94	0.87	0.00	1.09	1.00
her-noss	0.81	0.63	0.50	0.81	0.63
sai-3a46	0.76	0.51	0.53	0.94	0.88
sol-bisc	0.71	0.42	0.00	0.98	0.98
cod-farp	0.59	0.17	0.53	0.64	0.23
had-faro	0.47	0	0.63	0.47	0
sol-kask	0.40	0	0.60	0.54	0
sol-iris	0.26	0	0.71	0.38	0
cod-347d	0.26	0	0.47	0.46	0
had-rock	0.20	0	0.67	0.30	0

(*) FishStock is described in Table 2.
(1) computed using equation (17);
(2) computed using equation (16).

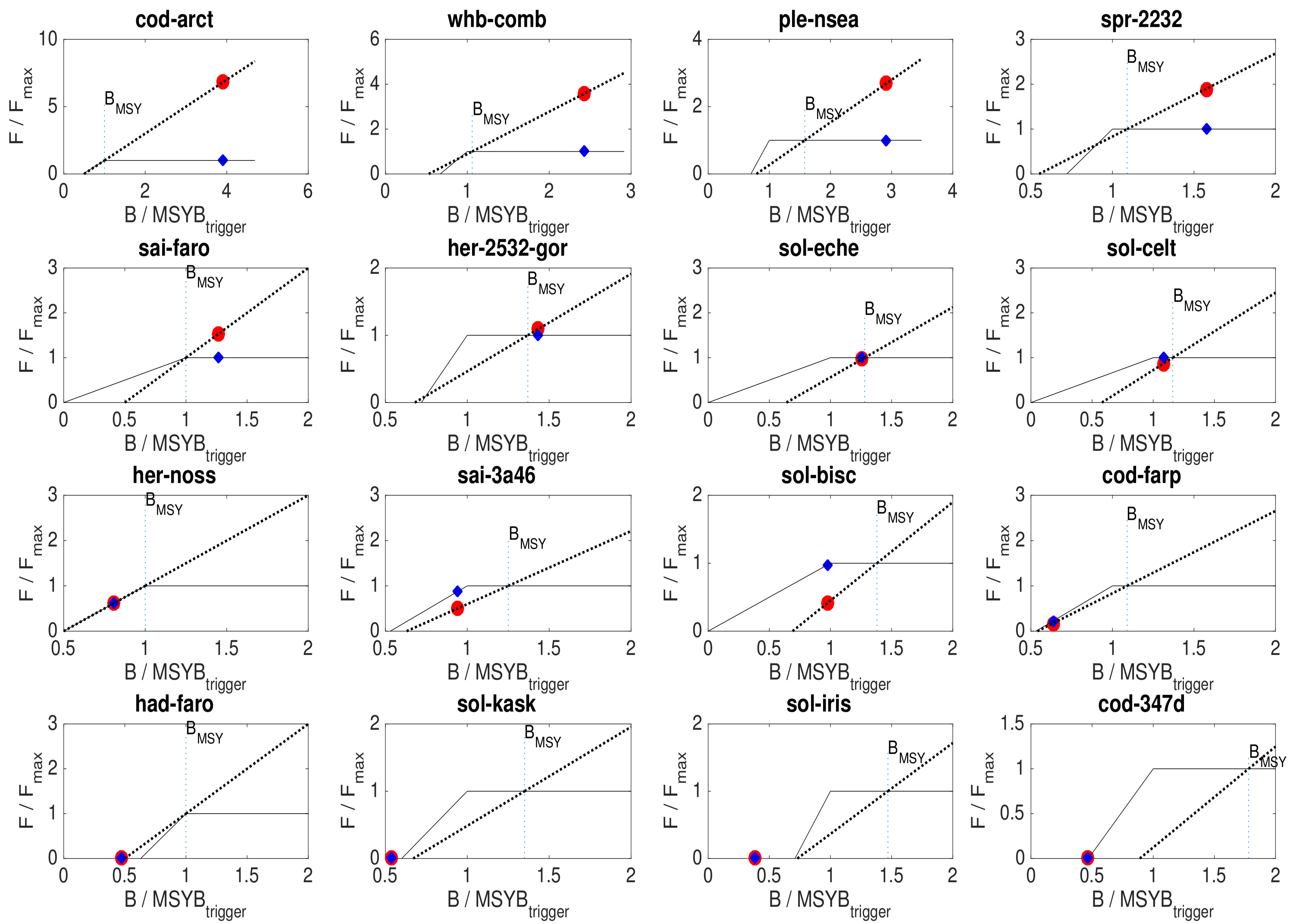

Figure 5: The solid line represents the ICES MSY advice rule and the dotted line the $3-\sigma$ HCR. The horizontal line is plotted at the B_{MSY} proxy level. The (large) red dot represents the F recommended for exploitation by the robust HCR. The blue diamond represents the F recommended for exploitation by the ICES MSY advice rule. Each stock is described in Table 2.

5 Conclusions

In this paper we show that scientific uncertainty can be treated analytically by using simple calculus. We reveal that constant effort HCRs are not robust under scientific uncertainty. Moreover, the best way to deal with scientific uncertainty is to increases the (endogenous) precautionary biomass levels obtained by using the reference points. That is, by contrast with standard practice –where the safety margin is taken as $B_{pa} = 1.4B_{lim} \simeq (1+\sigma)B_{lim}$– the sources of uncertainty can be decreased by defining intervals around the reference points.

But how robust are our findings? The class of perturbations considered is very general. As Hansen and Sargent (2008) point out –for Linear Quadratic (LQ) problems– they include unknown parameter values, misspecfication of higher moments of the error distribution, and various kinds of 'structured uncertainty'. Therefore, scientific uncertainty in our model includes all the sources of uncertainty classified by Francis and Shotton (1997), i.e. observation, model structure, process error and implementation errors. Our findings can therefore be applied in case studies where different sources of uncertainty are included to evaluate different management strategies (MSE) by using a simulation modeling approach.

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A Appendix

A.1 Proof of Lemma 1

For any accuracy level, η, equation (12) implies $\hat{\rho} > \rho = 0$. Thus, a robust HCR, given by equation (13), has $\lambda > 0$ and $F_t = F_{tar}$ and cannot be a robust precautionary rule. ■

A.2 Proof of Lemma 2

It is straightforward to check in equation (1) that when $\eta = 0$, $\hat{\rho}^2 = \rho^2$ and when $\eta \to \infty$, $\hat{\rho}^2 = 1$. Taking this into account λ increases with scientific uncertainty (and goes to infinity, $\lambda \to \infty$, when $\eta \to \infty$). Finally, it is clear that $B_{pa} = B_{tar} - F_{tar}/\hat{\rho}\beta\lambda$ increases when λ and $\hat{\rho}$ increases. ■

A.3 A rule of thumb

If managers are not concerned about misspecification ($\theta \to \infty$ or equivalently $\eta = 0$), the auto-correlation coefficient of recruitment is $\hat{\rho} = \rho$. In this context, selecting $\lambda = 0$ implies, according to (13), a standard deviation for the residuals of $\widetilde{\sigma}_\varepsilon = \frac{(B_{lim}-B_{tar})\sqrt{(1-\rho^2)}}{\sqrt{2}\text{erf}^{-1}(2v-1)}$ and the variance of the biomass is therefore given by $\sigma_B^2 = \widetilde{\sigma}_\varepsilon^2/(1-\rho^2)$.

However, if there is misspecification and nature chooses $\omega_{t+1} \neq 0$, then $\hat{\rho} > \rho$, $\lambda > 0$ and the biomass variance is instead $\sigma_B^2 = \widetilde{\sigma}_\varepsilon^2/(1+\beta\lambda)^2(1-\hat{\rho}^2)$. The risk of biomass dropping below B_{lim} is then given by

$$Pr(B \leq B_{lim}) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\widetilde{\sigma}_\varepsilon \text{erf}^{-1}(2v-1)}{\sigma_B(1-\rho^2)^{1/2}}\right)\right] = \frac{1}{2}\left[1 + (2v-1)(1+\beta\lambda)\left(\frac{1-\hat{\rho}^2}{1-\rho^2}\right)^{1/2}\right].$$

If the HCR is selected to guarantee that the risk of biomass dropping below B_{lim} is v,i.e. then the robust λ satisfies $Pr(B \leq B_{lim}) = v$, i.e. $1 - \beta\lambda = \left(\frac{1-\rho^2}{1-\hat{\rho}^2}\right)^{1/2}$. This condition can be expressed as $\beta\lambda = \left(\frac{1-\rho^2}{1-\hat{\rho}^2}\right)^{1/2} - 1 = \frac{\widehat{\sigma} - \sigma}{\sigma}$.

Table 2: Fish Stocks

FishStock	StockDescription	SGName
cod-arct	Cod (Gadus morhua) in Subareas I and II (Northeast Arctic)	Cod
whb-comb	Blue whiting in Subareas I-IX. XII and XIV (Combined stock)	Blue whiting
ple-nsea	Plaice Subarea IV (North Sea)	Plaice
spr-2232	Sprat in Subdivisions 22 - 32 (Baltic Sea)	Sprat
sai-faro	Saithe in Division Vb (Faroe Saithe)	Saithe
her-2532-gor	Herring in Subdivisions 25 - 29 (excluding Gulf of Riga) and 32	Herring
sol-eche	Sole in Division VIId (Eastern Channel)	Sole
sol-celt	Sole in Divisions VIIf. g (Celtic Sea)	Sole
her-noss	Herring in Subareas I. II. V and Divisions IVa and XIVa (Norwegian spring-spawning herring)	Herring
sai-3a46	Saithe in Subarea IV (North Sea) Division IIIa West (Skagerrak) and Subarea VI (West of Scotland and Rockall)	Saithe
sol-bisc	Sole in Divisions VIIIa.b (Bay of Biscay)	Sole
cod-farp	Cod (Gadus morhua) in Subdivision Vb1 (Faroe Plateau)	Cod
had-faro	Haddock in Division Vb	Haddock
sol-kask	Sole in Division IIIa and Subdivisions 22-24 (Skagerrak. Kattegat. and the Belts)	Sole
sol-iris	Sole in Division VIIa (Irish Sea)	Sole
cod-347d	Cod (Gadus morhua) in Subarea IV and Divisions VIId and IIIa West (North Sea. Eastern English Channel. Skagerrak)	Cod
had-rock	Haddock in Division VIb (Rockall)	Haddock

Source: ICES Dataset. Extraction date: July , 2015.

Table 3: Reference Points

FishStock	SSB(2014)	Flim	Fpa	Blim	Bpa	FMSY	MSYBtrigger	(Fpa-FMSY)/Fpa	BMSY (*)
cod-arct	1797496	0,74	0,4	220000	460000	0,4	460000	0,00	460000
whb-comb	5471000	0,48	0,32	1500000	2250000	0,3	2250000	0,06	2390625
ple-nsea	670372	0,74	0,6	160000	230000	0,25	230000	0,58	364167
spr-2232	901000	0,39	0,32	410000	570000	0,29	570000	0,09	623438
sai-faro	69868	0	0,3	0	55000	0,3	55000	0,00	55000
her-2532-gor	858701	0,52	0,41	430000	600000	0,26	600000	0,37	819512
sol-eche	10088	0,55	0,4	0	8000	0,29	8000	0,28	10200
sol-celt	2394	0,52	0,37	0	2200	0,31	2200	0,16	2557
her-noss	4066000	0	0,15	2500000	5000000	0,15	5000000	0,00	5000000
sai-3a46	188837	0,6	0,4	106000	200000	0,3	200000	0,25	250000
sol-bisc	12752	0,58	0,42	0	13000	0,26	13000	0,38	17952
cod-farp	25410	0,68	0,35	21000	40000	0,32	40000	0,09	43429
had-faro	16596	0,4	0,25	22000	35000	0,25	35000	0,00	35000
sol-kask	1072	0,92	0,49	1200	2000	0,32	2000	0,35	2694
sol-iris	1182	0,4	0,3	2200	3100	0,16	3100	0,47	4547
cod-347d	68528	0,65	0,86	70000	150000	0,19	150000	0,78	266860
had-rock	2743	0	0,4	6000	9000	0,2	9000	0,50	13500

Source: ICES Dataset. Extraction date: July , 2015. $\mathrm{B_{MSY}} = \left[1 + \frac{(\mathrm{F_{pa}} - \mathrm{F_{MSY}})}{\mathrm{Fpa}}\right] \times \mathrm{MSYB_{trigger}}$.