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Revisiting the Coase theorem

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Abstract. We provide a version of the Coase theorem within a general equilibrium framework. We consider an economy with other-regarding preferences, and where rights, licenses, or permissions are required to use, consume or transform some specific commodities. These permissions are initially allocated among consumers and, as the commodities, can be costlessly traded. In this scenario, we define different veto mechanisms and the corresponding core solutions that, naturally, result in the same set of efficient allocations. Our final result sets sufficient conditions on preferences and the requirement of rights to ensure that any equilibrium allocation belongs to the core and, in particular, is efficient.

Keywords: cap-and-trade, Coase theorem, competitive equilibrium, core, externalities, other-regarding preferences, rights, tradable licenses.

JEL Classification: D51, D00, D62.

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1 Introduction

In many situations, the individual preferences derived from the use of some goods depend not only upon the individual quantity consumed, but also upon the other individuals' consumption. If it is the case, we say that preferences are interdependent. One famous quotation from 165 BC., by the Roman playwright Terence, reads: *Homo sum, humani nihil a me alienum puto.*[1] Although we may find many interpretations of this sentence, one of them is that the behavior of others affects us. That is, the idea of considering preferences depending on the consumption of the others (other-regarding preferences) is far to be new.

In economic theory, the consideration of interdependent preferences is a way of formalizing externalities, which typically give rise to inefficiency problems. Indeed, intending to diminish the inefficiencies originated by externalities, several regulation systems have been proposed and analyzed in the literature.

This paper analyzes both market equilibria and the core for economies with externalities that, in general, represent market failure situations. To illustrate the kind of externalities and problems we focus on, we may think about greenhouse gases, air or water pollution, traffic congestion, and, in general, common resources involving the problem of overuse as roads, parking, swimming pools, university libraries, etc.

In this scenario, prices do not gather all the information. Since externalities are fundamentally about individual facing "wrong" prices for their actions, they are naturally a general equilibrium issue. The analysis of the price mechanism within a general equilibrium model may give some additional insights explaining how the prices can fail to incorporate "external" effects.

Arrow (1969) addressed this issue by defining a new commodity for each type of externality, each agent who produces it, and each person who suffers it. This new set of securities generates an expansion of the commodity space. In equilibrium, externalities are under the control of the price system leading to Pareto optimality. However, the Arrow model does not intend to reduce the negative effect of externalities.

Since the work of Coase (1960) the literature involving law and economics has paid attention to the role of property rights when they involve a social

[1] *I am human, I consider nothing human alien to me. This comes from his play Heauton Timorumenos.*

cost. Coase's analysis focusses on those economic activities which have harmful effects and discusses the legal delimitation of property rights suggesting a pricing system with liability for damage. He analyzes –and criticizes some consequences– Pigou's treatment in "The Economics of Welfare" arguing on the need of a change of approach on the basis that the cost of exercising a right should be compared with the loss which is suffered elsewhere in consequence of the exercise of that right.

To deal with the social costs involved in pollution of natural water, Dales (1968) has discussed a wide variety of arrangements concluding that a charging scheme lends itself easily to a market mechanism, whereas other methods –as taxes or a subsidy scheme – do not. Despite not being formalized, these ideas led to a proposal to establish a market in licenses – BOD bonds –to control water pollution from industrial sources in the Delaware estuary which will lead to an abatement of water pollution.[2,3]

Montgomery (1972) provided a solid economic analysis that gives a theoretical foundation for markets in tradable allowances to control pollution. A portfolio of licenses is defined in a way such that the possession of a specified allowance confers the right to carry out a certain average rate of emission of a specific pollutant. As in the previous literature, he assumes that prices of commodities (except those associated with pollution) are unaffected by undertaken control measures. Under this assumption, Montgomery characterizes a general economic model and shows the existence of an equilibrium that achieves externally given standards of environmental quality at least cost to the regulated firms. The equilibrium allocations need not be Pareto optimal but obtaining levels of environmental quality in an efficient manner.

Since Montgomery's contribution, many authors have considered decentralized economic models with a *cap-and-trade* mechanism and tradable allowances (rights or permits) for achieving environmental goals at many different locations.

[2]The creation of the Environmental Protection Agency (EPA) in 1970 and the passage of the Clean Water Act in 1972 enabled efforts to protect the Delaware to reach a new level of effectiveness.

[3]The Trump administration proposed, for 2018 and 2019 fiscal years, cuts to the EPA budget. Water-related programs run directly by the EPA would be slashed by 34 percent, hobbling efforts to prevent runoff pollution, monitor water quality, establish pollution limits, protect watersheds and wetlands, and pursue polluters. See https://environmentamerica.org/sites/environment/files/cpn/AME-022118-A9-REPORT/ROUGH-WATERS.html

This economic literature has influenced on applications to reduce pollutants both at the level of local institutions and at the level of the global political economy. For instance, during the commitment period of the Kioto's Protocol and Paris Agreement, industrialized countries had committed to reduce the emissions of gases – CO_2– responsible for global warming using markets of permits based on price equaling marginal abatement cost. However, the cash flows have thus far been meager.[4]

Others socio-ecological models, not confined to a timeless setting, to address problems as the *Tragedy of the Commons* analyze the harvesting strategies as a non-cooperative game. Under some reasonable assumptions, the corresponding Markov perfect Nash equilibria are compared with collectively optimal harvesting policies –see Dasgupta *et al.* (2019) and references therein. In particular, these authors conclude that, without property rights, open-access in the commons leads to their depletion in finite time even though it is in the collective interest to enforce a sustainable consumption policy.

Despite the variety of regulations to respond to externalities, there are two main types, namely, command and control regulations and market-based policies. The first regulate behavior directly, whereas the latter try to incentivize private decision-makers to change their behavior. It is known that quantity regulation may result not only in the non-existence of equilibrium but also in the non-existence of individually rational allocations.

These circumstances lead us to address the problems arising from negative externalities by establishing a market-based *cap-and-trade* mechanism. The model considers an exogenously given limiting cap and a portfolio of perfectly divisible tradable licenses or rights that are allocated among agents and will be required to use, consume, or transform some specific commodities. The total number of each permit to use or consume a given item susceptible to originate negative externalities will depend upon the cap. The establishment of these rights has implications regarding the feasibility of assignments. We define the set of feasible allocations by both the endowments –physical feasibility– and the specification of the total number of permissions required to allocate consumption bundles –rights feasibility. The rights feasibility aims to keep the negative effect of externalities below the cap.

The literature focussing on a general equilibrium framework pricing rights and

[4] See Holtsmark and Weitzman (2020).

commodities on a competitive basis is scarce. Exception are the contributions by Hurwicz (1995), and Chipman and Gouqiang (2012) –see also the references therein– that show equilibrium existence and Pareto optimality in particular models – see Section 6. We will follow Hervés-Beloso *et al.*(2012) that consider a general model of an economy with interdependent preferences where an exogenously given cap sets limits to the consumption of some specific commodities and a portfolio of tradable permits, required to use these commodities, is shared among agents. Under natural assumptions on endowments, preferences, and rights, this economy has a Nash-Walras equilibrium where the price system relates commodities and rights. However, due to externalities, the equilibrium allocations are not efficient in general.

To provide a version of the first welfare theorem for our model, we focus our attention on the core allocations. In economic environments with other-regarding preferences, the blocking mechanism involves conceptual difficulties since the payoff of each member in the deviating group depends on the reaction of the complementary coalition. Thus, when a coalition of agents aims to object the *status quo*, they must take into account the freedom to react of agents outside the coalition, and/or what the objecting coalition forecasts about the others' reaction. The different alternatives and requirements regarding feasibility give room to several definitions of core. Florenzano (1989, 1990) and Dufwenberg *et al.* (2011) have analyzed core solutions with non-ordered preferences and other-regarding preferences, respectively, although the objecting allocations may not be feasible. Yannelis (1991) was the first to have this point into account for economies, following the pioneering works by Aumann and Peleg (1960), and Aumann (1961) addressing games. Recent literature has extended the analysis paying attention to the condition of feasibility in the reaction of agents outside the blocking coalition. For instance, Graziano, Meo & Yannelis (2017, 2020) have adapted a variety of blocking mechanism to the study of stable sets with interdependent preferences and addressing the housing markets, respectively, and Di Pietro, Graziano & Platino (2020) characterize core notions, under other-regarding preferences, as zero points of social loss functions.

In a cap-and-trade economy, the requirement of permits may lead to the unfeasibility of some redistributions of the initial resources, in particular, the endowments of coalitions. This affects the results on the corresponding core solutions and taking it into account, we define the strong, prudent, and weak veto mechanisms leading to the cooperative solutions that we refer to as the pessimistic, cautious and optimistic core, respectively – see Hervés-Beloso and Moreno-García, 2016 –. A feasible allocation is efficient if it is not blocked by the coalition formed by all the participants in the economy. Since all the aforementioned veto systems coincide in the case of the big coalition, we have a consistent notion of efficiency.

To get efficiency or, more specifically, to guarantee that equilibrium allocations belong to some core, we follow the philosophy behind the Coase theorem, which suggests a legal -in addition to economic- interpretation of the externalities. To illustrate this point, let us consider, for example, that although Anne likes to drive on an empty road and dislikes to drive with traffic, from a legal point of view it is difficult to argue that Anne suffers a negative externality when the traffic does not prevent her driving at the maximum legal speed in that road. Following Coase, we assume a positive threshold below which the potential externality is negligible and elaborate on conditions of the model ensuring that rights–feasibility leads to admissible allocations regarding the given threshold. Also, we find properties on agents' interdependent preferences and the requirements of rights that allow us to obtain a strong version of the first welfare theorem by showing that the equilibrium belongs to every one of the cores we define and, in particular, is efficient.

The rest of this paper is organized as follows. In Section 2, we present a model of an economy with externalities and restrictions, given by the requirement of rights, to consume, use or transform some specific commodities. In Section 3, we present several notions of the core for the case of interdependent preferences and the requirement of rights. In Section 4, we specify a notion of equilibrium where rights are required. Once the cap is established, in Section 5 we detail natural conditions in the definition of rights ensuring that the limit is not exceeded. In Section 6, within a general equilibrium framework, we provide a revision of the neutrality and efficiency results attributed to Coase. In Section 7, we address some final remarks.

2 A model with externalities and licenses

Let us consider an economy with n consumers and ℓ commodities. Each agent $i \in N = \{1, \ldots, n\}$ has endowments $\omega_i \in \mathbb{R}^\ell_+$ and chooses a commodity bundle in the consumption set $\mathbb{R}^\ell_+$. An allocation $x = (x_1, \ldots, x_n)$ specifies a bundle x_i

for each consumer $i \in N$. Each individual has a preference relation that depends not only on her consumption but on allocations. In this way, externalities are represented by the fact that everyone is affected by the consumption of the others. We assume that the other-regarding preferences of each consumer $i \in N$ are represented by an utility function $\mathcal{V}_i : \mathbb{R}_+^{\ell n} \to \mathbb{R}$ which is given by

$$\mathcal{V}_i(x) = \mathcal{V}_i(x_i, x_{-i}),$$

where x_{-i} denotes the consumption of every agent except i.

We consider that a number k of commodities in $K \subset \{1, \ldots, \ell\}$ generate externalities and, in this case, $\mathcal{V}_i$ depends just on x_i and on $x_{-i}^K = (x_{hj}, j \in K, h \in N, h \neq i)$ where x_{hj} is the amount on commodity j that agent h chooses to use or consume.

Moreover, one could also consider the case where each commodity $j \in K$ may originate externalities only when the aggregate consumption of commodity j exceeds an exogenously given threshold.

Following Hervés-Beloso, Martínez, and Rivera (2012), we assume that a finite number k of different types of rights or permissions are established and are required to use, transform or consume those commodities $j \in K$ that generate externalities. The model aims to reflect a scenario where there are local, national or international agreements to set limits to the excessive use of some common facilities as parking or swimming pools, limits to the use of roads, limits to the capture of some fish species, limits to the use or transformation of some row materials that produce greenhouse effects, and air pollution or water pollution. Thus, the enforcement of these permissions is given exogenously by a mapping that specifies the amount of the different types of licenses required for each consumption plan. The model assumes that each type of rights is perfectly divisible and tradable and that the total amount of these permissions are distributed among consumers.

Formally, for each commodity $j \in K$ liable to generate negative externalities there is a total number $R_j \geq 0$ of rights. Each consumer $i \in N$ is endowed with $r_{ij} \geq 0$ rights or licenses of type j, thus $R_j = \sum_{i \in N} r_{ij} \in \mathbb{R}$. We denote $r_i = (r_{ij})_{j \in K} \in \mathbb{R}_+^k$ the rights' endowment of agent i. The total amount of licenses is $R = \sum_{i \in N} r_i \in \mathbb{R}_+^k$.

To obtain the bundle $z \in \mathbb{R}_+^\ell$ the model requires a quantity $\rho_j(z) \in \mathbb{R}_+$ of type j rights, with $\rho_j(0) = 0$. Thus, the mapping $\rho : \mathbb{R}_+^\ell \to \mathbb{R}_+^k$ defined by

$\rho(z) = (\rho_j(z))_{j \in K} \in \mathbb{R}^k_+$ is given exogenously, and determines the vector of rights required to get the consumption plan z.

Although one can consider that ρ depends only on the commodities generating externalities, the fact that ρ_j depends not only on the coordinate j but on the complete commodity bundle allows us to consider a wider variety of situations. To illustrate this point, note that if we consider the use of parking or a road, then the dimensions of vehicles become relevant (for example, the number of rights may differ for cars and trucks); for the case of a lake, the material used for fishing might determine the number of rights (the utilization of a fishing rod is not the same as a fishing net). Thus, the economy $\mathcal{E}$ is defined by

$$\mathcal{E} = (\mathbb{R}^\ell_+, \mathcal{V}_i, \omega_i, r_i, \rho)_{i=1,\dots,n}$$

The requirement of rights implies restrictions on agents' consumption sets. Agents may not be able to consume, use or transform certain quantities of commodities generating externalities (transform coal in electricity, for instance) even when these form part of their endowments. The consideration of rights also has implications with regard the feasibility conditions for allocations. An allocation $x = (x_1, \dots, x_n)$ is feasible in the economy $\mathcal{E}$ if

(i) $\sum_{i \in N} x_i \leq \sum_{i \in N} \omega_i$ (*physical feasibility*) and

(ii) $\sum_{i \in N} \rho(x_i) \leq R$ (*rights feasibility*).

Both conditions are stablished with inequality. This is so because rights feasibility aims to limit the consumption or transformation of commodities $j \in K$, and thus it may imply free-disposal, i.e., $\sum_{i \in N} x_{ij} < \sum_{i \in N} \omega_{ij}$ for some $j \in K$. On the other hand, note that there is no assumption relating ρ, R, and the endowments.[5] Then, $\sum_{i \in N} \rho(x_i) = R$ could result in an empty set of feasible allocations.

3 Some definitions of cores

The Coase theorem, as named and formulated by Stigler (1966), points out that complete property rights and zero (or negligible) transaction costs are needed to

[5]Section 5 analyzes conditions relating ρ, R, endowments, and an exogenous bound on consumption.

get an efficient market solution. To revisit the Coase theorem within a general equilibrium scenario we attempt a version of the first welfare theorem in its strong version for the economy $\mathcal{E}$. For this, we must have into account the presence of licenses and we face a conceptual problem which is the definition of the core. Given that each agent's preference relation depends on the consumption choices of other agents, there can be many definitions of the core.

Yannelis (1991), inspired by the work of Aumann (1961)[6], was the first to point out the relevance of the behavior of the complementary coalition in his definition of α-dominance and the corresponding α-core for economies with general preferences –and in particular for interdependent preferences. An allocation belongs to the α-core if it is feasible and no coalition of agents can redistribute their endowments making all its members better off, whatever the redistribution of the outsiders' endowments may be.

The consideration of rights has implications on the feasibility of attainable allocations for coalitions.[7] Given a coalition S and an allocation $x = (x_S, x_{-S})$, where x_S denotes the consumption bundles assigned to members of S whereas x_{-S} are the consumption bundles assigned to agents that do not belong to S, i.e., to agents in $N \setminus S$.

Definition 3.1 *An allocation $x_S = (x_i, i \in S)$ is attainable for the coalition S if it is physically and rights feasible for S, that is, (i) $\sum_{i\in S} x_i \leq \sum_{i\in S} \omega_i$, and (ii) $\sum_{i\in S} \rho(x_i) \leq \sum_{i\in S} r_i$.*

The rights feasibility, stated in condition (ii), imposes restrictions of importance on the assignments to be considered when defining cooperative solutions as it is the case of the core. These restrictions affect the allocations that coalitions can attain and, in particular, those attainable by the big coalition. In this way, the requirement of rights reduces the power of coalitions to block and also the set of feasible allocations in the economy $\mathcal{E}$. Note that if $x = (x_S, x_{-S})$ is a feasible allocation and x_S is attainable for S in $\mathcal{E}$, then x_{-S} is attainable for the coalition $N \setminus S$. However, if $x = (x_S, x_{-S})$ is feasible and z_S is attainable for S, then the allocations (z_S, x_{-S}) and (z_S, ω_{-S}) can fail to be feasible in $\mathcal{E}$.

[6]Aumann (1961) introduced the notion of α-core of a cooperative game without side payments.

[7]We have already addressed this point in a previous draft of this work –see Hervés-Beloso and Moreno-García (2016).

In economies without rights, physical feasibility defines attainable allocations for coalitions. If preferences are monotonic regarding the own consumption bundle, to define the core equality in condition (i) can be set without loss of generality. This is not the case in the economy $\mathcal{E}$ since the condition (ii) matters.

Next, we specify different veto systems that result in different core solutions.

Pessimistic Core. Consider that coalitions expect that the behavior of outsiders leads to the worst situation for them. That is, coalitions are pessimistic about what they can achieve. More precisely, an allocation x is strongly blocked by the coalition S if there exists an attainable allocation y_S for S such that $\mathcal{V}_i(y_S, y_{-S}) > \mathcal{V}_i(x)$, for every $i \in S$, and every allocation y_{-S} attainable for $N \setminus S$. That is, what the coalition can guarantee itself improves its members regardless of the actions of the outsiders.

A feasible allocation belongs to the pessimistic core if it is not strongly blocked by any coalition. For economies without rights, this is the α-core as defined by Yannelis.

Optimistic Core. Coalitions may also be optimistic and adopt a positive thinker behavior. If there is a possibility of improving they will be able to become better off. Thus, an allocation x is weakly blocked by the coalition S if there exists an attainable allocation y_S for S such that $\mathcal{V}_i(y_S, y_{-S}) > \mathcal{V}_i(x)$, for every $i \in S$, and some allocation y_{-S} attainable for $N \setminus S$.

A feasible allocation belongs to the optimistic core if it is not weakly blocked by any coalition.

Cautious Core. We can also consider cooperative solutions in-between. For instance, coalitions may be cautious and adopt a foresight behavior. Then, an allocation x is prudently blocked by the coalition S if for each attainable allocation y_{-S} for $N \setminus S$, there exists an attainable allocation y_S for S such that $\mathcal{V}_i(y_S, y_{-S}) > \mathcal{V}_i(x)$, for every $i \in S$.

A feasible allocation belongs to the cautious core if it is not prudently blocked by any coalition. This is an adaptation to the economy $\mathcal{E}$ of the β-core introduced by Aumann and Peleg (1960) for games.

We remark that the harder to block an allocation is, the larger the core we obtain. Then, we have: optimistic core $\subseteq$ cautious core $\subseteq$ pessimistic core. All of these core notions reduce to the one given by the standard blocking mechanism in the case of selfish models. Moreover, the strong, weak, and prudent veto are the same when the blocking coalition is the big coalition, leading to a consistent notion of efficiency. We remark that the set of efficient allocations depends on preferences and on the total endowments of commodities and rights but is independent of the initial distributions of both goods and rights.

We conclude that regarding the core, the impact of rights feasibility is two-fold; on the one hand, it reduces the set of potential blocking allocations and, on the other hand, the set of feasible allocations is also reduced. The former tends to make the core larger while the latter tends to make it smaller. Consequently, the total effect is not determined.

4 Equilibrium in economies with externalities and rights

In this section, we specify the equilibrium notion for the economy $\mathcal{E}$ defined in Section 2, following Hervés-Beloso *et al.* (2012).

To be precise, let $p \in \mathbb{R}_+^\ell$ denote the price of commodities, $q \in \mathbb{R}_+^k$ the price vector for rights and let Δ be the simplex of $\mathbb{R}_+^\ell \times \mathbb{R}_+^k$. For each $(p, q) \in \Delta \subset \mathbb{R}_+^\ell \times \mathbb{R}_+^k$, agent i's budget set is

$$B_i(p,q) = \left\{ x \in \mathbb{R}_+^\ell | p \cdot x + q \cdot \rho(x) \leq p \cdot \omega_i + q \cdot r_i \right\}$$

An equilibrium in $\mathcal{E}$ is a price vector (p, q) for commodities and rights and an allocation $x = (x_i, i \in N)$, such that,

(i) x is feasible in $\mathcal{E}$; that is, $\sum\limits_{i \in N} x_i \leq \sum\limits_{i \in N} \omega_i$ and $\sum\limits_{i \in N} \rho(x_i) \leq R$

(ii) for every individual i, the bundle x_i maximizes $\mathcal{V}_i(\cdot, x_{-i})$ on the budget set $B_i(p, q)$.

To guarantee equilibria existence, we assume the following hypotheses on the fundamentals of our economy:

(A.1) For every agent $i \in N$, the utility function $\mathcal{V}_i$ is continuous and $\mathcal{V}_i(\cdot, x_{-i}) : \mathbb{R}_+^\ell \to \mathbb{R}$ is locally non-satiated and quasi-concave.

(A.2) $\omega_i \in \mathbb{R}^{\ell}_{++}$ and $r_i \in \mathbb{R}^{k}_{++}$ for every $i \in N$.

(A.3) For each $j \in K, \rho_j$ is a continuous, convex function that is increasing in the commodity j.

Moreover, $\rho_j(\hat{z}, 0_j) = 0$, being $(\hat{z}, 0_j)$ any bundle with no consumption or transformation of the commodity j.

The continuity assumption (A.1) is standard in general equilibrium theory. Note also that quasi-concavity of agent i's utility is only required regarding her consumption. The interiority of endowments in (A.2) is required to guarantee the continuity of the budget correspondence. On the other hand, (A.3) states that the amount of rights of each type j is a continuous function that depends on the consumption plan and not only on commodity j. However, it takes the value 0 on consumptions that do not use or transform the commodity j. For instance, suppose that the commodity $j \in K$ is coal and agent i is endowed with commodity j, but her consumption plan, z_i has no consumption or transformation of coal, then $\rho_j(z_i) = 0$. The convexity condition aims to discourage high levels of consumption by requiring proportionally larger amounts of rights.

Under the above assumptions, equilibrium exists (see Hervés-Beloso *et al.*, 2012). Let $((p, q), x)$ be an equilibrium, where $x = (x_i, i \in N)$ is the equilibrium allocation. It is easy to see that the continuity of ρ and the local non-satiability of $\mathcal{V}_i$ imply that, for any bundle z such that $\mathcal{V}_i(x) \leqslant \mathcal{V}_i(z, x_{-i})$, then $p \cdot z \geqslant p \cdot \omega_i + q \cdot (r_i - \rho(z))$. This implies $p \cdot x_i = p \cdot \omega_i + q \cdot (r_i - \rho(x_i))$ and thus, in equilibrium we have

$$p \cdot \sum_{i \in N} (x_i - \omega_i) + q \cdot \sum_{i \in N} (\rho(x_i) - r_i) = 0.$$

Then, if $\sum_{i \in N} \rho_j(x_i) < R_j$ we have $q_j = 0$.

Moreover, at any equilibrium where $\sum_{i \in N} x_i^j < \sum_{i \in N} \omega_i^j$, that is, there is an effective cap on the consumption of the commodity j, the price of this commodity becomes null and the relevant price is the price of the rights.

To conclude this section, we state the following example that illustrates our model.

Consider an economy with three consumers, 1, 2 and 3, and two commodities, coal (c), gas (g). Consumer 1 has 1 unit of c and 2 of g and consumer 2 has 2 units of c and 1 of g. Consumer 3 has null endowments. That is, $\omega_1 = (1,2)$, $\omega_2 = (2,1)$, and $\omega_3 = (0,0)$. Individual i has a preference relation represented by the utility function $U_i(c_1, c_2, c_3, g_i) = c_i + g_i - \alpha(c_1 + c_2 + c_3)$, with $\alpha \in (0,1)$. Each consumer is endowed with two units of rights that are required for consumption, i.e., $r_1 = r_2 = r_3 = 2$.

Assume that one unit of either coal or gas can be transformed into one unit of energy. However, the use of coal produces more pollution than the use of gas. Agents' preferences are focussed on energy and suffer the externality that the transformation of coal in energy generates. Consumers leave their endowments in a clearinghouse or trading post where trade takes place. Along the trade, consumers take into account the rights required for consumption that, in this case, are defined as $\rho(c,g) = 2c+g$. Thus, consumer 1 can choose the consumption plan $(0,2)$ that requires $\rho(0,2) = 2$ units of rights and then it is in her budget set.

In this economy, the equilibrium prices for coal, gas and rights are $p_c = 0$, $p_g = 1$ and $q = \frac{1-\alpha}{1+\alpha}$, respectively. The equilibrium amount of energy is $9/2$ units using 3 units of gas and $3/2$ units of coal. To be precise, the set of equilibrium allocations is the family of bundles (c_1, g_1), (c_2, g_2) and (c_3, g_3), with market-clearing for gas, and such that:

$$g_1 \in [0,2], (1-\alpha)c_1 + g_1 = 2, (1-\alpha)c_2 + g_2 = \frac{3-\alpha}{2}, (1-\alpha)c_3 + g_3 = 1-\alpha.$$

Note that all the collection of bundles (c_i, g_i) that each consumer i can receive at equilibrium are indifferent for i. Moreover, there is an equilibrium allocation that does not depend on α and is given by $(c_1^*, g_1^*) = (0,2)$, $(c_2^*, g_2^*) = (1/2, 1)$ and $(c_3^*, g_3^*) = (1,0)$. Thus, the equilibrium utility levels are $2 - \frac{3}{2}\alpha$, $\frac{3}{2} - \frac{3}{2}\alpha$ and $1 - \frac{3}{2}\alpha$, respectively.

To show that the equilibrium is not efficient, take for instance $\alpha = 2/3$, and note that with the feasible allocation $(c_1, g_1) = (0, 2-\varepsilon)$, $(c_2, g_2) = (1/2 - \delta/2, 1+\varepsilon/2)$ and $(c_3, g_3) = (1-\delta/2, \varepsilon/2)$, all the three consumers become better off whenever $0 < \varepsilon < \delta \leqslant 1/2$ and $\varepsilon/\delta < 2/3$.

5 Relating rights and the cap

In previous sections, the total consumption – use or transformation –of each commodity $j \in K \subset \{1, \dots, \ell\}$ liable to generate negative externalities is bounded for feasible allocations through the total number of rights R_j and the function $\rho_j : \mathbb{R}^\ell_+ \to \mathbb{R}$. Assuming the objective of reducing the consumption of j to get it below a given cap $L_j > 0$, the general model given by the economy $\mathcal{E}$ would require to relate, for each $j \in K$, the cap L_j with R_j through ρ_j.

More precisely, in a given allocation $z = (z_1, \dots, z_j, \dots, z_n)$ the total consumption of commodity j is $\sum_{i \in N} z_{ij}$. Then the set of allocations fulfilling the cap is

$$\mathcal{C} = \{x \in \mathbb{R}^{\ell n}_+, \text{ such that, for each } j \in K; \sum_{i \in N} x_{ij} \le L_j\},$$

The aim of this section is to analyze the primitives of $\mathcal{E}$, namely, the number n of agents, the portfolio of permits $R = (R_j)_{j \in K}$, and the mapping $\rho = (\rho_j)_{j \in K}$ that drive to implement the cap $L = (L_j)_{j \in K}$, i.e., conditions in which rights-feasibility of an allocation embodies that the cap is fulfilled. That is, whenever $x = (x_1, \dots, x_i, \dots, x_n) \in \mathbb{R}^{\ell n}_+$ is such that $\sum\limits_{i \in N} \rho(x_i) \le R$, then $x \in \mathcal{C}$.

Consider the easiest case where the required permits for a bundle z are given by a linear mapping depending only on the consumption of each commodity j, i.e., $\rho_j(z) = \rho_j(z_1, \dots, z_j, \dots, z_\ell) = \alpha_j z_j$. The rights feasibility condition for an allocation $x = (x_i, i \in N)$ is $\sum\limits_{i \in N} \rho_j(x_i) = \alpha_j \sum\limits_{i \in N} x_{ij} \le R_j$, for each $j \in K$. Thus, if the total number of type j rights is $R_j = \alpha_j L_j$, the allocation x belongs to $\mathcal{C}$, and any point in $\mathcal{C}$ is feasible regarding rights.

For the general case, the rights of type j required for a bundle z may depend on each coordinate of z but it is sensible to assume that it strongly depends on z_j and the linear approximation is $\alpha_j z_j$ for some number α_j. Consider, for example, the use of a highway where the number of permits required depends on miles traveled given by z_j, and the dimension of the vehicle stated by z_h; in this case $\alpha_j z_j$ would represent the permits required for a car traveling z_j miles and we can assume $\rho_j(z) \ge \alpha_j z_j$. Thus, when the type j permits required to consume any bundle z depend linearly on the coordinate z_j, that is, $\rho_j(z) \ge \alpha_j z_j$– which is consistent with the convexity of ρ_j, – setting $R_j = \alpha_j L_j$ for each j, the rights feasibility implies the effectiveness of the cap. The following proposition shows this with more generality.

Proposition 5.1 *Let $\rho_j(z_1, \dots, z_j, \dots, z_\ell) \ge g_j(z_j)$, where g_j is any strictly increasing convex real function whit $g_j(0) = 0$. Given a cap $L = (L_1, \dots L_k) \in \mathbb{R}^k_+$,* *define the total number of rights of type j by $R_j = n \cdot g_j\left(\frac{L_j}{n}\right)$, for each $j \in K$.*[8]

Then, $\sum_{i=1}^{n} x_{ij} \leq L_j$, for all $j \in K$, for any feasible allocation $x \in \mathbb{R}_+^{\ell n}$.

Proof. Note that if $\sum_{i\in N} x_{ih} > L_h$ for some $h \in K$, by convexity and strict monotonicity of g_h, we have

$$\sum_{i\in N} \rho_h(x_i) \geq \sum_{i\in N} g_h(x_{ih}) \geq n g_h\left(\frac{\sum_{i\in N} x_{ih}}{n}\right) > n g_h\left(\frac{L_h}{n}\right) = R_h,$$

a contradiction with the feasibility condition.

Q.E.D.

To stress the relation between the number of rights and the cap, for each $j \in K$, consider the problem:

$$\max_{x_{ij}, i\in N} \quad \sum_{i\in N} x_{ij}$$

$$s.t. \quad \sum_{i\in N} \rho_j(x_i) = \sum_{i\in N} g_j(x_{ij}) = R_j,$$

where each $x_{ij} > 0$ and g_j is a strictly convex regular function.

For each j, the associated Lagrangian function is

$$F_j(x) = \sum_{i\in N} x_{ij} - \lambda(\sum_{i\in N} g_j(x_{ij}) - R_j).$$

First order conditions give $\frac{\partial F_j(x)}{\partial x_{ij}} = 1 - \lambda g_j^{'}(x_{ij}) = 0$, for all $i \in N$. Thus, $\lambda = \frac{1}{g_j^{'}(x_{ij})}$ for all i and then, $g_j^{'}(x_{ij}) = g_j^{'}(x_{hj})$ for all $i, h \in N$. As $g_j^{'}$ is strictly increasing we obtain that $x_{ij} = x_{hj} = \hat{x}_j$ for all $i, h \in N$. The candidates for solving the problem are those allocations in which all agents use identical amount, $\hat{x}_j$, of commodity j. If the cap for for commodity j is L_j, and we set $R_j = n \cdot g_j\left(\frac{L_j}{n}\right)$, the allocations where $x_{ij} = \hat{x}_j = \frac{L_j}{n}$ solve the problem. Thus, $\sum_{i\in N} x_{ij} < L_j$ for any feasible allocation $x = (x_1, \ldots, x_i, \ldots, x_n)$, with $x_{ij} \neq x_{hj}$ for some $i, h \in N$.

In the case where $\rho_j(z) = g_j(z_j)$, the convexity of g_j implies convexity of ρ_j, required to obtain budget correspondences with convex values. Moreover, if g_j is strictly convex and $x \neq 0$, then $g_j((1 + \frac{\alpha}{100}) \cdot x) > (1 + \frac{\alpha}{100}) \cdot g_j(x)$, for any $\alpha > 0$.

[8]This is w.l.o.g. changing the scale of g_j or the rights. If R_j is already given, by normalizing g_j one obtains $R_j = n \cdot g_j\left(\frac{L_j}{n}\right)$. If g_j is given, then we change the units of rights. In the linear case, $\alpha_j = \frac{R_j}{L_j}$.

Thus, increasing the consumption of commodity j by a given proportion, say α percent, requires to increase the rights more than α percent. For instance, if $g_j(x) = x^2$, to increase the consumption of x by 10% requires an increase of more than 12% in rights, and doubling consumption would require quadrupling the number of rights. Thus, strict convexity discourages large consumption. This could be relevant when tackling fishing in a lake where the locals traditionally fish sustainably. To face the free-entry and avoid the tragedy of the commons by setting a cap-and-trade mechanism, a suitable strictly convex function g will prevent large fishing companies from entering.

Several functions in the same conditions as ρ can be considered. Let $\hat{\rho}, \tilde{\rho}$ such that $\hat{\rho} \leq \tilde{\rho}$, then $\tilde{\mathcal{F}} \subseteq \hat{\mathcal{F}}$, where $\hat{\mathcal{F}}$ and $\tilde{\mathcal{F}}$ denote the sets of rights-feasible allocations for $\hat{\rho}$ and $\tilde{\rho}$, respectively. To analyze this situation, let us consider for a type j of rights $\hat{\rho}_j(z) = \hat{g}_j(z) = az_j$ and $\tilde{\rho}_j(z) = \tilde{g}_j(z_j)$, where $\tilde{g}_j$ is any real convex function with $\tilde{g}_j(0) = 0$. To ensure that the cap L_j is fulfilled for feasible allocations let $\hat{g}_j(\frac{L_j}{n}) = \tilde{g}_j(\frac{L_j}{n}) = a\frac{L_j}{n}$. Note that, for any such a function $\tilde{g}_j(z_j) = \hat{g}_j(z_j) = az_j$ if $z_j \leq \frac{L_j}{n}$, and $\tilde{g}_j(z_j) \geq az_j + b\left(z_j - \frac{L_j}{n}\right)$ for some $b > 0$ if $z_j > \frac{L_j}{n}$. Note that a $\hat{\rho}$ feasible allocation x such that $\sum_{i\in N} x_{ij} = L_j$ and $x_{hj} \neq x_{ij}$ for some i, h cannot be $\tilde{\rho}$ feasible. On the other hand, if the allocation y is $\tilde{\rho}$ feasible, $\sum_{i\in N} \tilde{\rho}_j(y_i) = \sum_{i\in N} \tilde{g}_j(y_{ij}) \leq R_j = aL_j$. Assume that agent h uses $2\frac{L_j}{n}$ units of commodity j, then $aL_j = \sum_{i\in N} \tilde{g}_j(y_{ij}) \geq a\sum_{i\in N} y_{ij} + b\frac{L_j}{n}$. Thus, $\sum_{i\in N} y_{ij} \leq L_j - \frac{bL_j}{an}$. Therefore, the total feasible consumption of the commodity j depends strongly on the convex function $\tilde{\rho}$.

6 Revisiting the Coase theorem

The Coase Theorem –as it was christened by Stigler– suggests that property rights traded in a free market lead towards a situation in which each right ends in the hands of the agents who value them the most. This final 'efficient' re-allocation of rights is independent of the initial distribution of the property titles. See Parisi (2007).

Paul Samuelson disagrees with this interpretation of Coase's works. On the first page of his paper Some uneasiness with the Coase Theorem (1995), he writes *... Ronald Coase never wrote IT down. When rash folk do try, Palgrave tells me, it "probably" turns out to be "false" or a "tautology"; and the something called "Coase Proposition" is to be apprehended by a series of examples.*

Hurwicz (1995) analyzed the validity of the Coase Theorem in a pure exchange economy with two agents – a polluter and a pollutee– and two commodities – the 'money' and pollution. According to Coase's ideas, in equilibrium, the total amount of pollution will be independent of the initial assignment of legal rights between the two agents. Assuming positive money holdings of both agents and preferences 'parallels' concerning to the money, Hurwicz showed that the set of Pareto optima exhibits a constant level of pollution. Years later, Chipman and Gouqiang (2012), revisiting Hurwicz's work, obtain necessary and sufficient conditions for the validity of this neutrality result which are slightly weaker than Hurwicz's sufficient conditions. Thus, the requirement of a particular class of functional form of the two preferences highlights that, even in the easy cases, the economic analysis only supports the neutrality result in some examples.

However, Samuelson (1995) gives some room to elaborate on the economic analysis viewpoint on Coase's ideas. On page 4, he writes … *If the polar cases, the easy cases, are defective in their purported analysis, then the subject is still in a primitive stage of development.* Samuelson suggests that an alternative property-grant could make for Pareto-Optimality and ends his paper stressing that allocations of these permits – and how they are to be defined – matters mightily.

On the other hand, the first line in the Introduction of Coase's (1960) article, "The Problem of Social Cost" –that originates the so-called Coase Theorem – reads: *This paper is concerned with those actions of business firms which have harmful effects on others.* Thus, despite the words externality or externalities do not appear throughout the paper, it addresses externalities represented by other-regarding preferences. When dealing with harmful effects on others, Coase focusses on legal consequences and writes on page 19 … *a person may make use of his property or … conduct his affairs at the expense of some harm to his neighbors. He may operate a factory whose noise and smoke cause some discomfort to others, so long as he keeps within reasonable bounds.*

This Section analyzes this Coase's viewpoint of the problem in the economy

$$\mathcal{E} = (\mathbb{R}^{\ell}_{+}, \mathcal{V}_i, \omega_i, r_i, \rho)_{i=1,\dots,n}$$

studied in the previous sections. To follow the aforementioned Coase's words, assume that the harmful effects on others, i.e., externalities, are originated only for a class of allocations that keep "within reasonable bounds" denoted by $\mathcal{B}$ the.

We may interpret that the law defines $\mathcal{B}$ and, following Coase, if an allocation is in $\mathcal{B}$, then there is no externality and individuals are not –legally– affected by the actions or consumption of the others.

To formalize these ideas, individual preferences are defined as follows:

$$\mathcal{V}_i(x) = \begin{cases} U_i(x_i) \text{ if } x \in \mathcal{B} \\ V_i(x_i, x_{-i}) \text{ otherwise.} \end{cases}$$

where, for each $i \in N = \{1, \cdots n\}$, the functions U_i and V_i are defined on $\mathbb{R}^{\ell}_{+}$, and $\mathbb{R}^{\ell n}_{+}$, respectively.

For example, the *tragedy of the commons* would vanish if the total catch of fish in the lake is below a threshold that allows the normal reproduction of fish. In this case, $\mathcal{B}$ would be the set of allocations where the total catches of fish are below the threshold.

Consider that externalities appear only when the aggregate consumption of some commodity $j \in K$ exceeds a certain level, namely, L_j. In this case, the set $\mathcal{B}$ will be the set $\mathcal{C}$ of allocation that fulfill the cap

$$\mathcal{B} = \mathcal{C} = \{x \in \mathbb{R}^{\ell n}_{+}, \text{ such that } \sum_{i=1}^{n} x_{ij} \leq L_j; j \in K\}.$$

Given the cap L, and assuming the fundamentals ρ and R_j of economy $\mathcal{E}$ according with Proposition 5.1, every feasible allocation in $\mathcal{C}$ fulfills the cap.

Note that the total feasible consumption of any commodity j liable to produce negative externalities relies on the mapping ρ. Depending on ρ only special feasible allocations will lead the same total consumption of j –see Section 5. Thus, according to Hurwicz (1985) and Chipman and Gouqiang (2012) the neutrality result, attributed to Coase, is not generally true. Going back to Samuelson (1995), the definition of permits to get a bundle matters.

To address efficiency within this general equilibrium framework with assignment of rights and a positive threshold below which the externality is negligible will require the following condition.

(A.4) Assume that $\mathcal{B} \subset \mathbb{R}^{\ell n}_{+}$ has interior points and that the function ρ and the total rights R are related in such a way that $\sum_{i \in N} \rho(x_i) \leq R$ implies that the allocation $x = (x_i, i \in N)$ is in the interior of $\mathcal{B}$.

Note that the set of allocations fulfilling the cap $\mathcal{C} = \{x \in \mathbb{R}^{\ell n}_{+} | \sum_{i \in N} x_{ij} \leq L_j, j \in K\}$, where $L_j > 0$ for every $j \in K \subset \{1, \ldots, \ell\}$, has interior points. If we take any $\hat{L} \ll L$ and, following Proposition 5.1, design ρ and R of the economy $\mathcal{E}$ according with $\hat{L}$, then $\mathcal{B} = \mathcal{C}$ fulfills (A.4).

In which follows we obtain a version of the first welfare theorem in its strong form. That is, we show that the equilibrium belongs to the core and, in particular, is efficient.

For it, let $\hat{\mathcal{E}}$ be the economy with rights that coincides with the economy $\mathcal{E}$ except that preferences for each agent i are selfish and given by U_i instead of $\mathcal{V}_i$. That is,

$$\hat{\mathcal{E}} = (\mathbb{R}^{\ell}_{+}, U_i, \omega_i, r_i, \rho)_{i=1,\ldots,n}$$

Theorem 6.1 *Under assumptions (A.1)-(A.4) we have the following statements:*

(i) The set of equilibria in the economy $\mathcal{E}$ with externalities coincides with the set of equilibria in the economy $\hat{\mathcal{E}}$ without externalities.

(ii) An equilibrium in the economy with externalities belongs to the optimistic core that, in this case, coincides with the pessimistic core. In particular, the equilibrium is efficient.

Proof. To show *(i)* let (p, q, x) be an equilibrium in $\mathcal{E}$ and assume that it is not equilibrium in $\hat{\mathcal{E}}$. Then there is an agent h and a bundle z such that $z \in B_h(p, q)$ and $U_h(z) > U_h(x_i)$. If $(z, x_{-h}) \in \mathcal{B}$ we obtain a contradiction. Otherwise, $z_\lambda = \lambda z + (1 - \lambda)x_h \in B_h(p, q)$ for every $\lambda \in [0, 1]$ and, since by (A.4) x is in the interior of $\mathcal{B}$, we have that (z_λ, x_h) belongs to $\mathcal{B}$ for λ small enough. This implies $\mathcal{V}_h(z_\lambda, x_{-h}) = U_h(z_\lambda)$ and, by convexity of preferences, $U_h(z_\lambda) > U_h(x_h) = \mathcal{V}_h(x)$ which is a contradiction.

To show the converse, let (p, q, x) be an equilibrium in $\hat{\mathcal{E}}$ that it is not equilibrium in $\mathcal{E}$. Then there is an agent h and a bundle z such that $z \in B_h(p, q)$ and $\mathcal{V}_h(z, x_{-h}) > \mathcal{V}_h(x_h, x_{-h}) = U_h(x_h)$. Taking z_λ as before and following the same argument we get a contradiction.

To show *(ii)*, assume that (p, q, x) is an equilibrium in $\mathcal{E}$ and x is not in the optimistic core. Then, there is y_S attainable for a coalition S such that for some feasible allocation $y = (y_S, y_{-S})$ we have $\mathcal{V}_i(y) > \mathcal{V}_i(x), \quad$ for every $i \in S$. Since

$y = (y_S, y_{-S})$ is rights-feasible, Assumption (A.4) guarantees that $\mathcal{V}_i(y) = U_i(y_i)$ and $\mathcal{V}_i(x) = U_i(x_i)$ for every i.

Then for every $i \in S$ we have that y_i does not belong to $B_i(p, q)$ and in this way, we get a contradiction with the feasibility of y_S for the coalition S.

Finally, we remark that assumption (A.4) implies that all the notions of core provided in Section 4 coincide provided that for every feasible allocation x we have $\mathcal{V}_i(x) = U_i(x_i)$ for every $i \in N$. The same occurs for any attainable allocation for a coalition $S \subset N$.

Q.E.D.

As we show in the example below, even with a threshold for externalities, without (A.4) we cannot ensure the efficiency of the market equilibrium.

A non-efficiency example. Consider an economy with two consumers (1 and 2) and two commodities (x and y). To consume the good y rights are necessary and are given by $\rho(x, y) = y$. Each agent is endowed with 2 units of every commodity. Agent 1 has 2 units of rights whereas agent 2 has 1 unit of rights. Preferences are represented by the following utility functions:

$$\mathcal{V}_1((x_1, y_1), (x_2, y_2)) = \begin{cases} x_1 y_1 & \text{if } \; y_1 + y_2 \leq 2.5 \\ x_1 y_1 - (y_1 + y_2 - 2.5) & \text{otherwise} \end{cases}$$

$$\mathcal{V}_2((x_1, y_1), (x_2, y_2)) = x_2 y_2.$$

Note that any allocation which distributes the total endowment of y is not feasible.

A Walrasian equilibrium is given by the consumption bundles $(5/2, 3/2)$ for consumer 1 and $(3/2, 3/2)$ for consumer 2, commodity prices $p_x = 1, p_y = 0$ and right price $q = 1$.

However this equilibrium is not efficient. To see this, note that the allocation which assigns the consumption bundle $\left(\frac{5}{2} - a, \frac{3}{2} + b\right)$ to consumer 1 and the bundle $\left(\frac{3}{2} + a, \frac{3}{2} - b\right)$ to consumer 2 is feasible. With this allocation every individual is better off whenever $b \in (0, 3/2)$ and a belongs to the interval $\left(\frac{3b}{3-2b}, \frac{5b}{3+2b}\right)$. Take, for instance, $a = 1/3$ and $b = 1/4$ and note that

$$\mathcal{V}_1\left(\left(\tfrac{13}{6}, \tfrac{7}{4}\right), \left(\tfrac{11}{6}, \tfrac{5}{4}\right)\right) = \tfrac{13}{6} \cdot \tfrac{7}{4} - (3 - 2.5) > \tfrac{5}{2} \cdot \tfrac{3}{2} - (3 - 2.5) = \mathcal{V}_1\left(\left(\tfrac{5}{2}, \tfrac{3}{2}\right), \left(\tfrac{3}{2}, \tfrac{3}{2}\right)\right)$$

$$\mathcal{V}_2\left(\left(\tfrac{13}{6}, \tfrac{7}{4}\right), \left(\tfrac{11}{6}, \tfrac{5}{4}\right)\right) = \tfrac{11}{6} \cdot \tfrac{5}{4} > \tfrac{3}{2} \cdot \tfrac{3}{2} = \mathcal{V}_2\left(\left(\tfrac{5}{2}, \tfrac{3}{2}\right), \left(\tfrac{3}{2}, \tfrac{3}{2}\right)\right)$$

We remark that in this example the threshold signaling externalities is 2.5 whereas permits –feasibility allows a higher consumption of the second commodity.

7 Conclusion and final remarks

In this paper, we have analyzed a general economy $\mathcal{E}$ with tradable permits and externalities represented by other-regarding preferences, where Nash-Walras equilibria do exist.

Due to externalities, a coalition objecting a *status quo* must have into account not only the freedom allowed to outsiders to react but also the rights feasibility of the reaction that, for instance, could impede its members the use of their own endowments. The different feasible reactions of the outsiders yield to consider several definitions of the core.

Given an exogenous cap, we drafted the parameters of $\mathcal{E}$ in such a way that every feasible allocation fulfills the cap. In this scenario, we have revised the neutrality and efficiency results attributed to Coase. Neutrality suggests that, in equilibrium, the total amount of pollution will be independent of the initial assignment of rights among agents. However, it is shown that the total amount of pollution–externalities– in feasible allocations for $\mathcal{E}$, particularly in Pareto optimal allocations, will depend on how permits are defined. Thus, according to Samuelson's skepticism, and with Hurwicz (1995), and Chipman and Gouqiang's (2012) results, neutrality is only fulfilled in particular economies. Regarding efficiency, the conditions in $\mathcal{E}$ ensuring that below the cap externalities are negligible, allow us to show that equilibrium allocations belong to the core of the economy.

The notions of blocking mechanisms we have stated are not all the veto systems that can be defined when preferences are interdependent. To finish the paper, we introduce a family of further core concepts.

Regarding property rights, in a recent paper, Balbuzanov and Kotowski (2019), propose the notion of exclusion core in economies with single-unit demand, indivisible goods, and no transfers. Their blocking mechanism rests upon the legal understanding of the property, that allows excluding others. In their definition, a coalition S blocks the allocation x via y if the members of S prefer y rather than x, and if an agent i becomes worse off with y than with the *status quo* x, then the property rights of the complementary coalition are not enough to obtain the bundle that i receives under the *status quo.* This exclusion veto mechanism can be adapted to our framework leading to a larger variety of core notions.

Moreover, we may define a family of *self-enforcing cores* as further cooperative solutions inspired by the refinement of Nash equilibrium that Bernheim, Peleg, and Whinston (1987) introduced and referred to as coalition-proof Nash equilibrium. This equilibrium relies on a criterion to distinguish viable deviations from those that are not: viable or sustainable deviations (self-enforcing in their terminology) are characterized by being invulnerable to further sustainable deviations. The notion of sustainable deviation is thus recursive. The coalition-proof Nash equilibrium, which is in between Nash and strong Nash equilibrium, requires that an agreement be immune to improving deviations which are self-enforcing.

We adapt the aforementioned criterion to the core as follows. A deviation or blocking assignment for a coalition S is a feasible allocation for S such that every member of S becomes better off than in the initial situation. When the coalition has just one member, then any deviation is self-enforcing. When a coalition S has more than one member, a blocking assignment y for S is self-enforcing if there is no proper sub coalition $T \subset S$ that blocks, via a self-enforcing deviation, the outcome defined by y.

A feasible allocation belongs to the self-enforcing core if it is not self-enforcing blocked.

For each of the previous notions of blocking, namely, strong, prudent, weak, and exclusion, we have a different concept of the self-enforcing core.

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