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Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity*

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Abstract

The large differences in income per capita across countries are mostly accounted for by differences in total factor productivity (TFP). What explains the differences in TFP across countries? Empirical evidence points to factor misallocation across heterogeneous production units as an important factor. We study factor misallocation in a model where establishment-level productivity is endogenous. In this framework, policy distortions not only misallocate resources across a given set of productive units, but also worsen the productivity distribution of establishments and this effect is substantial quantitatively. Reducing the dispersion in revenue productivity by half to the level of the U.S. benchmark in the model implies an increase in aggregate output and TFP by a factor of 7.8-fold. Improved factor allocation accounts for 38 percent of the gain, whereas the change in the productivity distribution accounts for the remaining 62 percent.

Keywords: distortions, misallocation, investment, endogenous productivity, establishments.
JEL codes: O1, O4.

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1 Introduction

A crucial question in economic growth and development is why some countries are rich and others poor. A consensus has emerged in the literature whereby the large differences in income per capita across countries are mostly accounted for by differences in labor productivity and in particular total factor productivity (TFP).[1] Hence, a key question is what explains differences in TFP across countries. A recent literature has emphasized the (mis)allocation of factors across heterogeneous production units as an important factor.[2] We study factor misallocation in a model where establishment-level productivity is determined endogenously. In this framework, policy distortions not only misallocate resources across a given set of productive units, but also worsen the productivity distribution in the economy.

A recent literature has emphasized the importance of the productivity distribution by considering variations of the growth model whereby the productivity distribution is endogenous.[3] We build on this literature by endogenizing the entire distribution of productivity as a function of the economic environment which is affected by policy distortions. In our framework, not only there is a tight mapping between abstract policy distortions and the empirical counterparts of dispersion in revenue products and factor misallocation, but also compared to the model with an exogenous distribution of productivity, the quantitative effect of empirically-plausible policy distortions is substantial. For instance, in our framework, the output gain from equalizing the dispersion in log revenue productivity (TFPR) in 1991 China to the dispersion in log TFPR in 1997 U.S. is 120%, which is substantially larger than the 41% static gain from reduced factor misallocation reported in Hsieh and Klenow (2009).

We develop a framework with heterogeneous production units that builds on Hopenhayn

[1]See, for instance, Klenow and Rodriguez-Clare (1997), Prescott (1998), and Hall and Jones (1999).

[2]See Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Guner et al. (2008), and Hsieh and Klenow (2009). See also surveys of the literature in Restuccia and Rogerson (2013), Restuccia (2013a), and Hopenhayn (2014).

[3]Some of the contributions on the endogenous productivity distribution include Restuccia (2013b), Bello et al. (2011), Acemoglu et al. (2013), Ranasinghe (2014), Bhattacharya et al. (2013), Gabler and Poschke (2013), Hsieh and Klenow (2014), Bento and Restuccia (2015), Guner et al. (2015), among others.

(1992) and Restuccia and Rogerson (2008). The framework is extended to allow for an endogenous determination of the distribution of establishment-level productivity. We use this framework to study the impact of policy distortions on misallocation and aggregate measured productivity and output. There is a large number of homogeneous households with standard preferences over consumption goods. Households accumulate physical capital and supply inelastically their endowment of one unit of time. The key elements of the model are on the production side. A single good is produced in each period. The production unit is the establishment. An establishment has access to a decreasing returns to scale production function with capital and labor as inputs. Establishments are heterogeneous with respect to total factor productivity. Establishments are subject to an exogenous exit rate but differently from the standard framework, the distribution of establishment-level productivity is not exogenous, rather it is determined by establishment's endogenous decisions on entry size and productivity investment over time. In other words, the level of productivity of entering establishments is determined endogenously in the model by the properties of the economic environment such as policy distortions, as is the evolution of their productivity over time. Following the literature, the economy faces policy distortions which, for simplicity, take the form of output taxes on individual producers. That is, each producer faces an idiosyncratic tax and it is the properties of policy distortions that generate misallocation in the model. Revenues collected from these taxes are rebated back to the households as a lump-sum transfer.

We characterize the closed-form solution of this model in continuous time. In particular, we solve in closed form for the stationary distribution of establishments which is an endogenous object that varies across economies. We show the equilibrium productivity distribution is a Pareto distribution with tail index that depends on policy distortions and on the investment response of incumbent establishments to distortions. This allows us to characterize the behavior of aggregate output and TFP across distortionary policy configurations as well as the size and productivity growth rate of establishments, the size distribution of establishments, among other statistics of interest.

To assess the quantitative properties of the model relative to the existing literature, we calibrate the model and provide a set of relevant quantitative experiments. We consider a benchmark economy with distortions that is calibrated to data for the United States. The key calibrated parameters in our analysis are the investment cost in establishment-level productivity, the variance in the distribution of productivity, and the size growth rate of establishments, which are targeted to data on the aggregate growth rate of TFP, the average employment growth of establishments, and the right tail index of the share of employment distribution in the U.S. data. We then perform quantitative analysis by exploring the implications of increased distortions for aggregate output and productivity.

Our main result is that the quantitative effect of policy distortions on aggregate output and TFP is substantially larger than in a model with an exogenous distribution of productivity. In particular, in an economy with double the dispersion in revenue products than the U.S. benchmark economy, reducing the dispersion to that of the benchmark implies a large increase in aggregate output of 7.8-fold, out of which only a 2.2-fold increase is due to improved factor allocation and the remaining 3.7-fold increase due to the improvement in the endogenous distribution of productivity. The large effect of policy distortions on the productivity distribution arises from both the effect of entry size and changes in life-cycle productivity investment by establishments. For instance, the reduction in misallocation in this economy implies a reduction in life-cycle growth in productivity from 4.7 percent in the benchmark economy to 2.6 percent. This large decline in life-cycle growth of establishments is consistent with the empirical evidence for the life cycle of plants in India and Mexico relative to the United States in Hsieh and Klenow (2014).

Our paper is related to a large and growing literature on misallocation and productivity. By studying the aggregate impact of policy distortions across countries our paper is closely linked to Restuccia and Rogerson (2008) but instead we consider the endogenous response of establishment-level productivity to distortions. As such, our paper is related to the growing literature on misallocation endogenizing the productivity distribution referenced previously. This literature has emphasized various separate channels such as life-cycle investment of plants, human capital accumulation of managers, step-by-step innovation, among others.[4] Within this literature, a closely related paper to ours is Hsieh and Klenow (2014) who consider the model of establishment innovation in Atkeson and Burstein (2010) to emphasize the life-cycle growth of establishment productivity and its response to distortions. We emphasize two key distinctions with our work. First, whereas in Hsieh and Klenow (2014) entering establishments draw their productivity from an exogenous and constant distribution across countries, entry size and hence productivity is a key equilibrium object in our framework that responds to policy distortions. We show that this element of the productivity distribution is essential in the quantitative impact of distortions on aggregate output. Second, we differ in the tools used to characterize the economy, in particular, we characterize the model in closed form using continuous time and Brownian motion processes. These tools are increasingly popular in the growth literature.[5] More closely linked, these tools were prominently used by the seminal work of Luttmer (2007) to study the size distribution of establishments in the United States, by Da-Rocha and Pujolas (2011) and Fattal (2014) to study the effect of policy distortions with stochastic productivity and entry/exit decisions, and by Gourio and Roys (2014) to study the productivity effects of size-dependent labor regulations, just to name a few. A key distinction of our work with this existing literature is the emphasis on the amplification effect of policies. We argue that a larger propagation effect of policies on aggregate output and productivity is essential in providing a more accurate assessment of the quantitative impact of specific policies such as firing taxes, size-dependent policies, among many others.

[4] See also Rubini (2014) for an analysis of changes in tariffs in the context of a trade model with endogenous establishment-level productivity.

[5] For instance, Lucas and Moll (2014), Benhabib et al. (2014), Buera and Oberfield (2014), among many others.

The paper proceeds as follows. In the next section and section 3, we describe the model and characterize the equilibrium solution. Section 4 characterizes aggregate output, measured TFP, and establishment size in the model as a function of distortions. In section 5, we calibrate a benchmark economy with distortions to data for the United States. Section 6 performs a series of quantitative experiments to assess the impact of increased policy distortions on aggregate output, TFP, and other relevant statistics. We conclude in section 7.

2 Economic Environment

We consider a standard version of the neoclassical growth model with producer heterogeneity as in Restuccia and Rogerson (2008). We extend this framework in order to allow establishments to invest on their productivity. As as a result, the extended framework generates an endogenous productivity distribution of establishments associated with the economic environment that may differ across countries. Time is continuous and the horizon is infinite. Establishments have access to a decreasing return to scale technology, pay a one-time fixed cost of entry, and die at an exogenous rate. Establishments hire labor and rent capital services in competitive markets. New entrants enter with a level of productivity z_e which is endogenous. We study a stationary equilibrium in which the economy grows at an exogenous rate. We then analyze policy distortions that affect the allocation of factors across establishments (static misallocation). In our framework, policy distortions also affect the establishment investment on productivity, the stationary productivity distribution of establishments, and therefore, aggregate measured TFP and output. We contrast the effects of policy distortions in the environment where the distribution of establishment productivity is exogenous. In what follows we describe the economic environment in more detail.

2.1 Baseline Model

There is an infinity-lived representative household with preferences over consumption goods described by the utility function,

$$\max \int_0^{+\infty} e^{-\rho t} u(c) dt,$$

where c is consumption and ρ is the discount rate. The household is endowed with one unit of productive time at each instant and $k_0 > 0$ units of the capital stock at date 0.

The unit of production in the economy is the establishment. Each establishment is described by a production function $f(z, k, n)$ that combines capital services k and labor services n to produce output. The function f is assumed to exhibit decreasing returns to scale in capital and labor jointly and to satisfy the usual Inada conditions. The production function is given by:

$$y = z^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma}, \quad \alpha, \gamma \in (0, 1), \quad 0 < \gamma + \alpha < 1, \quad \theta > 1, \tag{1}$$

where θ is a TFP normalization factor. Establishment productivity z is stochastic but establishments can invest in upgrading their productivity at a cost. Establishments also face an exogenous probability of death λ.

New establishments can also be created. Entrants must pay an entry cost c_e measured in units of output and as in the literature the expected value of entry satisfies the zero profit condition in equilibrium. Feasibility in the model requires:

$$C + I + Q = Y - E,$$

where C is aggregate consumption, I is aggregate investment in physical capital, Q is aggregate cost of investing in establishment productivity, E is the aggregate cost of entry, and Y

is aggregate output.

2.2 Policy Distortions

We introduce policies that create idiosyncratic distortions to establishment-level decisions as in Restuccia and Rogerson (2008). We model these distortions as idiosyncratic output taxes but none of our results are critically dependent on the particular source of distortions. While the policies we consider are hypothetical, there is a large empirical literature documenting the extent of idiosyncratic distortions across countries and a nice feature of our framework is that there is a tight mapping between the distortions we consider and empirical observations.[6] In our framework, distortions not only affect the allocation of resources across existing productive units, but also the investment decision in productivity thereby affecting the distribution of productive units in the economy. Specifically, we assume that each establishment faces its own policy distortion (idiosyncratic distortions) reflected as an output tax rate τ_y. In what follows, for simplicity in our algebraic expressions we rewrite distortions as $\tau = (1-\tau_y)^{\frac{1}{\theta(1-\alpha-\gamma)}}$. Note that this transformation implies that an establishment with no distortions $\tau_y = 0$ faces $\tau = 1$, whereas a positive output tax $\tau_y > 0$ implies $\tau < 1$ and an output subsidy $\tau_y < 0$ implies $\tau > 1$.

In order to generate dispersion in distortions across productive unites, we assume that τ follows a standard stochastic process, a Geometric Brownian motion,

$$d\tau = \mu_\tau \tau dt + \sigma_\tau \tau dw_\tau,$$

where μ_τ is the drift, σ_τ is the standard deviation and dw_τ is the standard Wiener process of the Brownian motion. In this specification σ_τ controls the dispersion of distortions across

[6]See for instance Hsieh and Klenow (2009), Bartelsman et al. (2013) and the survey in Restuccia and Rogerson (2013).

producers and hence the dispersion in marginal revenue products that is restricted to data.[7]

Establishment's productivity z follows a Brownian motion and establishments can invest in upgrading their productivity by choosing the drift of the Brownian motion x_z. The distortion τ affects the establishment decision of investing in productivity by changing the drift of the productivity process. Hence, in the presence of distortions, establishment productivity follows:

$$dz = \frac{x_z}{\tau} z dt + \sigma_z z dw_z.$$

For tractability, we assume that the output tax and productivity processes are uncorrelated, that is $E(dw_\tau, dw_z) = 0$. We note however that much of the quantitative literature has focused on what Restuccia and Rogerson (2008) call correlated distortions, distortions that apply more heavily on more productive establishments and have the potential to generate much larger negative aggregate productivity effects.

At the time of entry, the establishment-entry distortion τ_e is known and establishments enter with a productivity z_e that is determined in equilibrium and implies an expected value of entrants that satisfies a zero profit condition. In this economy, the relevant information for establishment's decisions is the joint distribution over productivity and distortions. We denote this joint distribution by $g(z, \tau)$.

A given distribution of establishment-level distortion and productivity may not lead to a balanced budget for the government. As a result, we assume that budget balance is achieved by either lump-sum taxation or redistribution to the representative household. We denote the lump-sum tax by T.

[7] As discussed earlier, our specification for distortions is reduced form and abstract as is standing in for the myriad of policies and institutions that effectively create a process for the dispersion in individual producer prices. See for example Buera et al. (2013).

3 Equilibrium

We focus on a stationary equilibrium where the number of establishments grows at an exogenous rate η. The stationary equilibrium is characterized by an invariant distribution of establishments $g(z,\tau)$ over productivity z and distortion τ and an entry productivity z_e. In the stationary equilibrium, the mass of establishments grows over time, however, the rank of the establishments' size distribution is constant. In the stationary equilibrium, the rental prices for labor and capital services are constant and we denote them by w and r. Before defining the stationary equilibrium formally, it is useful to consider the decision problems faced by incumbents, entrants, and consumers. We describe these problems in turn.

3.1 Incumbent establishments

Incumbent establishments maximize the present value of profits by making static and dynamic decisions. The static problem is to choose the amount of capital and labor services, whereas the dynamic problem involves solving for the amount of investment in establishment productivity. In what follows next, we describe these problems in detail.

Static problem At any instant of time an establishment chooses how much capital to rent k and how much labor to hire n. These decisions are static and depend on the establishment's productivity z, the establishment's distortion τ, the rental rate of capital r, and the wage rate w. Formally, the instant profit function $\pi(z,\tau)$ is defined by:

$$\pi(z,\tau) = \max_{k,n} (\tau z)^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma} - wn - rk,$$

from which we obtain the optimal demands for capital and labor:

$$n(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\gamma}} z^{\theta}\tau^{\theta}, \tag{2}$$

$$k(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}} z^{\theta}\tau^{\theta}. \tag{3}$$

For future reference, we redefine instant profits as a function of the optimal demand for factors:

$$\pi(z,\tau) = m(w,r) z^{\theta}\tau^{\theta}, \tag{4}$$

where $m(w,r) = (1-\alpha-\gamma)\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}}$ is a constant across establishments that depends on equilibrium prices.

Dynamic problem Incumbent establishments choose how much to invest in upgrading their productivity x_z. The cost of investing in upgrading productivity is in units of output, described by a cost function $q(\cdot)$ that is increasing and convex in the productivity parameter x_z, specifically we assume $q(x_z) = c_{\mu}\frac{x_z^{\theta}}{\theta}$. The optimal decision of upgrading productivity is characterized by maximizing the present value of profits subject to the Brownian motion governing the evolution of productivity and the Brownian motion governing the evolution of distortions. Formally, incumbent establishments solve the following dynamic problem:

$$W(z,\tau) = \max_{x_z} \left\{ m(w,r) z^{\theta}\tau^{\theta} - q(x_z) + \frac{1}{1+(\lambda+R)dt} E_{z,\tau} W(z+dz, \tau+d\tau) \right\},$$

$$\begin{aligned} s.t.\quad dz &= \tfrac{x_z}{\tau} z dt + \sigma_z z dw_z, \\ d\tau &= \mu_{\tau}\tau dt + \sigma_{\tau}\tau dw_{\tau,} \end{aligned}$$

where λ is the exogenous exit probability of establishments and R is the stationary equilibrium interest rate. Next, we define the Hamilton-Jacobi-Bellman of the stationary solution,

$$(\lambda + R)W(z,\tau) = \max_{x_z} \left\{ m(w,r)\tau^\theta z^\theta - \frac{c_\mu}{\theta} x_z^\theta + \frac{x_z}{\tau} W_z' + \frac{\sigma_z^2}{2} z^2 W_{zz}'' + \mu_\tau \tau W_\tau' + \frac{\sigma_\tau^2}{2} \tau^2 W_{\tau\tau}'' \right\}.$$

From the first order conditions, we find that the optimal investment rate x_z is a function of the distortion τ, the investment cost c_μ, and the marginal present value profits W_z',

$$x_z^{\theta-1} = \frac{W_z'}{c_\mu \tau}.$$

By guessing and verifying, we find that the optimal Hamilton-Jacobi-Bellman equation is given by $W(z,\tau) = A(w,r)z^\theta \tau^\theta$, where the constant $A(w,r)$ is the solution of the polynomial:

$$\left[\frac{(\lambda + R)}{(\theta - 1)} - \frac{\theta \mu_\tau}{(\theta - 1)} - \frac{\theta}{2}(\sigma_z^2 + \sigma_\tau^2) \right] A(w,r) - \left[\frac{\theta}{c_\mu} \right]^{\frac{1}{\theta - 1}} A(w,r)^{\frac{\theta}{\theta - 1}} = \frac{m(w,r)}{(\theta - 1)}. \tag{5}$$

Given the solution to this polynomial, the optimal investment rate is linear in τz, i.e.

$$x_z = \left[\frac{\theta A(w,r)}{c_\mu} \right]^{\frac{1}{\theta - 1}} \tau z. \tag{6}$$

In the following Lemma 1 we characterize formally the impact of distortions on the productivity drift.

Lemma 1. *Given a distortion τ, a productivity level z, and operating profits $m(w,r)$, the value function that solves the establishment dynamic problem is given by $W(z,\tau) = A(w,r)\tau^\theta z^\theta$, and the expected growth rate of establishment's productivity z follows Gibrath's law:*

$$\frac{dz}{z} = \left[\frac{\theta A(w,r)}{c_\mu} \right]^{\frac{1}{\theta - 1}} dt + \sigma_z dw_z,$$

where $A(w,r)$ is the solution of the polynomial in equation (5).

The proof of Lemma 1 is straightforward from equation (6). The implication of Lemma 1 is that the growth rate of productivity of individual establishments does not depend on the intrinsic characteristics of the establishment, that is, it does not depend on the establishment productivity z or the distortion τ. As a consequence Gilbrath's law holds and productivity growth does not depend on the establishment size. This implication of the model is supported by a large body of empirical evidence.[8] Moreover, for the purpose of our paper, this implication of the model is conservative in terms of the amplification effects that can be generated by policy distortions in our framework as it explicitly shuts down a channel that has been emphasized as important in the literature of endogenous investments in productivity. For example as emphasized in Bhattacharya et al. (2013) and Hsieh and Klenow (2014), correlated policy distortions can affect the productivity growth of more productive establishments leading to potentially larger negative output effects. We note however, that the growth rate of productivity can still differ across economies if distortions affect equilibrium wages, however, what the result in Lemma 1 implies is that the growth rate of productivity would not differ across establishments in the same economy. To provide some intuition regarding the establishment value function we characterize the implicit value function of an incumbent in Lemma 2.

Lemma 2. *Given an output tax τ, a productivity level z, and operating profits $m(w,r)$, the value function of an establishment that solves the dynamic problem is given by $W(z,\tau) = A(w,r)\tau^{\theta}z^{\theta}$, where the constant $A(w,r)$ is implicit given by the following expression:*

$$A(w,r) = \frac{m(w,r)}{\lambda + R - (\theta-1)\mu_z - \theta\mu_\tau - \dfrac{\theta(\theta-1)}{2}(\sigma_z^2+\sigma_\tau^2)}.$$

Proof See Appendix A.1.

[8] For more discussion, see Luttmer (2010).

From Lemma 2 is clear that increases in policy distortions σ_τ have a direct and an indirect impact on the value of incumbents. The direct impact is positive and the indirect impact through prices $m(w, r)$ and through establishments' investment in productivity μ_z can be positive or negative. Now, we can characterize the problem of entering establishments.

3.2 Entering establishments

Potential entering establishments face an entry cost c_e in units of output and make their entry decision knowing the output entering tax level τ_e. For tractability, we assume that entrants enter with the same level of productivity, denoted by z_e. The initial level of productivity is such that the value of entering establishments satisfies the usual zero profit condition:

$$W_e = W(z_e, \tau_e) - c_e.$$

Note that such a value of productivity z_e exists and is unique which follows from the fact that the value of entry W_e inherits the properties of the value of incumbent establishments characterized in Lemma 2. In addition, in the special case where the model is deterministic the value of entering is the same as in Restuccia and Rogerson (2008), which is the establishments' expected profit.

3.3 Stationary distribution of establishments

Given the optimal decisions of incumbents and entering establishments, we are now ready to characterize the stationary distribution $g(z, \tau)$ over productivity z and distortion τ. The first step to characterize this distribution is to rewrite the Brownian motions of productivity z and distortion τ as a function of s, where $s = \tau^\theta z^\theta$.[9] The resulting s Brownian motion is

[9]Note from the input demands discussed previously, the size of the establishment is proportional to s.

given by:

$$\frac{ds}{s} = \left[\theta\left(\mu_z(w,r) + \mu_\tau\right) + \frac{\theta(\theta-1)}{2}(\sigma_z^2 + \sigma_\tau^2)\right] dt + \theta(\sigma_z + \sigma_\tau) dw_s, \tag{7}$$

where the drift μ_s is equal to a weighted average of the output tax and the productivity Brownian motion, i.e. $\mu_s = \theta\left(\mu_z(w,r) + \mu_\tau\right) + \frac{\theta(\theta-1)}{2}(\sigma_z^2 + \sigma_\tau^2)$. It is important to remember that the drift of the productivity Brownian motion μ_z is an endogenous object and is given by the solution of the incumbent establishment's dynamic problem, $\mu_z = \left[\frac{\theta A(w,r)}{c_\mu}\right]^{\overline{\theta - 1}}$ $\frac{1}{\theta-1}$.
The standard deviation σ_s of the s Brownian motion is the weighted sum of the standard deviation of the output tax Brownian motion σ_τ and the standard deviation of the productivity Brownian motion σ_z, i.e. $\sigma_s = \theta(\sigma_z + \sigma_\tau)$.

In order to characterize the stationary distribution over size s, it is useful to rewrite the model in logarithms. Let x denote the logarithm of s, that is $x = \log(s/s_e)$, where s_e is the size in which establishments enter. Now we can rewrite the Geometric Brownian motion in equation (7) as a Brownian motion in the logarithm of s,

$$dx = \mu_x dt + \sigma_x dw_x,$$

where $\mu_x = \mu_s - \frac{1}{2}\sigma_s^2$, and $\sigma_x = \sigma_s$. Let $M(x,t)$ denote the number density function of establishments, i.e. the mass of size x establishments at time t. At time t, the total number of establishments is equal to $M(t) = \int_{-\infty}^{+\infty} M(x,t)dx$.

The establishments productivity process can be modeled by a *modified* Kolmogorov-Fokker-Planck equation of the form:

$$\frac{\partial M(x,t)}{\partial t} = -\mu_x \frac{\partial M(x,t)}{\partial x} + \frac{\sigma_x^2}{2}\frac{\partial^2 M(x,t)}{\partial x^2} - \lambda M(x,t) + B(0,t), \tag{8}$$

where λ is the death rate of establishments and the function $B(0,t)$ are the new establishments that enter at t and have size 0, after the renormalization. The solution of this problem is discussed in Gabaix (2009), and we solve by applying Laplace Transforms methods.[10]

We are interested in a stationary distribution for the number density function, i.e. solutions that are separable in time t and are of the form $M(x,t) = M(t)f(x)$ and $B(0,t) = M(t)b\delta(x-0)$, where b is the establishment entry rate at point $x = 0$ and $\delta(\cdot)$ is a Dirac delta function which is equal to 1 at the entry, normalized to zero, and is equal to zero everywhere else.[11] Therefore, we can rewrite the *modified* Kolmogorov-Fokker-Planck equation equation (8) as:

$$\frac{M'(t)}{M(t)}f(x) = \eta f(x) = -\mu_x f'(x) + \frac{\sigma_x^2}{2}f''(x) - \lambda f(x) + b\delta(x-0), \tag{9}$$

where $\frac{M'(t)}{M(t)}$ is the separation rate denoted by η and $M(t) = e^{\eta t}M(0)$ in the balanced growth path. We normalize $M(0) = 1$. We assume four boundary conditions:

$$\lim_{x\to+\infty} f(x) = 0, \qquad \lim_{x\to+\infty} f'(x) = 0, \tag{10}$$

$$\lim_{x\to-\infty} f(x) = 0, \qquad \lim_{x\to-\infty} f'(x) = 0, \tag{11}$$

and

$$f(x) \geq 0, \qquad \int_{-\infty}^{+\infty} f(x)dx = 1. \tag{12}$$

The first four boundary conditions (10) and (11) guarantee that the stationary distribution

[10] The paper Da-Rocha et al. (2016) shows that the Double Pareto is a particular solution in frameworks with inaction.

[11] Mathematically we can express this by using a Dirac delta function that is equal to infinity at the point on which new firms enter and zero otherwise. Let the function $b(0)$ can be described by:

$$\hat{b}(0) = b\delta(x-0),$$

where δ denotes the Dirac delta function:

$$\delta(x) = \begin{cases} +\infty & \text{if} \quad x = 0, \\ 0 & \text{if} \quad x \neq 0. \end{cases}$$

is bounded, and equations (12) guarantee that f is a pdf. The boundary constraints restricts the separation rate η, by integrating (9) we find:

$$\eta \int_{\infty}^{+\infty} f(x)dx = \left(-\mu_x f(x) + \frac{\sigma_x^2}{2} f'(x)\right)\Big|_{-\infty}^{+\infty} - \lambda \int_{-\infty}^{+\infty} f(x)dx + \int_{-\infty}^{+\infty} b\delta(x-0)dx$$

and applying the boundary conditions and using the Dirac delta function, we find that growth rate of establishments η is equal to:

$$\eta = b - \lambda.$$

The expression for η has a very intuitive interpretation, it states that growth rate of establishments η is equal to the net entry rate $(b - \lambda)$. After some algebraic manipulation from equation (9), we find that the stationary distribution must satisfy the following differential equation:

$$f''(x) - \frac{2\mu_x}{\sigma_x^2} f'(x) - \frac{2(\lambda+\eta)}{\sigma_x^2} f(x) = -\frac{2b}{\sigma_x^2}\delta(x-0), \tag{13}$$

subject to the boundary conditions and to $f(\cdot)$ be a pdf. We can now characterize the stationary (log) size distribution, which is a double Pareto, with endogenous tail index, ξ, and endogenous net entry rate, $b - \lambda$ at $x = 0$. Formally, Lemma 3 characterizes the stationary distribution.

Lemma 3. *Given wages w and rental rate of capital r, the stationary size distribution associated with the output tax rate Geometric Brownian Motion is an double Pareto:*

$$g(s) = \begin{cases} C\left(\frac{s}{s_e}\right)^{-(\xi_- + 1)} & \textit{for} \quad s < s_e. \\ \\ C\left(\frac{s}{s_e}\right)^{-(\xi_+ + 1)} & \textit{for} \quad s \geq s_e. \end{cases}$$

where the tail indexes ξ_+ is the positive root and ξ_- is the negative root that solves the characteristic equation $\frac{\sigma_s^2}{2}\xi^2 + \left(\mu_s - \frac{\sigma_s^2}{2}\right)\xi - (\lambda+\eta) = 0$ and $C = \frac{-\xi_- \xi_+}{s_e(\xi_+ - \xi_-)}$. Moreover, the *average size $\overline{s}$ is given by:*

$$\overline{s} \;=\; s_e \frac{-\xi_- \xi_+}{(\xi_+ - 1)(1 - \xi_-)}.$$

Proof See Appendix A.2.

We leave the poof of Lemma 3 to the Appendix. Lemma 3 characterizes the endogenous distribution that is a function of establishments' investment in productivity μ_s and entry size s_e. We can also calculate from Lemma 3, the expected average growth rate of establishments, given by:

$$\frac{\overline{s}}{s_e} \;=\; \frac{\eta + \lambda}{\eta + \lambda - \mu_s}.$$

An important implication of the model is with respect to the size of new entrants s_e. An increase in policy distortions (an increase in σ_τ) which increases misallocation, produces an increase μ_s and a decrease in entry size s_e, generating a larger distance between an incumbent average size and the size of news entrants. An increase in distortions also increases the left tail of the distribution of establishments.

3.4 Household's problem

The household problem is standard and essentially help us pin down the stationary interest rate R. As such, the process for capital accumulation in this model follows the standard neoclassical growth model. The stand-in household seeks to maximize lifetime utility subject to the law of motion of wealth given by:

$$(RK + w + T + \Pi - bc_e - c)\, dt,$$

where w is the wage rate, R is the is the interest rate which in equilibrium is the rental price of capital minus capital depreciation ($R = r - \delta_k$), T is the lump-sum tax levied by the government, Π is the total profit from the operations of all establishment, bc_e is the entry cost and c is consumption.

We assume that households have log utility, $u(c) = \log(c)$, and we characterize the equilibrium interest rate by solving the household's problem. We define total wealth as:

$$a = K + \frac{w}{R} + \frac{T}{R} + \frac{\Pi}{R} - b\frac{c_e}{R},$$

and we rewrite the law of motion of wealth as $da = (Ra - c)dt$. The household solves the following Hamilton-Jacobi-Bellman equation:

$$\rho V(a) = \max_c \left\{ \log(c) + [Ra - c] V'(a) \right\}.$$

Lemma 4 establishes that in the stationary equilibrium the interest rate R is equal to the discount rate ρ.

Lemma 4. *In the stationary equilibrium the interest rate is equal to the discount rate $R = \rho$.*

3.5 Stationary equilibrium

We assume that population grows at an exogenous rate that is the same as the net growth rate of establishments $b = \eta + \lambda$. This assumption guarantees that wages w are constant in the stationary equilibrium. Since the growth in population is exogenous, we define the stationary equilibrium in per capita terms, i.e. $N = 1$. Establishments average size $\overline{s}$, establishments entry size s_e, and establishments investment in productivity, μ_z are endogenous in the model.

Definition Given the exogenous growth rate of establishments and GDP per capita η and initial capital stock k_0, a stationary equilibrium for this economy is a stationary distribution

$g(\cdot)$, a net entry rate $b = \eta + \lambda$, a value function for incumbents $\{W(\cdot)\}$, a policy function for new entrants $\{z_e\}$, policy functions $\{k(\cdot), n(\cdot), x_z(\cdot), c(\cdot)\}$, prices $\{r, w\}$, and transfer $\{T\}$ such that:

i) Given prices and transfer, the households' policy function $\{c(\cdot)\}$ solves the household dynamic problem.

ii) Given prices, the incumbents' policy functions $\{k(\cdot), n(\cdot)\}$ solve the incumbents' static problem.

iii) The incumbents' policy function $\{x_z(\cdot)\}$ together with the value function $\{W(\cdot)\}$ solve the incumbents' dynamic problem.

iv) The stationary distribution $\{g(\cdot)\}$ solve the Kolmogorov forward equation.

v) The entering establishments' policy function $\{z_e\}$ solves the free-entry condition.

vi) Market Clearing:

  a) Capital: $K = \int_0^{+\infty} k(s, w, r) g(s) ds$

  b) Labor: $N = \int_0^{+\infty} n(s, w, r) g(s) ds$

vii) The government budget constraint is satisfied, $T = \int_0^{+\infty} \tau_y y(s) g(s) ds$.

The stationary equilibrium is a fixed point in measure and it is very simple to compute. From the household's problem, we solve for the stationary interest rate R and hence pin down the rental rate of capital r. From the incumbents' static problem, we solve the labor and capital demand as a function of prices $\{r, w\}$ and policies $\{\tau\}$. Given the solution of the static problem, incumbents solve the dynamic problem of investing in productivity. The solution of this problem is a policy function $\{x_z(\cdot)\}$ that determines the Geometric Brownian motion for productivity of the entire economy. Given the Geometric Brownian motion for productivity, we solve for the stationary distribution $g(\cdot)$ that solves the Kolmogorov forward equation. After solving for the stationary distribution $g(\cdot)$, the entry rate at the minimum entry size, s_e must solve the free-entry condition, and markets must clear. There are two market clearing conditions: capital and labor. Capital market clearing is straightforward. Labor market clearing guarantees that labor demand is equal to labor supply.[12]

4 Output, TFP, and Establishment Size

We characterize the impact of policy distortions on output and TFP using the well-known concept of revenue total factor productivity TFPR, which was disseminated in the context of the macro development literature by Hsieh and Klenow (2009). First, we note that in our model an establishment's TFPR is given by:

$$\text{TFPR} \;=\; \frac{y}{k^{\alpha}n^{\gamma}} \propto \frac{1}{(1-\tau_y)} = \frac{1}{\tau^{\theta(1-\alpha-\gamma)}} = \tau^{-\theta(1-\alpha-\gamma)}.$$

Since policy distortions follow a Geometric Brownian motion, we can use the same methodology as in Lemma 3 to find the stationary distribution of τ and TFPR.[13]

We calculate aggregate capital and aggregate labor by integrating the demand of capital and labor from the the establishments' static problem (equations 2 and 3),

$$N = \left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha-\gamma}} \overline{s},$$

[12]Equilibrium conditions can be found in the Appendix.

[13]The distribution of distortions $g_\tau(\tau)$ is a Double Pareto. Therefore, log TFPR follows a Double Exponential with roots $\xi_{TFPR,-}$ and $\xi_{TFPR,+}$ that solve the characteristic equation $\frac{\sigma^2_{TFPR}}{2}\xi^2 + \left(\mu_{TFPR} - \frac{\sigma^2_{TFPR}}{2}\right)\xi - (\lambda+\eta) = 0$ where $\mu_{TFPR} = -\theta(1-\alpha-\gamma)\mu_\tau - [\theta(1-\alpha-\gamma)+1]\frac{\sigma_\tau}{2}$ and $\sigma^2_{TFPR} = \theta^2(1-\alpha-\gamma)^2\sigma^2_\tau$.

$$K=\left[\left(\frac{\alpha}{r}\right)^{1-\gamma}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}}\overline{s},$$

capital and labor demands are functions of prices and average establishment size $\overline{s}$. The establishments' output is not a function of the establishment size. Misallocation implies that establishments of the same size may produce different amounts of output. That is,

$$y(z,\tau)=\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}}\frac{s}{\tau^{\theta(1-\alpha-\gamma)}}.$$

Now we can calculate aggregate output Y, which after some algebraic manipulation is given by:

$$Y=\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}}\int_0^{+\infty}z^{\theta}g_z(z)dz\int_0^{+\infty}\tau^{\theta(\alpha+\gamma)}g_\tau(\tau)d\tau,$$

aggregate output depends on the static TFP gains, $E\tau^{\theta(\alpha+\gamma)}=\int_0^{+\infty}\tau^{\theta(\alpha+\gamma)}g_\tau(\tau)d\tau$, from equalizing TFPR within industries, and from the (endogenous) productivity distribution, $Ez^{\theta}=\int_0^{+\infty}z^{\theta}g_z(z)dz$.

After solving for the endogenous distribution of productivity $g_z(\cdot)$ and output taxes $g_\tau(\cdot)$ following the same methodology as in Lemma 3, we explicitly obtain aggregate output Y in the model to be proportional to:[14]

$$Y\propto\left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{\tau^{\theta(\alpha+\gamma)}}}\right)\frac{1}{(1-\tau_{y,e})}\right]\left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z^{\theta}}}\right)\frac{s_e}{\overline{s}^{\gamma/(1-\alpha)}}\right], \tag{14}$$

where we use the fact that $z_e^{\theta}\tau_e^{\theta}=s_e$, and $(1-\tau_{y,e})=\tau_e^{-\theta(1-\alpha-\gamma)}$. From equation (14) we emphasize that aggregate output Y depends on two key terms. The first term in square

[14] Both distributions are double Pareto with drifts $\mu_{z^{\theta}}=\theta\mu_z+\theta(\theta-1)\frac{\sigma_z^2}{2}$ and $\mu_{\tau^{\theta(\alpha+\gamma)}}=\theta(\alpha+\gamma)\mu_\tau+\theta(\alpha+\gamma)(\theta(\alpha+\gamma)-1)\frac{\sigma_\tau^2}{2}$; and standard deviations $\sigma_{z^{\theta}}=\theta\sigma_z$ and $\sigma_{\tau^{\theta(\alpha+\gamma)}}=\theta(\alpha+\gamma)\sigma_\tau$.

brackets represents the static output gain from equalizing TFPR across establishments,

$$\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{\tau^{\theta(\alpha+\gamma)}}}\right)\frac{1}{(1-\tau_{y,e})}.$$

The second term represents the output gain from the change in the (endogenous) productivity distribution,

$$\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z^{\theta}}}\right)\frac{s_e}{\overline{s}^{\gamma/(1-\alpha)}}.$$

Notice that in this endogenous component of productivity the first element relates to the growth in productivity investment over the life-cycle of establishments and the second element relates to the effect on the entry size and hence establishments' productivity. In our quantitative analysis that follows we emphasize the relative importance of all these terms in accounting for income differences in our model. For completeness, we compute measured TFP following standard practice as:

$$\text{TFP}=\frac{Y}{K^{\alpha/(\alpha+\gamma)}N^{\gamma/(\alpha+\gamma)}}. \tag{15}$$

5 Calibration

Our main objective is to study the quantitative impact of policy distortions on aggregate TFP and GDP per capita in an economy that is relatively more distorted than the United States in the same spirit of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). For this reason, we calibrate a benchmark economy to U.S. data.

We start by selecting a set of parameters that are standard in the literature. These parameters have either well-known targets which we match or the values have been well discussed in the literature. Following the literature, we assume decreasing returns in the establishment-level production function and set $\alpha+\gamma = 0.85$, e.g., Restuccia and Rogerson (2008). Then we split it between α and γ by assigning 1/3 to capital and 2/3 to labor, implying $\alpha = 0.283$ and $\gamma = 0.567$. We set the annual exit rate λ to be 10 percent, which is in line to the estimates in the literature, e.g., Davis et al. (1998). We set the discount rate ρ to match a real interest rate of 4 percent and the depreciation rate of capital δ to 7 percent to match a capital to output ratio of 2.5. To calibrate the exogenous growth rate of the mass of establishments η, we use the equilibrium implication of the model that the aggregate growth rate of TFP over time is proportional to the growth rate of the mass of establishments. Since the growth rate of TFP in the United States in the last 100 years is roughly equal to 2 percent, we set η equal to 0.02. We normalize $\tau_e = 1$ for the benchmark economy.

We calibrate the remaining parameters by solving the equilibrium of the model and making sure the equilibrium statistics match some targets. The remaining 6 parameters to calibrate are $(\sigma_z^2, \mu_\tau, \sigma_\tau^2, c_e, c_\mu, \theta)$. We construct the following 6 statistics in the model and match with the corresponding targets in the data:

(1) Standard deviation of log revenue total factor productivity (TFPR):

$$\text{SD log TFPR} = \sqrt{\frac{1}{\xi^2_{TFPR,-}} + \frac{1}{\xi^2_{TFPR,+}}}.$$

(2) Employment growth rate:

$$\mu_s = \left[\theta\left(\mu_z + \mu_\tau\right) + \frac{\theta(\theta-1)}{2}(\sigma_z^2 + \sigma_\tau^2)\right].$$

(3) Standard deviation of log employment:

$$\sigma_s = \theta(\sigma_z + \sigma_\tau).$$

(4) Productivity growth rate:

$$\mu_z = \left[\frac{\theta}{s_e}\frac{c_e}{c_\mu}\right]^{\dfrac{1}{\theta - 1}}.$$

(5) Average establishment size:

$$\overline{s} = s_e \frac{\eta + \lambda}{\eta + \lambda - \mu_s}.$$

(6) Cumulative Distribution Function (CDF) of establishment size:

$$F(s \leq \overline{s}) = 1 - \left(\frac{-\xi_- \xi_+}{\xi_+ - \xi_-}\right) e^{-(\xi_+)\log(\overline{s}/s_e)}.$$

The six parameters are selected simultaneously but some parameters have a first-order impact on some targets so we discuss them in turn. The policy distortions parameters μ_τ and σ_τ help matching both the standard deviation of log of TFPR in the United States from Hsieh and Klenow (2009) and the employment growth over the life cycle of plants in the United States from Hsieh and Klenow (2014).

Table 1: Calibration to U.S. Data

Parameter	Value	Target	
c_e	1.7919	Average establishment size	21.85
θ	1.1610	(CDF) % small establishments	92.5%
c_μ	0.2553	Productivity growth rate	4.7%
σ_z^2	0.4639	Zipf's law, ξ_+	1.059
σ_τ^2	0.3764	SD log TFPR	0.49
μ_τ	-0.0741	Employment growth rate	4.7%

We calibrate c_μ to match the annual measured productivity growth rate of establishments of 4.7 percent from Hsieh and Klenow (2014) for the United States. The standard deviation of productivity σ_z is calibrated to match the standard deviation of employment across establishments compatible with Zipf's law, so we set ξ_+ equal to 1.059 from Gabaix (2009). The entry rate c_e and the TFP normalization factor θ are calibrated to match the average establishment size and the share of small establishments in the United States in 1997 from Hsieh and Klenow (2009). The implied parameters values from this procedure are summarized in Table 1. In the next section we assess the quantitative impact of increased policy distortions in this model.

6 Quantitative Experiments

We quantify the impact of policy distortions on productivity investment, aggregate output, aggregate TFP, and other relevant variables by comparing these statistics in distorted economies relative to the benchmark economy. We highlight the quantitative impact of policy distortions in our model with investment in establishment-level productivity and an endogenous distribution of productivity relative to a version of the model where the distribution of productivity is exogenous as in Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). We show that empirically-plausible policy distortions generate substantial negative effects on aggregate output and TFP. Distortions also reduce the growth of establishments consistent with the empirical evidence in Hsieh and Klenow (2014). Hence, in our framework, policy distortions generate differences in output per capita across countries that are closer in line with evidence relative to the existing literature with exogenous distributions of productivity.

We quantify the impact of changes in policy distortions via changes in the dispersion in distortions σ_τ that create misallocation. Because changes in the dispersion in distortions affect the total amount of taxes in the economy, we follow the literature in adjusting the level of taxes to isolate the impact of dispersion, in particular we choose the level of entry taxes τ_e so that the static output gains from eliminating distortions is the same for the economy with the level of distortions in China reported in Hsieh and Klenow (2009). We note, however, that our quantitative results are nearly identical if we instead keep τ_e constant as in the benchmark economy, so this adjustment is inconsequential for our results. This is consistent with the findings in the literature whereby aggregate taxes are relatively unimportant and what matters is the dispersion in taxes across individual producers.[15] We quantify the contribution to our results of the endogenous component of the distribution of establishment-level TFP.

6.1 Changes in policy distortions σ_τ

Policy distortions generate large negative effects on aggregate output and TFP in our model. Table 2 reports the results for economies that differ in the dispersion in output taxes across establishments creating misallocation. We compute economies that feature dispersion in log TFPR that are comparable in magnitude to those estimated by Hsieh and Klenow (2009) for the United States, India, and China. We also compute results for three other economies that are relatively more distorted than China and India. Dispersion in log(TFPR) is 0.49 in the benchmark economy, 0.67 in India, and 0.74 in China. Table 2 reports relative aggregate output and measured TFP for each economy relative to that of the benchmark economy. Hence, the results reported are comparable to the exercise in Hsieh and Klenow (2009) of calculating the aggregate output gains from reducing the dispersion in marginal revenue products in China and India to the level observed in the United States.[16]

Our results are quite striking. For instance, the economies with dispersion in distortions of 0.67 and 0.74, as documented in Hsieh and Klenow (2009) for China and India, have aggregate output that is 45.6% and 34.1% of that in the benchmark economy. Economies with larger dispersion in distortions feature much lower relative output, 12.9% and 2.5% of the benchmark economy. We find similar quantitative effects for aggregate measured TFP.

[15] See, for instance, Restuccia and Rogerson (2008).

[16] Notice also that while Hsieh and Klenow (2009) report TFP gains, these gains are calculated as changes in aggregate output, however, in their static setting with constant factors and number of firms, TFP and output gains are identical.

Table 2: Effects of Changes in TFPR dispersion σ_τ

	SD(logTFPR)					
	0.49	0.67	0.74	0.85	1.00	1.40
Relative Output Y	1.000	0.457	0.342	0.225	0.129	0.025
Relative TFP	1.000	0.450	0.334	0.218	0.122	0.021
Entry size $s_e/\bar{s}$	0.611	0.499	0.454	0.389	0.302	0.096
Life-cycle investment μ_z	0.047	0.040	0.037	0.033	0.026	0.004

Notes: Output Y and total factor productivity (TFP) are reported relative to the benchmark economy. Entry size $s_e/\bar{s}$ is defined as the size of entrants relative to the average incumbent. Life-cycle investment μ_z is the growth rate of establishment productivity. Hsieh and Klenow (2009) report that the standard deviation of log TFPR is 0.49 for the United States in 1997, 0.67 in India in 1991, and 0.74 in China in 1998.

These results represent substantial decreases in output and TFP compared to the effects from static misallocation reported for instance in Hsieh and Klenow (2009). These larger effects on aggregate output and TFP arise in our model because in more distorted economies, establishments enter relatively small (low relative entry size $s_e/\bar{s}$) and their productivity does not growth as much (low μ_z) relative to the benchmark economy. Whereas the size of entrants is 61.1% of the average incumbent in the benchmark economy, entrants are only 30% of the incumbent size in the economy with double the dispersion in distortions than the benchmark economy. Similarly, while the annual growth in productivity is 4.7% in the benchmark economy, in the economy with double the dispersion the growth in productivity is 2.6%. This effect of distortions on the life cycle of firms is consistent with the evidence in Hsieh and Klenow (2014) where the growth in productivity of firms is found to be much lower in India and Mexico than in the United States.

6.2 Amplification

To illustrate the quantitative importance of the endogenous distribution in amplifying the negative impact of policy distortions on aggregate output and to relate our results with the gains from reallocation in Hsieh and Klenow (2009), in Table 3 we report the gains in aggregate output that arise in each economy when eliminating the dispersion in TFPR in the distorted economy relative to the gains of eliminating distortions in the benchmark economy. We decompose the total effect in aggregate output between the static gains from factor misallocation and the change in the endogenous distribution of productivity. This decomposition follows our characterization of aggregate output between the static effect of factor misallocation and the effect on the endogenous distribution of productivity in equation (14). Hence, the total output gain in Table 3 is the product of the static gains from factor misallocation and the gains from the change in the endogenous distribution.

Table 3: Changes in TFPR Dispersion σ_τ

	SD(logTFPR)					
	0.49	0.67	0.74	0.85	1.00	1.40
Relative output gains from reduced misallocation:						
Static	1.00	1.40	1.56	1.81	2.17	3.16
Endogenous Distribution	1.00	1.57	1.87	2.45	3.68	12.80
Total	1.00	2.19	2.93	4.44	7.76	40.49

Notes: Hsieh and Klenow (2009) report that the standard deviation of log TFPR is 0.49 in the United States, 0.67 in India, and 0.74 in China. We report the results for three other economies that are more distorted than China and India. Static gains refer to the output gains from reducing log TFPR dispersion (i.e., factor misallocation) to the level in the benchmark economy, holding aggregate factors and the productivity distribution of establishments constant. The static gains match up with the output gains for China and India relative to the United States in Hsieh and Klenow (2009). Endogenous Distribution refers to the change in the endogenous productivity distribution and Total refers to the overall impact on aggregate output. These terms follow the decomposition of aggregate output in equation (14).

In the economy with dispersion of log TFPR of 0.67, the output gains from eliminating distortions relative to the gains from eliminating distortions in the benchmark economy is 2.2-fold, that is, aggregate output in this economy would increase by 120% when eliminating distortions relative to the corresponding increase in the benchmark economy. Alternatively, this is the increase in aggregate output that results from reducing the dispersion in distortions in this economy to the level of the benchmark economy. Notice that the total increase in output is much larger than that reported in Hsieh and Klenow (2009) for the impact of factor misallocation in India. According to Hsieh and Klenow (2009, Table VI), the output gains in 1991 India from equalizing TFPR relative to 1997 U.S. gains is 41.4%. In our model, the corresponding increase in aggregate output of reducing dispersion in India to the level of the U.S. is 40% (Static gains 1.40 in Table 3). Hence, the increase in output arising from the reduction in factor misallocation is very close to that estimated empirically. But in our model the distribution of productivity changes endogenously generating a larger increase in output. As a result, the endogenous distribution generates a substantial amplification effect over and above the gains from eliminating static misallocation. This amplification effect on output is substantial. For the log TFPR 0.67 economy, whereas the static gain from reducing factor misallocation is 40%, the change in the endogenous distribution of productivity increases aggregate output by 57%. To put it differently, the change in the endogenous distribution accounts for 58% (log(1.57)/log(2.19)) of the gains in aggregate output from reducing misallocation to the levels in the benchmark economy. In the economy with dispersion in log TFPR of 1.4, the total increase in aggregate output from the reduction in the dispersion of distortions is 40.5-fold with the endogenous distribution accounting for 67% of this increase.[17]

6.3 Life-cycle vs. entry effects

We can further decompose the effect of the endogenous distribution on aggregate output in the model into the change in the life cycle of establishments through endogenous investment in productivity and into the entry productivity of establishments affecting entering establishment size. Table 4 reports the quantitative effect in relative aggregate output from these two terms as emphasized in equation (14) for all the economies considered.

[17] An amount of log TFPR dispersion of 1.4 is plausible among very poor countries. For instance, Restuccia and Santaeulalia-Llopis (2015) document static output gains from eliminating misallocation in the agricultural sector in Malawi that is a factor of 3.6-fold, hence of similar order of magnitude for the static gains in this economy.

Table 4: Changes in TFPR Dispersion σ_τ

	SD(logTFPR)					
	0.49	0.67	0.74	0.85	1.00	1.40
Relative Output Y	1.00	0.46	0.34	0.23	0.13	0.03
Life-cycle Effect $\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z\theta}}\right)$	1.00	0.75	0.67	0.58	0.48	0.31
Entry Effect $\left(\frac{s_e}{\bar{s}\gamma/(1-\alpha)}\right)$	1.00	0.86	0.80	0.71	0.59	0.25

Notes: Hsieh and Klenow (2009) report that the standard deviation of log TFPR is 0.49 in the United States, 0.67 in India, and 0.74 in China. We report the results for three other economies that are more distorted than China and India. Life-cycle and entry effects are reported relative to the benchmark economy and refer to the component of aggregate output due to the life-cycle component of productivity investment and entry size as described in equation (14), Section 4.

In Table 4, the product of the life-cycle and entry effects corresponds to the effect of the endogenous distribution emphasized earlier. So the remaining effect on output is the impact of static factor misallocation. For instance, in the economy with a dispersion in log TFPR of 0.67, the effect of policy distortions on establishment investment over the life cycle generates an aggregate output that is 75% of the benchmark economy, while the entry effect generates an output that is 86% that of the benchmark economy. In other words, the effect of life-cycle investment accounts for 38% (log(0.75)/log(0.46)) of the lower aggregate output, whereas the distortion in entry size accounts for 20% (log(0.86)/log(0.46)). The remaining 42% is due to the static effect of factor misallocation. The contribution of these factors to aggregate output differences vary with the magnitude of distortions. For the most distorted economy in Table 4, 32% of the lower relative aggregate output is due to life-cycle investment, 37% due to the entry effect, and the remaining 31% due to static factor misallocation. We conclude that policy distortions have a substantial negative impact on aggregate output not only through factor misallocation but also through the effects of distorted incentives for life-cycle investment and entry size that affect the productivity distribution in the economy.

7 Conclusions

We developed a tractable dynamic model that endogenizes the distribution of establishment-level productivity across economies. The model tractability allows us to find closed-form solutions that are useful in identifying the response of distortions on aggregate output via factor misallocation, entry distortions as well as life-cycle growth. In this framework, policy distortions not only generate differences in factor misallocation as emphasized in a large literature, but also on the distribution of establishment-level productivity. We showed that empirically-reasonable policy distortions have substantial negative effects on aggregate output and TFP in this economy, effects that are orders of magnitude larger than in models with exogenous distributions.

We have considered policy distortions that are uncorrelated to establishment-level productivity and nonetheless have found that these distortions have substantial negative effects on aggregate TFP. Since the literature has emphasized substantially larger productivity impacts of correlated idiosyncratic distortions, it would be interesting to explore the implications of correlated distortions in our framework. This requires a non-trivial extension of the theory and for this reason we leave this important exploration for future work. We also think that it would be interesting to explore specific policies or institutions such as size-dependent policies, firing taxes, financial frictions, among others in the context of this model with endogenous establishment-level TFP. These explorations of specific policies in our framework may help in reconciling the empirically large effects found in the literature relative to models with exogenous distributions. Broadly speaking, we argue that our framework with an endogenous productivity distribution may be better suited to explain the data. As a result, further progress aimed to broaden the empirical mapping of the model to the data may provide useful insights. We leave these interesting and important extensions for future work.

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A Appendix

This appendix presents the proofs of Lemma 2 and Lemma 3 and the algorithm to compute the stationary equilibrium.

A.1 Proof Lemma 2

The constant $A(w,r)$ solves the following polynomial equation:

$$\left[\frac{(\lambda+R)}{(\theta-1)} - \frac{\theta\mu_\tau}{(\theta-1)} - \frac{\theta}{2}(\sigma_z^2+\sigma_\tau^2)\right] A(w,r) - \left[\frac{\theta}{c_\mu}\right]^{\dfrac{1}{\theta-1}} A(w,r)^{\dfrac{\theta}{\theta-1}} = \frac{m(w,r)}{(\theta-1)}. \tag{16}$$

To provide intuition regarding the value function of the firm, we rewrite the implicit value function. From Lemma 1 we know that $A(w,r) = \frac{c_\mu}{\theta}\mu_z^{(\theta-1)}$, after substituting and rewriting the non-linear term, we find:

$$\left[\frac{\theta}{c_\mu}\right]^{\dfrac{1}{\theta-1}} A(w,r)^{\dfrac{\theta}{\theta-1}} \quad = \quad \mu_z A(w,r).$$

After this simplification, we rewrite equation (16) finding the following expression for $A(w,r)$:

$$A(w,r) = \frac{m(w,r)}{\lambda + R - \theta\mu_\tau - (\theta-1)\mu_z - \dfrac{\theta(\theta-1)}{2}(\sigma_z^2+\sigma_\tau^2)},$$

where μ_z depends on $A(w,r)$. ■

A.2 Proof Lemma 3

The stationary pdf is the solution of the boundary-value problem that consists of solving

$$\begin{array}{lll} f''(x) - \gamma_1 f'(x) - \gamma_2 f(x) = 0 & \text{if} & x \neq 0, \\ f''(x) - \gamma_1 f'(x) - \gamma_2 f(x) = -\gamma_3\delta(x-0) & \text{if} & x = 0, \end{array}$$

where the constants γ_1, γ_2, and γ_3 are given by

$$\gamma_1 = \frac{2\mu_x}{\sigma^2} < 0, \quad \gamma_2 = \frac{2(\lambda+\eta)}{\sigma_x^2} > 0, \quad \gamma_3 = \frac{2\hat{b}}{\sigma_x^2} > 0.$$

We solve the boundary-value problem using Laplace transforms.[18] By applying Laplace transforms in equation (9), we obtain:

$$(s^2 - \gamma_1 s - \gamma_2)\mathscr{L}[f(x)] - (s-\gamma_1)f(0) - f'(0) = -\gamma_3\mathscr{L}[\delta(x-0)]. \tag{17}$$

Using the boundary condition $f(0) \geq 0$ and $\mathscr{L}[\delta(x-0)] = 1$ we find:

$$(s^2 - \gamma_1 s - \gamma_2)Y(s) = f'(0) + (s-\gamma_1)f(0) - \gamma_3, \tag{18}$$

where

$$Y(s) = \frac{f'(0) - \gamma_3 + (s-\gamma_1)f(0)}{(s^2 - \gamma_1 s - \gamma_2)}.$$

We obtain the solution by solving the Laplace inverses when $x \neq 0$ given by:

$$\begin{aligned}
\mathscr{L}^{-1}\left[\frac{1}{(s-r_1)(s-r_2)}\right] &= \frac{1}{(r_1-r_2)}\left(e^{r_1x} - e^{r_2x}\right), \\
\mathscr{L}^{-1}\left[\frac{(s-\gamma_1)}{(s-r_1)(s-r_2)}\right] &= \frac{1}{(r_1-r_2)}\left[(r_1-\gamma_1)e^{r_1x} - (r_2-\gamma_1)e^{r_2x}\right],
\end{aligned}$$

where the two roots (one positive, and one negative) are given by $r = \dfrac{\gamma_1 \pm \sqrt{\gamma_1^2 + 4\gamma_2}}{2}$. We can rewrite the final solution for this case as:

$$\begin{aligned}
f(x) &= \tfrac{f'(0)}{(r_1-r_2)}\left(e^{r_1x} - e^{r_2x}\right) + \tfrac{f(0)}{(r_1-r_2)}\left[(r_1-\gamma_1)e^{r_1x} - (r_2-\gamma_1)e^{r_2x}\right] \quad \text{if} \quad x \neq 0, \\
f(x) &= \tfrac{f'(0)-\gamma_3}{(r_1-r_2)}\left(e^{r_1x} - e^{r_2x}\right) + \tfrac{f(0)}{(r_1-r_2)}\left[(r_1-\gamma_1)e^{r_1x} - (r_2-\gamma_1)e^{r_2x}\right] \quad \text{if} \quad x = 0.
\end{aligned}$$

When $x \neq 0$ (that is $\forall x \in (-\infty, 0) \cup (0, \infty)$), we have

$$f(x) = \begin{cases} C_1 e^{r_1x} + C_2 e^{r_2x}, & \text{if} \quad x < 0, \\ \\ C_1 e^{r_1x} + C_2 e^{r_2x}, & \text{if} \quad x > 0, \end{cases}$$

[18] Laplace transforms are given by

$$\begin{aligned}
\mathscr{L}[f'(x)] &= s\mathscr{L}[f(x)] - f(0), \\
\mathscr{L}[f''(x)] &= s^2\mathscr{L}[f(x)] - sf(0) - f'(0).
\end{aligned}$$

where

$$
\begin{aligned}
C_1 &= \frac{1}{(r_1 - r_2)}\left[f'(0) + f(0)(r_1 - \gamma_1)\right], \\
C_2 &= \frac{-1}{(r_1 - r_2)}\left[f'(0) + f(0)(r_2 - \gamma_1)\right],
\end{aligned}
$$

and $r_1 > 0$ and $r_2 < 0$. When $x > 0$ in order to $f(\cdot)$ be a pdf, it is necessary that $C_1 = 0$ and

$$
f'(0) = -f(0)(r_1 - \gamma_1) \Rightarrow C_2 = \frac{-1}{(r_1 - r_2)}\left[f(0)(\gamma_1 - r_1) + f(0)(r_2 - \gamma_1)\right] = f(0).
$$

Symmetrically when $x < 0$ we need $C_2 = 0$. Therefore

$$
f'(0) = -f(0)(r_2 - \gamma_1) \Rightarrow C_1 = \frac{1}{(r_1 - r_2)}\left[f(0)(\gamma_1 - r_2) + f(0)(r_1 - \gamma_1)\right] = f(0),
$$

and

$$
f(x) = \begin{cases} f(0)e^{r_1 x} & \text{if} \quad x < 0, \\ \\ f(0)e^{r_2 x} & \text{if} \quad x \geq 0, \end{cases}
$$

where $f(0) = \left(\frac{r_1 r_2}{r_2 - r_1}\right)$. Finally we need to prove that: 1) for $x > 0$, $f'(0) = -f(0)(r_1 - \gamma_1)$ (i.e. $C_1 = 0$), and 2) for $x < 0$, $f'(0) = -f(0)(r_2 - \gamma_1)$ (i.e. $C_2 = 0$); Given that when $x > 0$ $f'(0) = r_2 f(0)$ (and when $x < 0$ $f'(0) = r_1 f(0)$) this is equivalent to show that

$$
(r_2 + r_1)f(0) = \left(\frac{\gamma_1 - \sqrt{\gamma_1^2 + 4\gamma_2}}{2} + \frac{\gamma_1 + \sqrt{\gamma_1^2 + 4\gamma_2}}{2}\right) f(0) = f(0)\gamma_1.
$$

When $x = 0$ we have

$$
f(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x},
$$

where

$$
\begin{aligned}
C_1 &= \frac{1}{(r_1 - r_2)}\left[f'(0) - \gamma_3 + f(0)(r_1 - \gamma_1)\right], \\
C_2 &= \frac{-1}{(r_1 - r_2)}\left[f'(0) - \gamma_3 + f(0)(r_2 - \gamma_1)\right].
\end{aligned}
$$

Therefore

$$f(0) = C_1 + C_2 = \frac{1}{(r_1 - r_2)}\left[r_1 f(0) - r_2 f(0)\right] = f(0).$$

Using $s = s_e e^x$, we can recover the size distribution $g(s)$. That is

$$g(s) = \frac{1}{s} f(\ln(s/s_e)) = \begin{cases} f(0)\dfrac{s^{r_1-1}}{s_e^{r_1}} & \text{if} \quad s < s_e, \\ \\ f(0)\dfrac{s^{r_2-1}}{s_e^{r_2}} & \text{if} \quad s \geq s_e. \end{cases}$$

Note that this solution is equivalent to the guess and verify solution obtained by solving the characteristic equation $\frac{\sigma_s^2}{2}\xi^2 + \left(\mu_s - \frac{\sigma_s^2}{2}\right)\xi - (\lambda + \eta) = 0$ with $r_1 = -\xi_-$ and $r_2 = -\xi_+$.

Finally, average establishment size $\overline{s}$ is given by

$$\overline{s} = s_e \frac{-\xi_- \xi_+}{(\xi_+ - 1)(1 - \xi_-)} = s_e \frac{\eta + \lambda}{\eta + \lambda - \mu_s}. \blacksquare$$

A.3 Stationary Equilibrium

Formally μ_z, A, s_e, $\overline{s}$, μ_s and w are obtained by solving the following 6 equations.

(1) Productivity growth rate is endogenous in distortions, that is establishment's investment, μ_z satisfies:

$$\mu_z = \left[\frac{\theta A(w,r)}{c_\mu}\right]^{\dfrac{1}{\theta - 1}}. \tag{19}$$

(2) Establishment's value function, A satisfies:

$$\left[\frac{(\lambda + R)}{(\theta - 1)} - \frac{\theta \mu_\tau}{(\theta - 1)} - \frac{\theta}{2}\sigma_\tau^2\right] A(w,r) - \left[\frac{\theta}{c_\mu}\right]^{\dfrac{1}{\theta - 1}} A(w,r)^{\dfrac{\theta}{\theta - 1}} = \frac{m(w,r)}{(\theta - 1)}, \tag{20}$$

where $m(w,r) = (1 - \alpha - \gamma)\left[\left(\frac{\alpha}{r}\right)^\alpha \left(\frac{\gamma}{w}\right)^\gamma\right]^{\frac{1}{1-\alpha-\gamma}}$.

(3) The establishment size growth rate is equal to:

$$\mu_s = \theta\left(\mu_z(w,r) + \mu_\tau\right) + \frac{\theta(\theta - 1)}{2}(\sigma_z^2 + \sigma_\tau^2). \tag{21}$$

(4) Minimum establishment's size is compatible with free entry:

$$c_e = A(w, r)s_e. \tag{22}$$

(5) The average endogenous size is given by:

$$\frac{\overline{s}}{s_e} = \frac{\eta + \lambda}{\eta + \lambda - \mu_s}. \tag{23}$$

(6) Labour demand, N, is compatible with labour supply:

$$\left[\left(\frac{\alpha}{r} \right)^{\alpha} \left(\frac{\gamma}{w} \right)^{(1-\alpha)} \right]^{\frac{1}{1-\alpha-\gamma}} \overline{s} = 1. \tag{24}$$